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EUCLIDE'S ELEMENTS;

The whole Fifteen Books compendiouſly Demonſtrated

By Mr. ISAAC BARROW Fellow of *Trinity Colledge* in CAMBRIDGE.

And Tranſlated out of the Latin.

Καθαρμοὶ ψυχῆς λογικῆς εἰσιν αἱ μαθηματικαὶ ἐπιστῆμαι. *Hierocl.*

LONDON,
Printed by R. DANIEL, for WILLIAM NEALAND Bookſeller in *Cambridge*; and
are to be ſold there, and at the Crown in
Duck-Lane, 1660.

183. g. 57.

BIBLIOTHECA · BODLEIANA

THE AUTHOR'S PREFACE.

IN order to the Reader's ſatisfaction concerning the Book put into his hand, I am to advertiſe him of ſome few things, and that according to the nature of the Work, briefly; as followeth. My Undertaking aimed principally at two Ends. The firſt of which was to conjoin the greateſt Compendiouſneſs of Demonſtration with as much Perſpicuity as the quality of the ſubject would admit; that ſo the Volume might bear no bigger bulk then would render it conveniently portable. Which I have ſo farr attained, that though poſſibly ſome other perſon might with greater curioſity, yet (I preſume) none could with more conciſeneſſe have demonſtrated moſt propoſitions; eſpecially, ſince I have altered nothing in the number and order of the Propoſitions, nor taken the liberty to leave out any one of *Euclide's* as leſſe neceſſary, or to reduce certain of the eaſieſt into the Claſſis of Axiomes. Which notwithſtanding ſome have done; as that moſt accurate Geometrician *Andr. Tacquet*, whom I mention the rather, becauſe I eſteem it ingenuous to acknowledge ſome things taken from him. And, indeed, I ſhould have attempted nothing after his moſt elegant Edition, had it not pleaſed that learned Perſon to publiſh only Eight of *Euclide's* Books illuſtrated by his paines, either ſlighting or undervaluing the other Seven as leſſe relating to the Elements of Geometry. But I had a different Purpoſe from the beginning; not to compoſe Elements of Geometry any-wiſe at my diſcretion, but to demonſtrate *Euclide* himſelf, and all of him, and that with all poſſible brevity. For as for Foure of his Books, the Seventh, Eighth, Ninth and Tenth, although they do not ſo neerly pertain to the Elements of Plane and Solid Geometry, as the Six Firſt & the Two ſubſequent; yet no man that ha's arriv'd to any meaſure of skill in Geometry is ignorant how exceedingly uſefull they are in Geometricall matters, aſwell in regard of the very neer alliance between Arithmetick and Geometrie, as for the knowledge of Commenſurable and Incommenſurable Magnitudes which is highly important to the underſtanding both of Plane and Solid Figures. And the noble Theory of the Five Regular Bodies, contained in the Three Laſt Books, could not be omitted without prejudice & injury; ſince our Author of theſe Elements, being a Sectator of *Plato*'s Schole, is reported to have compiled the whole Syſteme only in reference to that Contemplation; which *Proclus* atteſteth in theſe words, Ὅθεν δὴ καὶ τῆς συμπάσης στοιχειώσεως τέλος προεστήσατο τὴν τῶν καλουμένων Πλατωνικῶν σχημάτων σύστασιν. Moreover, I was eaſily induc'd to believe, that it would be acceptable to all Lovers of theſe Sciences to have the Intire work of *Euclide* by them, as it is uſually cited and recommended by all men. Wherefore I determin'd to leave out no Book or Propoſition of thoſe which are found in *Peter Herigon*, whoſe footſteps I became neceſſitated to follow cloſely by having reſolved to make uſe for the moſt part of the Schemes of his Book, upon a foreſight that my ſpeedy departure out of *England*, would not allow me time to deſcribe New, although I ſometimes deſired ſo to doe. Upon the ſame account alſo I purpoſed to uſe generally no other then *Euclide's* own Demonſtrations, contracted into a more ſuccinct form, ſaving perchance in the Second and Thirteenth, & ſparingly in the Seventh, Eighth, and Ninth Books, where it ſeem'd convenient to vary ſomething from him. So that it may be reaſonably hoped that in this Particular our own Deſign and the Wiſhes of the Studious are in ſome manner ſatisfi'd.

The other End aimed at, was in favour of Their deſires who more affect Symbolical then Verbal Demonſtrations. In which kind, ſeeing moſt of our own Nation are accuſtomed to the Notes of Mr. *Oughtred*, I eſteemed it more convenient to make uſe of them principally throughout. For no man hitherto that I know of, ſaving only *Peter Herigon*, ha's attempted to ſet forth and interpret *Euclide* according to this way. The Method of which moſt learned Perſon, though in many other reſpects very excellent, and exactly accommodated to his peculiar purpoſe, ſeem'd to me notwithſtanding doubly defective. Firſt, in that, whereas of ſeverall Propoſitions brought to the proving of ſome one Theoreme or Probleme the Latter do's not alwaies depend on the Former, yet when they do cohere one with another, and when not, cannot readily enough be known, either from their order or any other way; whence it not ſeldome comes to paſſe, that through the want of Conjunctions and Adjectives, *Ergo, rurſus*, &c. there ariſes difficulty and occaſion of doubting, eſpecially to ſuch as are but little vers'd therein. And in the next place, it oftentimes falls out that the ſaid Method cannot avoid ſuperfluous repetitions, whereby the Demonſtrations become ſometimes prolixe, and ſometimes perplex'd and intricate. All which Inconveniences are eaſily remedied in our Way by the intermingling of Words and Signes at diſcretion. And thus much may ſuffice to be premiſed concerning the Intent and Method of this Compendium. I ſhall not alledge in favour of my ſelf the ſcantneſſe of time allotted to this Work, nor the avocations of affairs, nor the ſcarcity of Helps to this ſort of Studies amongſt us (as I might not untruly) out of fear leſt my Performance ſhould not throughly pleaſe every body: But I wholly ſubmit to the faire Cenſure and Judgement of the Ingenuous Reader, what I have undertaken for the advantage of his Studies; to be approved, if he find it ſerviceable thereunto; or, if otherwiſe, rejected.

Ad amiciſſimum Virum, *I. C.* de *EUCLIDE* contracto, Εὐφημισμός.

FActum bene! didicit Laconice loqui
Senex profundus, & aphoriſmos induit.
Immenſa dudum margo commentarii
Diagramma circuit minutum; utque Inſula
Problema breve natabat in vaſto mari.
Sed unda jam detumuit; & gloſſa arctior
Stringit Theoremata: minoris anguli
Lateribus ecce totus Euclides *jacet,*
Incluſus olim velut Homerus in nuce;
Pluteoque ſarcina modo qui incubuit, levis
En fit manipulus. Pelle in exigua latet
Ingens Matheſis, matris utero Hercules,
In glande quercus, vel Ithaca Eurus in pila.
Nec mole dum decreſcit, uſu fit minor;
Quin auctior jam evadit, & cumulatius
Contracta prodeſt erudita pagina.
Sic ubere magis liquor è preſſo effluit;
Sic pleniori vaſa inundat ſanguinis
Torrente cordis Syſtole; ſic fuſius
Procurrit æquor ex Abylæ anguſtiis.
Tantilli operis ars tanta referenda unice eſt
BAROVIANO *nomini, ac ſolertiæ.*
Sublimis euge mentis ingenium potens!
Cui invium nil, arduum eſſe nil ſolet;
Sic uſque pergas proſpero conamine,
Radiuſque multum debeat ac abacus tibi;
Sic creſcat indies feracior ſeges,
Simili colonum germine aſſiduo beans.
Specimen futuræ meſſis hic ſiet labor,
Magnæque famæ illuſtria hæc præludia.
Iuvenis dedit qui tanta, quid dabit ſenex?

Car. Robotham, *CANTAB.*
Coll. Trin. Sen. Soc.

The Explication of the Signes or Characters.

=	Signifies	Equall.
⊏		Greater.
⊐		Leſſer.
+		More, or to be added.
−		Leſſe, or to be ſubtracted.
−:		The Difference, or Exceſſe; Alſo, that all the quantities which ſollow, are to be ſubtracted, the Signes not being changed.
x		Multiplication, or the Drawing one ſide of a Rectangle into another. The ſame is denoted by the Conjunction of letters; as AB = A x B.
√		The Side or Root of a Square, or Cube, &c.
Q & q		A Square.
C & c		A Cube.
Q.Q.		The ratio of a ſquare number to a ſquare number.

Other Abbreviations of words, where ever they occurr, the Reader will without trouble underſtand of himſelf; ſaving ſome ſew, which, being of leſſe generall uſe, we referr to be explained in their own places.

THE FIRST BOOK
OF
EUCLIDE'S
ELEMENTS.

Definitions.

I. A Point is that which hath no part.

II. A Line is a longitude without latitude.

III. The ends, or limits, of a line are points.

IV. A right line is that which lyes equally betwixt it's points.

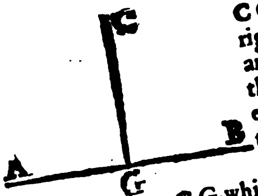
V. A Superficies is that which hath only longitude and latitude.

VI. The extremes, or limits, of a ſuperficies are lines.

VII. A plaine ſuperficies is that which lyes equally betwixt it's lines.

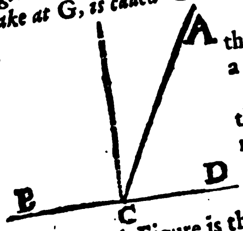
VIII. A plaine Angle is the inclination of two lines the one to the other, the one touching the other in the ſame plain, yet not lying in the ſame ſtrait line.

IX. And if the lines which contein the angle be right lines, it is called a right-lined angle.



X. When a right line C G ſtanding upon a right line A B, makes the angles on either ſide thereof, C G A, C G B, equall one to the other, then both thoſe equall angles are right angles; and the right line C G, which ſtandeth on the other, is termed a Perpendicular to that (A B) whereon it ſtandeth.

Note. *When ſeverall angles meet at the ſame point (as at* G) *each particular angle is deſcribed by three letters; whereof the middle letter ſheweth the angular point, and the two other letters the lines that make that angle: As the angle which the right lines* C G, A G *make at* G, *is called* C G A, *or* A G C.



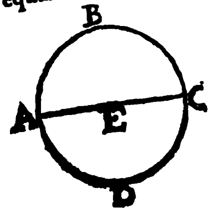
XI. An obtuſe angle is that which is greater then a right angle; as A C B.

XII. An acute angle is that which is leſſe then a right angle; as A C D.

XIII. A Limit, or Term, is the end of any thing.

XIV. A Figure is that which is conteined under one or more terms.

XV. A Circle is a plain figure conteined unde one line, which is called a Circumference; unto which all lines drawn from one point within the f gure, and falling upon the circumference thereo are equall the one to the other.



XVI. And that poi is called the Centre the circle.

XVII. A Diam of a circle is a right drawn through the tre thereof, and endi the circumference ther ſide, dividing the circle into two equall parts.

XVIII. A Semicircle is a figure which is conteined under the diameter and under that part of the circumference which is cut off by the diameter.

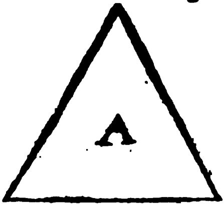
In the circle E A B C D, E *is the centre*, A C *the diameter*, A B C *the ſemicircle.*

XIX. Right-lined figures are ſuch as are conteined under right lines.

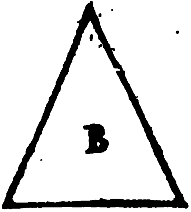
XX. Three-ſided or Trilateral figures are ſuch as are conteined under three right lines.

XXI. Four-ſided or Quadrilateral figures are ſuch as are conteined under four right lines.

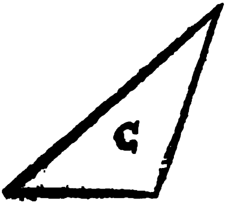
XXII. Many-ſided figures are ſuch as are conteined under more right lines then four.



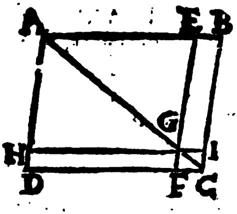
XXIII. Of trilateral figures, that is an Equilateral Triangle, which hath three equal ſides; as the Triangle A.



XXIV. Iſoſceles is a Triangle which hath onely two ſides equall; as the Triangle B.



XXV. Scalenum is a Triangle whoſe three ſides are all unequall; as C.



XXXVI. In a parallelogram A B C D, when a diameter A C, and two lines E F, H I parallel to the ſides, cutting the diameter in one and the ſame point G, are drawn, ſo that the parallelogram be divided by them into four parallelograms; thoſe two, D G, G B, through which the diameter paſſeth not, are called Complements; and the other two, H E, F I, through which the diameter paſſeth, the Parallelograms ſtanding about the diameter.

A Probleme is, *when ſomething is propoſed to be Done or effected.*

A Theoreme is, *when ſomething is propoſed to be Demonſtrated.*

A Corollary *is a conſectary, or ſome conſequent truth gained from a preceding demonſtration.*

A Lemma *is the demonſtration of ſome premiſe, whereby the proof of the thing in hand becomes the ſhorter.*

Poſtulates or Petitions.

1. FRom any point to any point to draw a right line.

2. To produce a right line finite, ſtrait forth continually.

3. Upon any centre, and at any diſtance, to deſcribe a circle.

Axiomes.

1. THings equall to the ſame third, are alſo equall one to the other;

As A=B=C. Therefore A=C. Or therefore all, A, B, C, are equall the one to the other.

Note. *When ſeverall quantities are joyned the one to the other continually with this mark =, the firſt quantity, is by vertue of this axiome equall to the laſt, & every one to every one: In which caſe we often abſtain* *from citing the axiome, for brevities ſake; although the force of the conſequence depend thereon.*

2. If to equall things you adde equall things, the wholes ſhall be equall.

3. If from equall things you take away equall things, the things remaining will be equall.

4. If to unequall things you adde equall things, the wholes will be unequall.

5. If from unequall things you take away equall things, the remainders will be unequall.

6. Things which are double to the ſame third, or to equall things, are equall one to the other. Underſtand the ſame of triple, quadruple, &c.

7. Things which are half of one and the ſamething, or of things equall, are equall the one to the other. Conceive the ſame of ſubtriple, ſubquadruple, &c.

8. Things which agree together, are equall one to the other.

The converſe of this axiome is true in right lines and angles, but not in figures, unleſſe they be like.

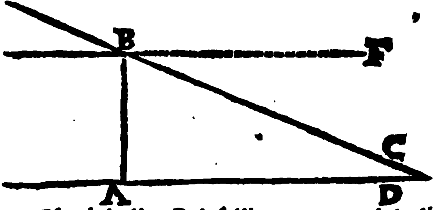
Moreover, magnitudes are ſaid to agree, when the parts of the one being applyed to the parts of the other, they fill up an equall or the ſame place.

9. Every whole is greater then it's part.

10. Two right lines cannot have one and the ſame ſegment (or part) common to them both.

11. Two right lines meeting in the ſame point, if they be both produced, they ſhall neceſſarily cut one the other in that point.

12. All right angles are equall the one to the other.



13. If a right line B A falling on two right lines

AD,

CB, make the internall angles on the ſame ſide, BAD, ABC, leſſe then two right angles, thoſe two right lines produced ſhall meet on that ſide, where the angles are leſſe then two right angles.

14. Two right lines do not contein a ſpace.

15. If to equall things you adde things unequall, the exceſſe of the wholes ſhall be equall to the exceſſe of the additions.

16. If to unequall things equall be added, the exceſſe of the wholes ſhall be equall to the exceſſe of thoſe which were at firſt.

17. If from equall things unequall things be taken away, the exceſſe of the remainders ſhall be equall to the exceſſe of what was taken away.

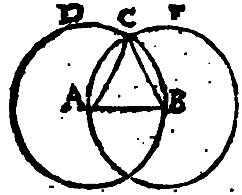
18. If from things unequall things equall be taken away, the exceſſe of the remainders ſhall be equall to the exceſſe of the wholes.

19. Every whole is equall to all it's parts taken together.

20. If one whole be double to another, and that so double which is taken away from the firſt to that which is taken away from the ſecond, the remainder of the firſt ſhall be double to the remainder of the ſecond.

The Citations are to be underſtood in this manner; When you meet with two numbers, the firſt ſhewes the Propoſition, the ſecond the Book; as by 4. 1. you are to underſtand the fourth Propoſition of the firſt Book; and ſo of the reſt. *Moreover, ax. denotes Axiome, poſt. Poſtulate, def. Definition, ſch. Scholium, cor. Corollary.*

PROPOSITION I.



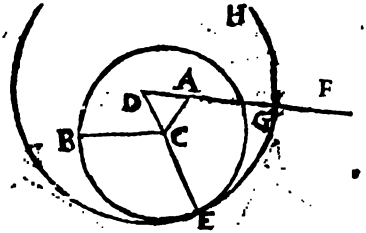
U*Pon a finite right line given* A B,*to deſcribe an equilateral triangle* ACB.

From the centres A and B,at the diſtance of AB,or B A, *a* deſcribe two circles a 3 poſt. cutting each other in the point C; from whence *b* draw two right lines CA, b 1. poſt. CB. Then is A C *c* = A B *c* = B C *d* = A C. c 15. def. d 1. ax. *e* Wherefore the Triangle A C B is equilateral. e 23. def. *Which was to be Done.*

Scholium.

After the ſame manner upon the line A B may be deſcribed an Iſoſceles triangle, if the diſtances of the equall circles be taken greater or leſſe then the line A B.

PROP. II.



At a point given A, *to make a right line* A G *equall to a right line given* B C.

From the centre C, at the diſtance of C B, *a* deſcribe the circle C B E. *b* join A C; upon which a 3 poſt. b 1. poſt. *c* raiſe the equilateral triangle A D C. *d* Produce c 1. 1. d 2. poſt. DC to E. From the centre D, at the diſtance of DE, de-e 2.poſt. deſcribe the circle DEH; & let DA be produced to the point G in the circumference thereof. Then AG = CB.

f 15.def. g conſtr. h 3. ax. k 15 def. l 1.ax, For DG [f] = DE, and DA [g] = DC. Wherefore AG [h] = CE [k] = BC [l] = AG. *Which was to to be Done.*

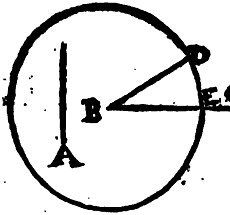
The putting of the point A within or without the line BC varies the caſes; but the conſtruction, and the demonſtration, are every where alike.

Schol.

The line AG might be taken with a pair of compaſſes; but the ſo doing anſwers to no poſtulate, as *Proclus* well intimates.

PROP. III.

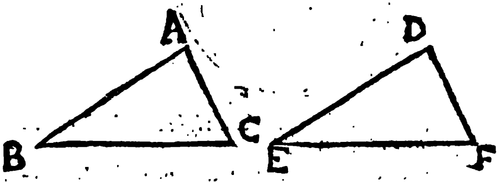
Two right lines, A and BC, being given, from the greater BC to take away the right line BE equall to the leſſer A.



a 3. 1. At the point B [a] draw the right line BD = A. The circle deſcribed from the center B at the diſtance of BD ſhall cut off BE [b] = BD [c] = A [d] = BE. *Which was to be Done.*

b 15.def. c conſtr. d 1.ax,

PROP. IV.

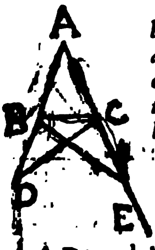


If two triangles BAC, EDF, *have two ſides of the one* BA. AC *equall to two ſides of the other* ED, DF, *each to it's correſpondent ſide (that is,* BA = ED, *and*

and A C = D F) *and have the angle* A *equall to the*
☞ *angle* D *contained under the equall right lines; they ſhall have the baſe* B C *equall to the baſe* E F; *and the triangle* B A C *ſhall be equall to the triangle* E D F; *and the remaining angles* B, C, *ſhall be equall to the remaining angles* E, F, *each to each, under which the equall ſides are ſubtended.*

If the point D be applyed to the point A, and
the right line D E plac'd upon the right line A B,
the point E ſhall fall upon B, becauſe D E *a* = A B. a *hyp.*
alſo the right line D F ſhall fall upon A C, becauſe
the angle A = D. moreover, the point F ſhall fall up-
on the point C, becauſe A C *a* = D F. Therefore the
right lines E F, B C ſhall *b* agree, becauſe they have the b 14. *ax.*
ſame termes, & ſo conſequently are equall. Where-
fore the triangles B A C, D E F, and the angles
B, E, as alſo the angles C, F, do agree, and are e-
quall. *Which was to be Demonſtrated.*

PROP. V.



The angles A B C, A C B, *at the baſe of an Iſoſceles triangle* A B C, *are equall one to the other: And if the equall ſides* A B, A C *be produc'd, the angles* C B D, B C E, *under the baſe, ſhall be equall one to the other.*

a Take A E = A D; and *b* join a 3. 1.
C D, and B E. b 1 *poſt.*

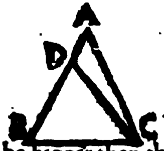
Becauſe, in the triangles A C D,
A B E, are A B *c* = A C, and A E c *hyp.*
d = A D; and the angle A common to them both, d *conſtr.*
e therefore is the angle A B E = A C D, and the angle e 4. 1.
A E B *e* = A D C, and the baſe B E *e* = C D; alſo E C f 3. *ax.*
f = D B. Therefore in the triangles B E C, B D C g 4. 1.
g ſhall be the angle E C B = D B C. *Which was to be*
Dem. Alſo therefore the angle E B C = D C B. but the h *before.*
angle A B E *h* = A C D; therefore the angle A B C k 3. *ax.*
k = A C B. *Which was to be Dem.*

Ca-

Corol.

Hence, Every equilateral triangle is alſo equiangular.

PROP. VI.



If two angles ABC, ACB *of a triangle* ABC *be equall the one to the other, the ſides* AB, AC *ſubtended under the equal angles, ſhall alſo be equall one to the other.*

If the ſides be not equall, let one be bigger then the other, ſuppoſe BA $\sqsubset$ CA. a Make BD = CA, and b draw the line CD.

a 3. 1. b 1 poſt.

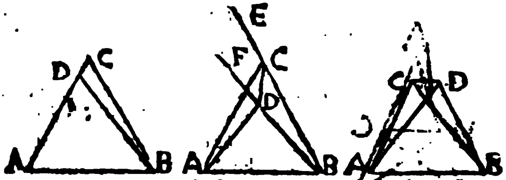
c ſuppoſ. d hyp. e 4. 1. f 9 ax.

In the triangles DBC, ACB, becauſe BD c = CA, and the ſide BC is common, & the angle DBC d = ACB, the triangles DBC, ACB e ſhall be equall the one to the other, a part to the whole. f *Which is impoſſible.*

Corol.

Hence, Every equiangular triangle is alſo equilateral.

PROP. VII.



Upon the ſame right line AB *two right lines being drawn* AC, BC, *two other right lines equall to the former,* AD, BD, *each to each (viz* AD = AC, *and* BD = BC) *cannot be drawn from the ſame points* A, B, *on the ſame ſide* C, *to ſeverall points, as* C *and* D, *but onely to* C.

1. *Caſe.* If the point D be ſet in the line AC, it is plain that AD is a not equall to AC.

a 9. ax.

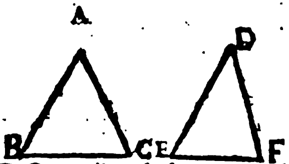
2. *Caſe.* If the point D be placed within the triangle ACB, then draw the line CD, and produce BDF, and BCE. Now you would have AD = AC, then

the angle ADC *b* $=$ ACD; as alſo, becauſe BD *c* $=$ BC, the angle FDC $=$ *b* ACD. therefore is the angle FDC $\sqsubset$ *d* ACD. that is, the angle FDC $\sqsubset$ ADC. *d Which is impoſſ.* b 5. 1. c ſuppoſ. d 9. ax.

3 *Caſe*. If D falls without the triangle ACB, let CD be joyned.

Again, the angle ACD *e* $=$ ADC, and the angle BCD *e* $=$ BDC. *f* Therefore the angle ACD $\sqsubset$ BDC, *viz.* the angle ADC $\sqsubset$ BDC. *Which is impoſſible.* Therefore, &c. e 5. 1. f 9. ax.

PROP. VIII.



If two triangles ABC, DEF *have two ſides* AB, AC *equall to two ſides* DE, DF, *each to each*, *and the baſe* BC *equall to the baſe* EF, *then the angles conteined under the equall right lines ſhall be equall*, viz. A *to* D.

Becauſe BC *a* $=$ EF, if the baſe BC be laid on the baſe EF, *b* they will agree: therefore whereas AB *c* $=$ DE, and AC $=$ DF, the point A will fall on D (for it cannot fall on any other point, by the precedent propoſition) and ſo the ſides of the angles A and D are coincident; *d* wherefore thoſe angles are equall. *Which was to be Dem.* a hyp. b ax. 8. c hyp. d 8. ax.

Coroll.

1. Hence, Triangles mutually equilateral are alſo mutually *x* equiangular. x 4. 1.
2. Triangles mutually equiſateral *x* are equall one to the other.

If the angles A B C, A B D be equall, *a* then they make two right angles; if unequall, then from the point B *b* let there be erected a perpendicular B E. Becauſe the angle A B C *c* $=$ to a right $+$ A B E, and the angle A B D *d* $=$ to a right $-$ A B E, therefore ſhall be A B C $+$ A B D *e* $=$ to two right angles $+$ A B E $-$ A B E $=$ 2 right angles. *Which was to be Dem.*

a def. 10.
b 11. 1.
c 19. *ax.*
d 3. *ax.*
e 2. *ax.*

Corollaryes.

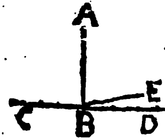
1. Hence, if one angle ABD be right, the other ABC is alſo right; if one acute, the other is obtuſe, and ſo on the contrary.

2. If more right lines then one ſtand upon the ſame right line at the ſame point, the angles ſhall be equall to two right.

3. Two right lines cutting each other make angles equall to four right.

4. All the angles made about one point make 4. right; as appears by Coroll. 2.

P R O P. XIV.

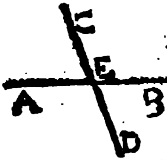


If to any right line A B, *and a point therein* B, *two right lines, not drawn from the ſame ſide, do make the angles* A B C, A B D *on each ſide equall to two right, the lines* C B, B D *ſhall make one ſtrait line.*

If you deny it, let C B, B E make one right line; then ſhall be the angle A B C $+$ A B E *a* $=$ 2 right angles *b* $=$ A B C $+$ A B D. *Which is c Abſurd.*

a 13. 1.
b hyp.
c 9. *ax.*

P R O P. XV.

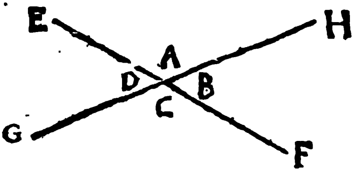


If two right lines A B, C D *cut through one another, then are the two angles which are oppoſite, viz.* C E B, A E D, *equall one to the other.*

For the angle A E C $+$ C E B *a* $=$ to 2 right angles $=$ A E C $+$ A E D; *b* therefore C E B $=$ A E D. *Which was to be Done.*

a 13. 1.
b 3 *ax.*

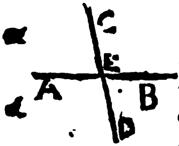
Schol. 1.



If to any right line G H, and in it a point A, two right lines being drawn E A, A F, and not taken on the ſame ſide, make the verticall (or oppoſite) angles D and B equall, thoſe right lines E A, A F, do meet directly and make one ſtrait line.

For 2 right angles are *a* equall to the angle D + A *a* = B + A. *b* therefore E A, A F are in a ſtrait line. *Which was to be Dem.* a 13. 1. b 14. 1.

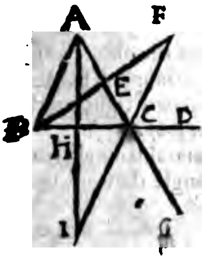
Schol. 2.



If four right lines E A, E B, E C, E D, proceeding from one point E, make the angles vertically oppoſite equall the one to the other, each two lines, A E, E B, and C E, E D, are placed in one ſtrait line.

For becauſe the angle A E C + A E D + C E B + D E B *a* = to 4 right angles, therefore the angle A E C + A E D *b* = C E B + D E B = to two right angles. *c* Therefore C E D & A E B are ſtrait lines. *Which was to be Dem.* a 4 cor. 13. 1. b hyp. & 2 ax. c 14. 1.

PROP. XVI.



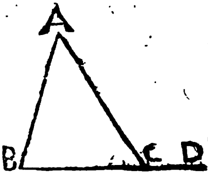
One ſide B C *of any triangle* A B C *being produc'd, the outward angle* A C D *will be greater then either of the inward and oppoſite angles* C A B, C B A.

Let the right lines A H, B E *a* biſect the ſides A C, B C; *b* produce E F = B E, and H I *b* = a 10. 1. & 1. poſt. b 3. 1.

b = A H. and join F C, and I C; and produce A C G.

c conſtr. Becauſe C E c = E A, and E F c = E B, and the
d 15. 1. angle F E C d = B E A, the angle E C F e ſhall be
e 4. 1. equall to E A B. By the like argument is the angle I C H = A B H. Therefore the whole angle ACD
f 15. 1. (f B C G) g is greater then eirher the angle C A B or
g 9. ax. A B C. *Which was to be Dem.*

PROP. XVII.

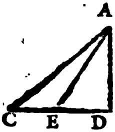


Two angles of any triangle A B C, which way ſoever they be taken, are leſſe then two right angles.

Let the ſide B C be produced. Becauſe the angle ACD
a 13. 1. + A C B a = 2 right angles,
b 16. 1. and the angle ACD b ⊏ A, c therefore A + ACB
c 4 ax. ⊐ then two right angles. After the ſame manner is the angle B+ACB ⊐ then two right. Laſtly, the ſide A B being produced, the angle A + B will be alſo leſſe then two right angles. *Which was to be Dem.*

Coroll.

1. Hence it follows that in every triangle, wherein one angle is either right or obtuſe, the two others are acute angles.



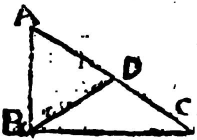
2. If a right line A E make unequall angles with another right line D, one acute A E D, the other obtuſe A E C, a perpendicular A D let fall from any point A to the other line C D, ſhall fall on that ſide the acute angle is of.

For if A C, drawn on the ſide of the obtuſe angle, be a perpendicular , then in the triangle ABC ſhall A E C + A C E be greater then two right angles.
* 17. 1. * *Which is contrary to the precedent prop.*

3. All the angles of an equilateral triangle, and the 2 angles of an Iſoſceles triangle that are upon the baſe, are acute.

PROP.

PROP. XVIII.

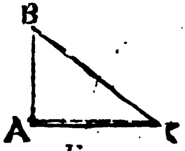


The greateſt ſide AC *of every triangle* ABC *ſubtends the greateſt angle* ABC.

From AC *a* take away AD = AB, and join BD. *b* Therefore is the angle ADB = ABD. But ADB *c* ⊏ C; therefore is ABD ⊏ C; *d* therefore the whole angle ABC ⊏ C. After the ſame manner, ſhall be ABC ⊏ A. *Which was to be Dem.*

a 3. 1.
b 5. 1.
c 16. 1.
d 9. ax.

PROP. XIX.

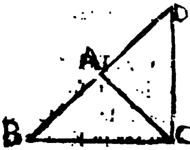


In every triangle ABC, *under the greateſt angle* A *is ſubtended the greateſt ſide* BC.

For if AB be ſuppoſed equall to BC, then will be the angle A *a* = C, which is contrary to the Hypotheſis: and if AB ⊏ BC, then ſhall be the angle C *b* ⊏ A, which is againſt the Hypotheſis. Wherefore rather BC ⊏ AB; and after the ſame manner BC ⊏ AC. *Which was to be Dem.*

a 5. 1.
b 18. 1.

PROP. XX.

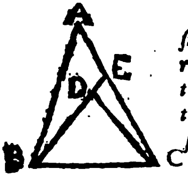


Of every triangle ABC *two ſides* BA, AC, *any way taken, are greater then the ſide that remains* BC.

Produce the line BA, *a* & take AD = AC, & draw the line DC. *b* then ſhall the angle D be equall to ACD, *c* therefore is the whole angle BCD ⊏ D; *d* therefore BD (*e* BA + AC) ⊏ BC. *Which was to be Dem.*

a 3. 1.
b 5. 1.
c 9. ax.
d 19. 1.
e conſtr. & 2. ax.

PROP. XXI.

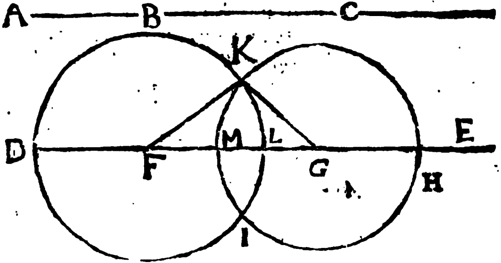


If from the utmoſt points of one ſide BC *of a triangle* ABC *two right lines* BD, CD *be drawn to any point within the triangle, then are both thoſe two lines ſhorter than the two other ſides of* *the triangle*, BA, CA; *but do contain a greater angle*, BDC.

Let BD be produced to E. Then is CE + ED [a] ⊏ CD. adde BD common to both, [b] then ſhall be BD + DE + EC ⊏ CD + BD. Again, BA + AE [a] ⊏ BE. [b] therefore BA + AC ⊏ BE + EC. Wherefore 1, BA + AC ⊏ BD + DC. 2. The angle BDC [c] ⊏ DEC [c] ⊏ A. Therefore the angle BDG ⊏ A. *Which was to be Dem.*

[a] 20. 1.
[b] 4. ax.
[c] 16. 1.

PROP. XXII.

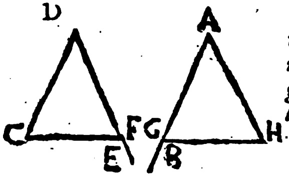


To make a triangle FKG *of three right lines* FK, FG, GK *which ſhall be equall to three right lines given* A, B, C. *Of which it is neceſſary that any two taken together be longer then the third.*

From the infinite line DE [a] take DF, FG, GH equall to the lines given A, B, C. Then if from the [b] centres F and G by the diſtances of FD and GH, two circles be drawn cutting each other in K, & the right lines KF, KG be joined, the triangle FKG ſhall be made, [c] whoſe ſides FK, FG, GK are equall to the three lines DF, FG, GH [d] that is to the three lines given A, B, C. *Which was to be Done.*

[a] 3. 1.
[b] 3. poſt.
[c] 15. def.
[d] 1. ax.

PROP. XXIII.



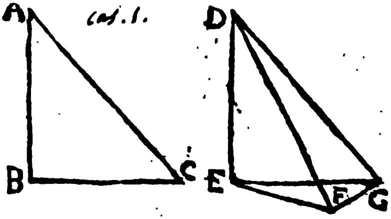
At a point A *in a right line given* AB, *to make a right-lined angle* A *equall to a right-lined angle given* D.

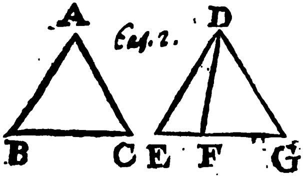
[a] Draw the right

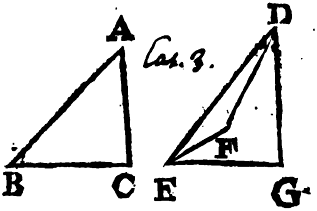
[a] 1. poſt.

line C F cutting the ſides of the angle given any wayes; *b* make AG = CD; upon AG *c* raiſe a triangle equilateral to the former C D F, ſo that A H be equall to D F, and GH to CF. then ſhall you have the angle A *d* = D. *Which was to be Done.* b 3. 1. c 22. 1. d 8. 1.

PROP. XXIV.







If two triangles A B C , D E F *have two ſides of the one triangle* AB, AC *equall to two ſides of the other triangle* DE, DF, *each to other, and have the angle* A *greater then the angle* EDF *conteined under the equall right lines , they ſhall alſo have the baſe* B C *greater then the baſe* EF.

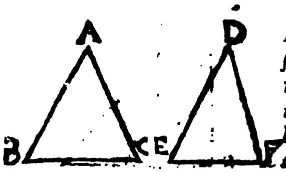
a Let the angle E D G be made equall to A, and the ſide DG *b* = DF *c* = AC ; & let EG, and FG be joyned. a 23. 1. b 3. 1. c hyp.

1. *Caſe.* If E G fall above E F; Becauſe AB *d* = D E, and A C *e* = DG, and the angle A *e* = EDG, *f* therefore is BC = E G. But becauſe DF *e* = DG, *g* therefore is the angle DFG = DGF ; *h* therefore is the angle DFG ⊏ EFG, and by conſequence the angle EFG *h* ⊏ EGF. *k* wherefore EG (BC) ⊏ EF. d hyp e conſtr. f 4. 1. g 5. 1. h 9. ax. k 19. 1.

2. *Caſe*: If the baſe EF falls in the ſame place with
l 9. ax. EG,[l] it is evident that EG (BC) $\sqsubset$ EF.

3. *Caſe*. If EG fall below EF, then becauſe DG +
m 21. 1. GE[m] $\sqsubset$ DF + FE, if from both DG, EF be taken
n 15. ax. away, which are equall, EG (BC) remains[n] $\sqsubset$ EF, *Which was to be Dem.*

PROP. XXV.

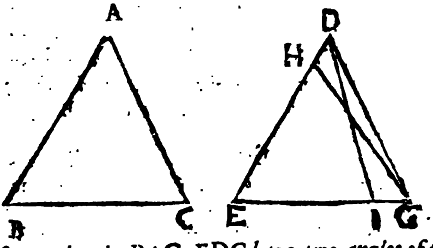


If two triangles ABC, DEF *have two ſides* AB, AC *equall to two ſides* DE, DF, *each to other, and have the baſe* BC *greater then the baſe* EF, *they ſhall alſo have the angle* A *contained under the equall right lines greater then the angle* D.

a 4. 1. For if the angle A be ſaid to be equall to D,[a] then is the baſe BC = EF, which is againſt the Hypotheſis; If it be ſaid the angle A $\sqsupset$ D, then[b] will
b 24. 1. be BC $\sqsupset$ EF, which is alſo againſt the Hypotheſis. Therefore BC $\sqsubset$ EF. *Which was to be Dem.*

$\angle A \sqsubset \angle D$.

PROP. XXVI.



If two triangles BAC, EDG *have two angles of the one* B, C *equall to two angles of the other* E, DGE, *each to his correſpondent angle, and have alſo one ſide of the one equall to one ſide of the other, either that ſide which lyeth betwixt the equall angles, or that which is ſubtended under one of the equall angles*; the other ſides al-

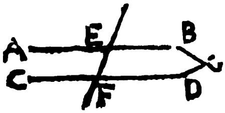
ſo of the one ſhall be equall to the other ſides of the other, each to his correſpondent ſide, and the other angle of the one ſhall be equall to the other angle of the other.

1. *Hypotheſis.* Let BC be equall to EG, which are the ſides that lye between the equall angles. Then I ſay BA = ED, and AC = DG, & the angle A = EDG. For if it be ſaid that ED ⊏ BA, then [a] let EH be made equall to BA, & let the line GH be drawn. a 3. 1.

Becauſe AB [b] = HE, and BC [c] = EG, and the angle B [c] = E, therefore ſhall be the angle EGH [d] = C [e] = DGE. [f] *Which is abſurd.* After the ſame manner let AC be equall to DG, [d] then will the angle A be equall to EDG. b ſuppoſ. c hyp. d 4. 1. e hyp. f 9 ax.

2. *Hyp.* Let AB be equall to ED. Then I ſay BC = EG, and AC = DG, and the angle A = EDG. For if EG be greater then BC, make EI = BC, and joyn the line DI. Now becauſe AB [g] = ED, and BC [h] = EI, and the angle G [g] = E; therefore will be the angle EID [k] = C [l] = EGD. [m] *Which is abſurd.* Therefore is BC = EG, and ſo as before AC = DG, and the angle A = EDG. *Which was to be Dem.* g hyp. h ſuppoſ. k 4. 1. l hyp. m 16. 1.

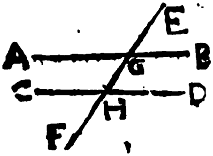
PROP. XXVII.



If a right line EF *falling upon two right lines* AB, CD *make the alternate angles* AEF, DFE, *equall the one to the other, then are the right lines* AB, CD *parallel.*

If AB, CD be ſaid not to be parallel, produce them till they meet in G. which being ſuppoſed, the outward angle AEF will be [a] greater then the inward angle DFE, to which it was equall by Hypotheſis. *Which is repugnant.* a 16. 1.

PROP. XXVIII.

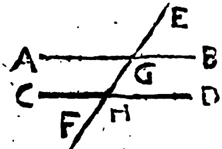


If a right line EF *falling upon two right lines* AB, CD *make the outward angle* AGE *of the one line equall to* CHG *the inward and oppoſite angle* *of the other on the ſame ſide, or make the inward angles on the ſame ſide* AGH, CHG *equall to two right angles, then are the right lines* AB, CD *parallel.*

Hyp. 1. Becauſe by Hypotheſis the angle AGE
= CHG, [a] therefore are BGH, CHG alternate
a 15. 1. angles and equall: And [b] therefore are AB and CD
b 27. 1. parallel.

Hyp. 2. Becauſe by Hypotheſis the angle AGH
+ CHG = to two right, [a] = AGH + BGH,
a 13. 1. [b] thence is CHG = BGH; and [c] therefore AB, CD
b 3. ax. are parallel, *Which was to be Dem.*
c 27. 1.

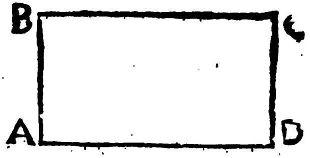
P R O P. XXIX.



If a right line EF *fall upon two parallels,* AB, CD, *it will both make the alternate angles* DHG, AGH *equall each to other, and the outward angle* BGE *equall to the inward and oppoſite angle on the ſame ſide* DHE, *as alſo the inward angles on the ſame ſide* AGH, CHG *equall to two right angles.*

It is evident that AGH + CHG = 2 right an-
gles; [a] otherwiſe AB, CD would not be parallel,
a 13 ax. which is contrary to the Hypotheſis: But moreover
b 13. 1. the angle DHG + CHG [b] = 2 right; therefore is
c 13. ax. DHG [c] = AGH [d] = BGE. *Which was to be Dem.*
d 15. 1.

Coroll.

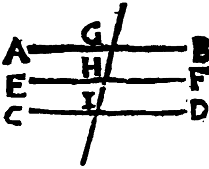


Hence it follows that every parallelogram AC having one angle right A, the reſt are alſo right.

For A + B [a] =
a 29. 1. 2 right angles. Therefore, whereas A is right, [b] muſt
b 3. ax. B be alſo right. By the ſame argument are C and D
right angles.

PROP.

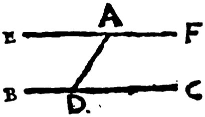
PROP. XXX.



Right lines (AB, CD) parallel to one & the ſame right line EF, are alſo parallel the one to the other.

Let G I cut the three right lines given any wayes. then becauſe AB , EF are parallel , will be the angle AGI *a* = EHI. alſo becauſe CD a 29. 1.
and EF are parallel , will be the angle EHI *a* = b 1 ax.
DIG. *b* Therefore the angle AGI = DIG. *c* whence c 27. 1.
AB and CD are parallel. *Which was to be Dem.*

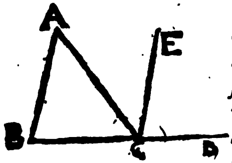
PROP. XXXI.



From a point given A to draw a right line A E parallel to a right line given BC.

From the point A draw a right line AD to any point of the given right line; with which at the point thereof
a A make an angle DAE = ADC. *b* then will AE a 23. 1. b 27. 1.
and BC be parallel. *Which was to be Done.*

PROP. XXXII.



Of any triangle ABC one ſide BC being drawn out, the outward angle ACD ſhall be equall to the two inward oppoſite angles A, B. and the three inward angles of the triangle, A, B, ACB, ſhall be equall to two right angles.

From C *a* draw CE parallel to BA. Then is the a 31. 1.
angle A *b* = ACE , and the angle B *b* = ECD. b 29. 1.
Therefore A + B *c* = ACE + ECD *d* = ACD. c 2. ax. d 19. ax.
Which was to be Dem.

I affirm ACD + ACB *e* = 2 right angles; e 13. 1.
f therefore A + B + ACB = 2 right angles. *Which* f 1. ax.
was to be Dem,

Coroll.

1. The three angles of any triangle taken together

are equall to three angles of any other triangle taken together. From whence it follows,

2. That if in one triangle, two angles (taken ſeverally, or together) be equall to two angles of another triangle (taken ſeverally, or together) then is the remaining angle of the one equall to the remaining angle of the other. In like manner, if two triangles have one angle of the one equall to one of the other, then is the ſum of the remaining angles of the one triangle equall to the ſumme of the remaining angles of the other.

3. If one angle in a triangle be right, the other two are equall to a right. Likewiſe, that angle in a triangle which is equall to the other two, is it ſelf a right angle.

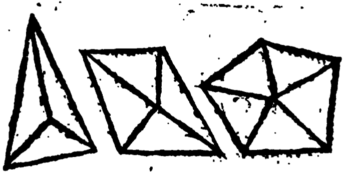
4. When in an Iſoſceles the angle made by the equall ſides is right, the other two upon the baſe are each of them half a right angle.

5. An angle of an equilateral triangle makes two third parts of a right angle. For $\frac{1}{3}$ of two right angles is equall to $\frac{2}{3}$ of one.

Schol.

By the help of this propoſition you may know how many right angles the inward and outward angles of a right-lined figure make; as may appear by theſe two following Theoremes.

THEOREME. I.



All the angles of a right-lined figure do together make twice as many right angles, bating four, as there are ſides of the figure.

From any point within the figure let right lines be drawn to all the angles of the figure, which ſhall reſolve the figure into as many triangles as there are ſides of the figure. Wherefore, whereas every triangle affords two right angles, all the triangles taken together will make up twice as many right angles as there are ſides. But the angles about the ſaid point within the figure make up four right; therefore, if from the angles of all the triangles you take away the angles which are about the ſaid point, the remaining angles, which make up the angles of the figure, will make twice as many right angles, bating four, as there are ſides of the figure. *Which was to be Dem.*

Coroll.

Hence, All right-lined figures of the ſame ſpecies have the ſums of their angles equall.

THEOREME II.

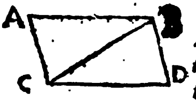
All the outward angles of any right-lined figure, taken together, make up four right angles.

For all the ſeverall inward angles of a figure with the ſeverall outward angles of the ſame make two right angles; therefore all the inward angles together with all the outward, make twice as many right angles as there are ſides of the figure; But (as it was now ſhewn) all the inward angles with four right make twice as many right as there are ſides of the figure; therefore the outward angles are equall to 4 right angles. *Which was to be Dem.*

Coroll.

All right-lined figures of whatſoever ſpecies have the ſummes of their outward angles equall.

PROP. XXXIII.

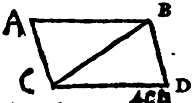


If two equall and parallel lines AB, CD *be joyned together with two other right lines* A C, B D, *then are thoſe lines alſo equall & parallel.*

Draw a line from C to B. Now becauſe AB and CD are parallel, and the angle ABC [a] = BCD; a 29. 1. and alſo by hypotheſis AB = CD, and the ſide CB

common , therefore is A C [b] = B D, and the angle
b 4. 1. ACB [b] = DBC. [c] whence alſo AC, BD are parallel.
c 27. 1.

PROP. XXXIV.



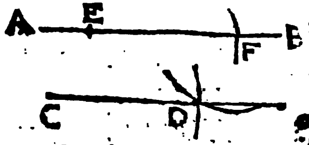
In parallelograms, as ABCD, *the oppoſite ſides* AB , CD, *and* AC, BD, *are equall each to other ; and the oppoſite angles* A, D, *and* ABD , ACD *are alſo equall ; and the diameter* BC *biſects the ſame.*

a hyp. Becauſe AB, CD [a] are parallel, [b] therefore is the angle ABC = BCD. Alſo becauſe AC, BD are [a] parallel , [b] therefore is the angle A C B = C B D ;
b 29 1.
c 2 ax. [c] therefore the whole angle ACD = ABD. After the ſame manner is A = D. Moreover becauſe the angles ABC , ACB lye at each end of the ſide CB,
d 26 1. and are equall to BCD, CBD, [d] therefore is AC = BD, and AB [d] = CD , and ſo the triangle ABC = CBD. *Which was to be Dem.*

Schol.

Every four-ſided figure ABDC *having the oppoſite ſides equall, is a parallelogram.*

For by 8. 1. the angle ABC = BCD; [a] wherefore
a 27. 1. AB, CD are parallel. In like manner is the angle BCA = CBD ; [a] wherefore AC, BD are alſo parallel.
b 35. def. 1. [b] Therefore ABCD is a parallelogram. *Which was to be Dem.*

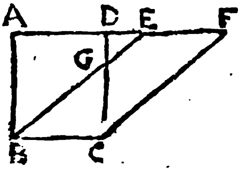


From hence may be learned how to draw a parallel C D to a right line given AB , through a point aſſigned C.

Take in the line AB any point, as E. From the centres E and C at any diſtance draw two equall circles EF, CD. From the centre F by the ſpace of EC draw a circle FD , which ſhall cut the former circle CD in the point D. Then ſhall the line drawn CD be parallel to AB. for as it was before demonſtrated, CEFD is a parallelogram.

PROP.

PROP. XXXV.



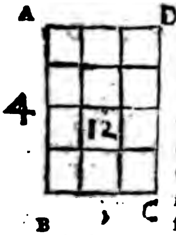
Parallelograms, BCDA, BCFE, *which ſtand upon the ſame baſe* BC, *and between the ſame parallels* AF, BC, *are equall one to the other.*

For AD [a] = BC [a] = EF. adde DE common to both, [b] then is AE = DF. But alſo AB [a] = DC, and the angle A [c] = CDF. [d] Therefore is the triangle ABE = DCF. take away DGE common to both triangles, [e] then is the Trapezium ABCD = EGCF. adde BGC common to both, [f] then is the parallelogram ABCD = EBCF. *Which was to be Dem.*

[a] 34. 1.
[b] 2. ax.
[c] 29. 1.
[d] 4. 1.
[e] 3. ax.
[f] 2. ax.

The demonſtration of any other caſes is not unlike, but much more plain and eaſy.

Schol.

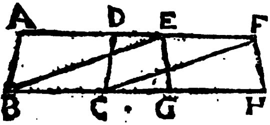


If the ſide AB of a right-angled parallelogram ABCD be conceived to be carried along perpendicularly through the whole line BC, or BC through the whole line AB, the Area or content of the Rectangle ABCD ſhall be produc'd by that motion. Hence a rectangle is ſaid to be made by the drawing or multiplication of two contiguous ſides. For examples ſake; let AB be ſuppoſed four foot, and BC three: draw 3 into 4, there will be produced 12 ſquare feet for the Area of the Rectangle.

This being ſuppoſed, the dimenſion of any parallelogram (*EBCF) is found out by this theoreme. For the Area thereof is produced from the altitude BA drawn into the baſe BC. So the Area of the parallelogram AC = EBCF, is made by the drawing of BA into BC, therefore, &c.

* *See the fig. of* prop. 35.

PROP. XXXVI.

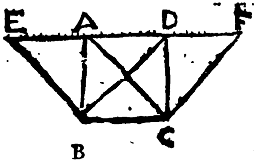


Parallelograms, BCDA, GHFE, *ſtanding upon equall baſes* BC, GH, *and betwixt the ſame parallels* AF, BH, *are equall one to the other.*

a hyp.
b 34. 1.
c 33. 1.
d 35. 1.

Draw BE, & CF. Becauſe BC $a = $ GH $b =$ EF, *c* therefore is BCFE a parallelogram. Whence the parallelogram BCDA $d =$ BCFE $d =$ GHFE. *Which was to be Dem.*

PROP. XXXVII.

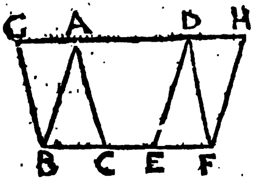


Triangles, BCA, BCD, *ſtanding upon the ſame baſe* BC, *and between the ſame parallels* BC, EF, *are equall one to the other.*

a 31. 1.
b 34. 1.
c 35. 1. and 7. ax.

a Draw BE parallel to CA, *a* and CF parallel to BD. Then is the triangle BCA $b = \frac{1}{2}$ of the parallelogram BCAE $c = \frac{1}{2}$ BDFC $b =$ BCD. *Which was to be Dem.*

PROP. XXXVIII.



Triangles, BCA, EFD, *ſet upon equall baſes* BC, EF, *and between the ſame parallels* GH, BF, *are equall the one to the other.*

a 34. 1.
b 36. 1. and 7. ax.
c 34. 1.

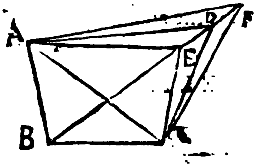
Draw BG parallel to CA, & FH parallel to ED. Then is the triangle BCA $a = \frac{1}{2}$ Pgr. BCAG $b = \frac{1}{2}$ EDHF $c =$ EFD. *Which was to be Dem.*

Schol.

If the baſe BC be greater then EF, then is the triangle BAC ⊏ EDF, and ſo on the contrary.

PROP.

PROP. XXXIX.

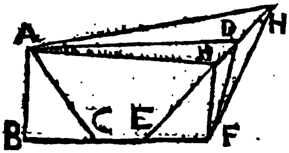


Equall triangles BCA, BCD, *ſtanding on the ſame baſe* BC, *and on the ſame ſide, they are alſo between the ſame parallels* AD, BC.

If you deny it, let another line AF be parallel to BC; and let CF be drawn. Then is the triangle CBF [a] = CBA [b] = CBD. [c] *Which is abſurd.*

a 37. 1.
b hyp.
c 9. ax.

PROP. XL.

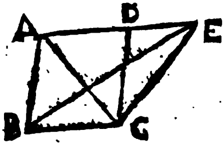


Equall triangles BCA, EFD, *ſtanding upon equall baſes* B C, E F, *and on the ſame ſide, they are betwixt the ſame parallels.*

If you deny it, let another line AH be parallel to BF, and let FH be drawn. Then is the triangle EFH [a] = BCA [b] = EFD. [c] *Which is abſurd.*

a 38. 1.
b hyp.
c 9. ax.

PROP. XLI.



If a Pgr. A B C D *have the ſame baſe* B C *with the triangle* B C E, *and be between the ſame parallels* A E, B C, *then is the Pgr.* ABCD *double to the triangle* BCE.

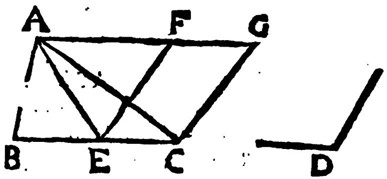
Let the line A C be drawn. Then is the triangle BCA [a] = BCE. therefore is the Pgr. ABCD [b] = 2 BCA [c] = 2 BCE. *Which was to be Dem:*

a 37. 1.
b 34. 1.
c 6. ax.

Schol.

From hence may the Area of any triangle BCE be found. for whereas the Area of the Pgr. ABCD is produced by the altitude drawn into the baſe, therefore ſhall the Area of a triangle be produced by half *the altitude drawn into* the baſe, or half the baſe drawn into the altitude. as if ſo be the baſe BC be 8, and the altitude 7, then is the area of the triangle BCE 28.

PROP. XLII.

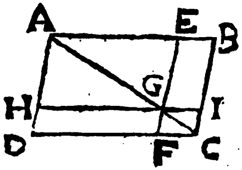


To make a Pgr. ECGF *equall to a triangle given* ABC *in an angle equal to a right-lined angle given* D.

a 31. 1. Through A [a] draw AG parallel to BC. [b] make
b 23. 1. the angle BCG = D. [c] biſect the baſe BC in E,
c 10. 1. and draw EF parallel to CG. then is the probleme reſolved.

For AE being drawn, the angle ECG is equall to
d 38. 1. D by conſtruction, and the triangle BAC [d] = 2
e 41. 1. AEC [e] = Pgr. ECGF. *Which was to be Done.*

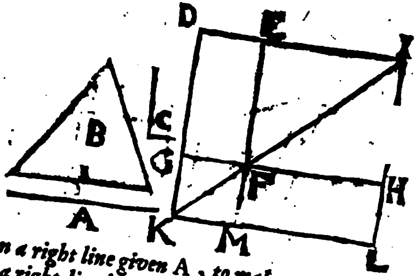
PROP. XLIII.



In every Pgr. ABCD, *the complements* DG, GB *of thoſe Pgrs* HE, FI, *which ſtand about the diameter, are equall one to the other.*

For the triangle ACD
a 34. 1. [a] = ACB, and the triangle AGH [a] = AGE, and
b 3. ax. the triangle GCF [a] = GCI. [b] Therefore the Pgr. DG = BG. *Which was to be Dem.*

PROP. XLIV.

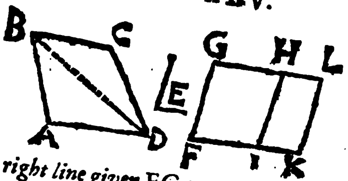


Vpon a right line given A, *to make a Parallelogram* FL *at a right-lined angle given* C, *equall to a triangle given* B.

a Make a Pgr. FD = to the triangle B, so that a 42. 1.
he angle G F E may be equall to C. Produce G F
ill FH be equall to the line given A. through H
draw IL parallel to EF, which let DE produced
eet in the point I, let DG drawn forth meet with b 31. 1.
right line drawn from I in the point K.
rough K b draw K L parallel to G H, with which
E F drawn out meet at M, and I H at L. Then
ll FL be the Pgr. required.

For the Pgr. FL c = FD = B, d and the angle c 43. 1.
H = GFE = C. *Which was to be Done.* d 15. 1.

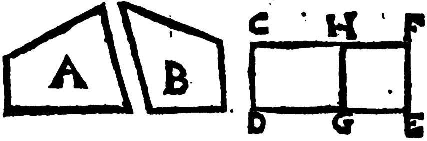
PROP. XLV.



a right line given FG *to make a Pgr.* FL *equall* *t-lined figure given* ABCD, *at a right-lined* *uen* E.

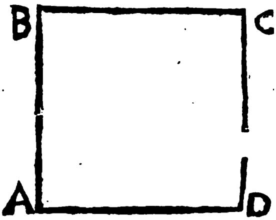
Reſolve the right-lined figure given into triangles
a 44.1. BAD, BCD. then *a* make a Pgr. FH = BAD, ſo that the angle F may be equall to E. FI being produced, *a* make on HI the Pgr. IL = BCD. Then is
b 19.ax. the Pgr. FL *b* = FH + IL *c* = ABCD. *Which was*
c *conſtr.* *to be Done.*

Schol.



Hence is eaſily found the exceſſe, HE, whereby any right-lined figure, A, exceeds a leſſe right-lined figure, B; namely, if to ſome right line, CD, both be applyed, Pgr. DF = A, and DH = B.

PROP. XLVI.



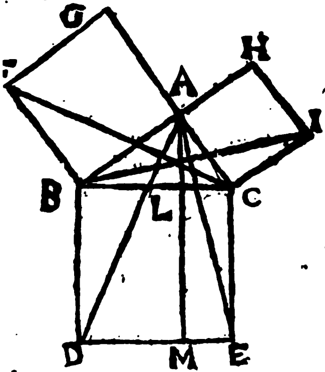
Upon a right line given AD *to deſcribe a ſquare* AC.

a 11. 3. *a* Erect two perpendiculars AB, DC, *b* e-
b 3. 1. quall to the line given AD; then joyn BC, & the thing required is done.

c *conſtr.* For, whereas the angle A+D *c* = 2 right, *d* there-
d 28. 1. fore are AB, DC parallel. But they are alſo *e* equall;
e *conſtr.* *f* therefore AD, BC are both parallel and equall;
f 34. 1. therefore the figure AC is a *Pgr.* and equilateral.
g *ſch.*29.1. Moreover the angles are all right, *g* becauſe one,
h 29.*def.* A, is right; *h* therefore AC is a ſquare. *Which was to be Done.*

After the ſame manner you may eaſily deſcribe a rectangle conteined under two right lines given.

PROP. XLVII.



In right angled triangles, BAC, *the ſquare* BE, *which is made of the ſide* BC *that ſubtends the right angle* BAC, *is equall to both the ſquares* BG, CH, *which are made of the ſides* AB, AC *conteining the right angle.*

Join AE, and D; and draw AM parallel to CE.

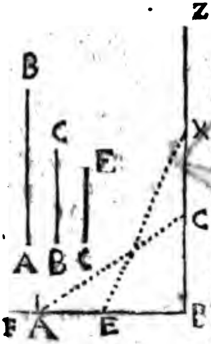
Becauſe the angle DBC [a] = FBA, adde the angle BC common to them both; then is the angle ABD : FBC. Moreover AB [b] = FB, and BD [b] = BC; herefore is the triangle ABD = FBC. But the Pgr. [d] = 2 ABD, and the Pgr. [d] BG = 2 FBC (for C is one right line by Hypotheſis, and 14.1.) refore is the Pgr. BM [e] = BG. By the ſame way rgument is the Pgr. CM = CH. Therefore is the le BE = BG + CH. *Which was to be Dem.*

a 15.ax.
b 29.def.
c 4.1.
d 41.1.
e 6.ax.

Schol.

is moſt excellent and uſefull theoreme hath ed the title of *Pythagoras* his theoreme, be e was the inventer of it. By the help of which dition and ſubſtraction of ſquares are per ; to which purpoſe ſerve the 2 following pro-

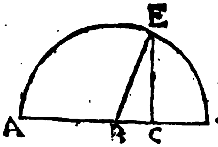
PROBLEME I.



Andr. Tacq. *To make one ſquare equall to any number of ſquares given.*

Let three ſquares be given, whereof the ſides are AB, BC, CE.
a 11. 1. [a] Make the right angle FBZ having the ſides infinite; and on them transfer BA and BC; join AC. then is ACq [b] = ABq + BCq.
b 47. 1. Then transfer AC from B to X, and CE the third ſide given from B to E; join EX. [b] Then is EXq = EBq (CEq) + BXq
c 2. ax. (ACq) [c] = CEq + ABq + BCq. *Which was to be Done.*

PROBLEME II.

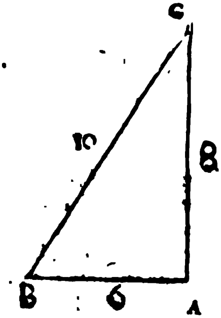


Two unequall right lines being given AB, BC, *to make a ſquare equall to the difference of the two ſquares of the given lines*, AB, BC.

From the centre B, at the diſtance of BA, deſcribe a circle; and from the point C erect a perpendicular CE meeting with the circumference in E; and draw BE.
a 47. 1. [a] Then is BEq
b 3. ax. (BAq) = BCq + CEq. [b] Therefore BAq − BCq = CEq. *Which was to be Done.*

PRO-

PROBLEME III.

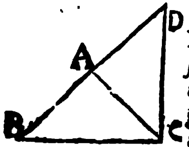


Any two ſides of a right-angled triangle ABC *being known, to find out the third.*

Let the ſides AB, AC encompaſſing the right angle, be, the one 6 foot, the other 8. Therefore, whereas ACq + ABq = 64 + 36 = 100 = BCq, thence is BC = √100 = 10. 47. 1.

Otherwiſe, let the ſides AB, BC be known, the one 6 foot, the other 10. Therefore ſince BCq − ABq = 100 − 36 = 64 = ACq, thence is AC = √64 = 8. *Which was to be Done.* 47. 1.

PROP. XLVIII.



If the ſquare made upon one ſide BC *of a triangle be equall to the ſquares made on the other ſides of the triangle,* AB, AC, *then the angle* BAC *comprehended under thoſe two other ſides of the triangle* AB, AC, *is a right angle.*

Draw to the point A in AC a perpendicular line DA = AB, and join CD.

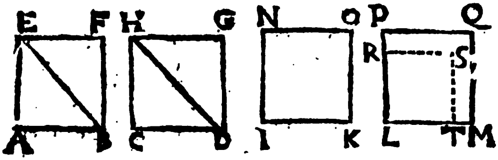
Now is [a] CDq = ADq + ACq = ABq + ACq = BCq. * Therefore is CD = BC. And therefore the triangles CAB, CAD are mutually equilaterall. Wherefore the angle CAB [b] = CAD[c] = a right angle. *Which was to be Dem.*

[a] 47. 1.
* *See the following Theor.*
[b] 8. 1.
[c] *conſtr.*

Schol.

We aſſumed in the demonſtration of the laſt propoſition, CD = BC, becauſe CDq was equall to BCq: our aſſumption we prove by the following theoreme.

THEOREME.



The ſquares AF, CG *of equall right lines* AB, CD *are equall one to the other: And the ſides* IK, LM *of equall ſquares* NK, PM, *are equall one to the other.*

1. *Hypotheſis.* Draw the diameters EB, HD. Then
a 34.1. it is evident that AF is [a] equall to the triangle EAB
b 4.1. & twice taken, and [b] equall to the triangle HCD
6. ax. twice taken, and equall to [a] CG. *Which was to be Dem.*

2. *Hyp.* If it may be, let LM be greater then IK.
a 46 1. Make LT = IK, and let LS be [a] equall to LTq.
b 1. part. Therefore is LS [b] = NK [c] = LQ. [d] *Which is Ab-*
c hyp. *ſurd.*
d 9. ax.

Coroll.

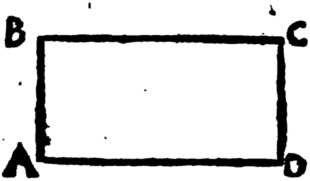
After the ſame manner any rectangles equilateral one to another, are demonſtrated to be alſo equall.

The End of the firſt Book.

THE

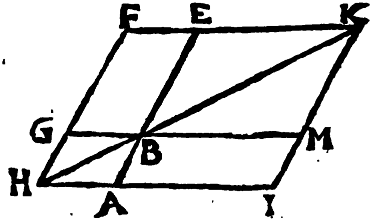
THE SECOND BOOK OF EUCLIDE'S ELEMENTS.

Definitions.



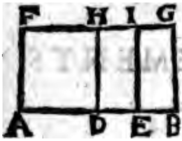
I. EVery right-angled Parallelogram ABCD is ſaid to be conteined under two right lines AB, AD comprehending a right angle.

Therefore when you meet with ſuch as theſe, the rectangle under BA, AD, *or for ſhortneſſe ſake the rectangle* BAD, *or* BA x AD (or ZA, for ZxA) *the rectangle meant is that which is conteined under the right lines* BA, AD *ſet at right angles.*



II. In every Pgr. FHIK, any one of thoſe parallelograms which are about the diameter together with its two complements is called a Gnomon. *As the Pgr.* FB + BI + GA (EHM) *is a Gnomon; and likewiſe the Pgr.* FB+BI+ EM(GKA) *is a Gnomon.*

PROP. I.



If two right lines A F, A B, *be given, and one of them* AB *divided into as many parts or ſegments as you pleaſe; the rectangle comprehended under the two whole right lines* AB, AF, *ſhall be equall to all the rectangles conteined under the whole line* A F *and the ſeverall ſegments*, AD, DE, EB.

a 11. 1. a Set AF perpendicular to AB. Through F a draw an infinite line FG perpendicular to AF. From the points D, E, B erect perpendiculars DH, EI, BG. Then is A G a rectangle comprehended under AF,
b 19 ax. 1. AB, and is b equall to the rectangles AH, DI, EG,
c 34. 1. that is (becauſe DH, EI, AF are equall) to the rectangles under AF, AD, under AF, DE, under AF, EB. *Which was to be Dem.*

Schol.

If two right lines given be both divided into how many parts ſoever, the product of the whole multiplied into it ſelf ſhall be the ſame with this of the parts multiplied into themſelves.

For let Z be $= A + B + C$, and $Y = D + E$;
a 1. 2. then, becauſe DZ a $= DA + DB + DC$, and EZ a $= EA + EB + EC$, and YZ a $= DZ + EZ$,
b 2. ax. b ſhall ZY be $= DA + DB + DC + EA + EB + EC$. *Which was to be Dem.*

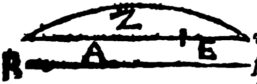
From hence is underſtood the manner of multiplying compounded right lines into compoundeds. For you muſt take all the Rectangles of the parts, and they will preſent you with the Rectangle of the wholes.

But whenſoever in the multiplication of lines into themſelves you meet with theſe ſignes − intermingled with theſe +, you muſt alſo have particular regard to the ſignes. For of + multiply'd into − ariſeth −; but of − into − ariſeth +. *ex. gr.* let +A be to be multiply'd into B − C; then becauſe +A is not affirmed of all B, but only of a part of it, whereby it exceeds C, therefore A C muſt remain denied ; ſo that the product will be $AB - AC$. Or us ; becauſe B conſiſts of the parts C and $B - C$, hence $AB = AC + A \times B - C$. take away AC om either, then $AB - AC = A \times B - C$. In like * 1. 2. anner if $-A$ be to be multiply'd into $B - C$, then eing by reaſon of the figure $-$, that A is not denied all B, but only of ſo much as it exceeds C, there- re AC muſt remain affirmed. whence the product ill be $-AB + AC$. Or thus; becauſe $AB = AC + \times B - C$; take away all throughout, and there will e $-AB = AC - A - \times B + C$; adde AC to either, d there will be $-AB + AC = A \times B - C$.

This being ſufficiently underſtood, the nine fol- wing propoſitions, and innumerable others of that ind, ariſing from the comparing of lines multi- ly'd into themſelves (which you may find done to our hand in *Vieta* and other Analytical writers) are emonſtrated with great facility, by reducing the atter for the moſt part to almoſt a ſimple work.

Furthermore, * it appears that the product of any * 19. ax. gnitude multiply'd into all the parts of any number equall to the product of the ſame multiply'd into he whole number: As $5A + 7A = 12A$, and $4 \times 5A + 4A \times 7A = 4A \times 12A$. Wherefore what here deliver'd of the multiplying of right lines in- themſelves, the ſame may be underſtood of the ultiplying of numbers into themſelves. ſo that hatſoever is affirmed concerning lines in the nine ollowing theoremes, holds good alſo in numbers; eeing they all immediately depend and are deriv'd om this firſt.

PROP. II.

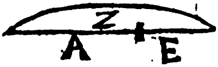


If a right line Z be divided any-wiſe into two parts, the rectangles comprehended under the whole ine Z and each of the ſegments A, E, are equall to the quare made of the whole line Z.

I

I ſay that $ZA + ZE = Zq$. For take $B = Z$;
a 1.2. a then is $BA + BE = BZ$. that is (becauſe $B = Z$) $ZA + ZE = Zq$. *Which was to be Dem.*

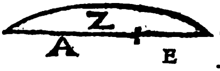
PROP. III.



If a right line Z be divided any-wiſe into two parts, the rectangle comprehended under the whole line Z and one of the ſegments E is equall to the rectangle made of the ſegments A, E, and the ſquare deſcribed on the ſaid ſegment E.

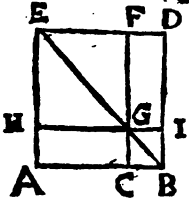
a 1.2. I ſay $ZE = AE + Eq$. a For $EZ = EA + Eq$.

PROP. IV.



If a right line Z be cut any-wiſe into two parts, the ſquare made of the whole line Z is equall both to the ſquares made of the ſegments A, E, and to twice a rectangle made of the parts A, E.

a 3.2. I ſay that $Zq = Aq + Eq + 2\,AE$. For $ZA \; a= Aq + AE$, and $ZE \; a= Eq + EA$. Therefore whereas
b 2.2. $ZA + ZE \; b= Zq$, c thence is $Zq = Aq + Eq +$
c 1.ax. $2\,AE$. *Which was to be Dem.*



Otherwiſe thus; Upon the right line AB make the ſquare AD, and draw the diameter EB; through C, the point wherein the line AB is divided, draw the perpendicular CF; and through the point G draw HI parallel to AB.

d 4 cor. 32. 1. Becauſe the angle $EHG = A$ is a right angle, and AEB is d half a right, e therefore is the remaining
e 32. 1. angle HGE half a right angle. Therefore is $HE \; f=$
f 6. 1. $HG \; g= EF \; g= AC$, ſo that HF h is the ſquare of
g 34. 1. the right line A C. After the ſame manner is C I
h 29. def. proved to be CBq. Therefore AG, GD are rectangles
k 19. ax. 1. under AC, CB. wherefore the whole ſquare $AD \; k=$ $ACq + CBq + 2\,ACB$. *Which was to be Dem.*

Corall.

1. Hence it appears that the Pgrs which are about the diameter of a ſquare are alſo ſquares themſelves.

2. That the diameter of any ſquare biſects it's angles.

3. That if $A = \frac{1}{2} Z$, then is $Zq = 4\, Aq$, and $Aq = \frac{1}{4} Zq$. As on the contrary, if it be ſo that $Zq = 4\, Aq$, then is $A = \frac{1}{2} Z$.

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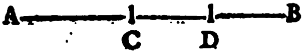
PROP. V.



If a right line AB *be cut into equall parts* AC, CB, *and into unequall parts* AD, DB, *the rectangle comprehended under the unequall parts* AD, DB, *together with the ſquare that is made of the difference of the parts* CD, *is equall to the ſquare that is made of the half line* CB.

I ſay that $CBq = ADB + CDq$.

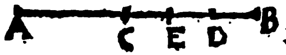
For theſe are all equall;
$$\left\{\begin{array}{l} CBq. \\ {}^{a}CDq + CDB + DBq + CDB. \\ CDq + {}^{b}CBD({}^{c}AC \times BD) + CDB. \\ CDq + {}^{d}ADB. \end{array}\right.$$

a 4. 2.
b 3. 2.
c hyp.
d 1. 2.

This theoreme is ſomewhat differently expreſs'd and more eaſily demonſtrated thus; *A Rectangle* [illegible] *of the ſumme and the difference of two right lines* A, E, *is equall to the difference out of them.*

For if $A + E$ be multiply'd into $A - E$, * there ariſeth $Aq - AE + EA - Eq = Aq - Eq$. *Which was to be Dem.*

Schol.



If the line AB be divided otherwiſe, (viz.) nearer to the point of biſection, in E; Then is $AEB \sqsubset ADB$.

For $AEB\ {}^{a}= CBq - CEq$. and $ADB\ {}^{a}= CBq - CDq$. Therefore, whereas $CDq \sqsubset CEq$, thence is $AEB \sqsubset ADB$. *W. W. to be Dem.*

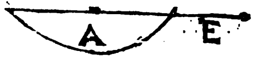
a 5. 2. & 3. ax.

Coroll.

1. Hence is $ADq + DBq \sqsubset AEq + EBq$. For
b 4. 2. $ADq + DBq + 2\,ADB \overset{b}{=} ABq \overset{b}{=} AEq + EBq + 2\,AEB$. Therefore becauſe $2\,AEB \sqsubset 2\,ADB$, thence is $ADq + DBq \sqsubset AEq + EBq$. *W. W. to be Dem.*

c 3. ax. 2. Hence is $ADq + DBq - AEq \overset{c}{+} EBq = 2\,AEB - 2\,ADB$.

PROP. VI.



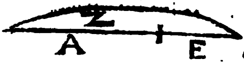
If a right line A be divided into two equall parts, and another right line E added to the ſame directly in one right line, then the rectangle comprehended under the whole and the line added, (viz. A+E,) and the line added E, together with the ſquare which is made of $\frac{1}{2}$ the line A, is equall to the ſquare of $\frac{1}{2}$ A+E taken as one line.

I ſay that $\frac{1}{4} Aq$ (^{a} $Q.\frac{1}{2}A$) $+ AE + Eq = Q.\frac{1}{2}A + E$.
a 4 & 3. Cor. 4. 2. For, $Q.\frac{1}{2}A + E = \frac{1}{4}Aq + Eq + AE$. *Which was to be Dem.*

Coroll.

Hence it follows that if 3 right lines E, $E + \frac{1}{2}A$, $E + A$ be in arithmeticall proportion, then the rectangle conteined under the extreme termes E, $E + A$, together with the ſquare of the difference $\frac{1}{2}A$, is equall to the ſquare of the middle term $E + \frac{1}{2}A$.

PROP. VII.



If a right line Z be divided any-wiſe into two parts, the ſquare of the whole line Z together with the ſquare made of one of the ſegments E, is equall to a double rectangle comprehended under the whole line Z and the ſaid ſegment E, together with the ſquare made of the other ſegment A.

a 4. 2. I ſay that $Zq + Eq = 2\,ZE + Aq$. For $Zq \overset{a}{=}$
b 3. 2. $Aq + Eq + 2\,AE$. and $2\,ZE \overset{b}{=} 2\,Eq + 2\,AE$. *W. W. to be Dem.*

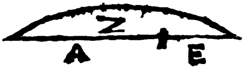
Co-

Coroll.

Hence it follows that the ſquare of the difference of any two lines Z, E, is equall to the ſquares of both the lines leſſe by a double rectangle comprehended under the ſaid lines.

[c] For $Zq + Eq - 2\,ZE = Aq = Q.\,Z - E$. c 7.2. and 3 ax.

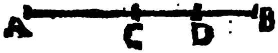
PROP. VIII.



If a right line Z be divided any-wiſe into two parts, the rectangle comprehended under the whole line Z and one of the ſegments E four times, together with the ſquare of the other ſegment A, is equall to the ſquare of the whole line Z and the ſegment E taken as one line Z+E.

I ſay that $4\,ZE + Aq = Q.\,Z + E$. For $2\,ZE$ [a] $= Zq + Eq - Aq$. Therefore $4\,ZE + Aq = Zq + Eq + 2\,ZE$ [b] $= Q.\,Z + E$. *W. W. to be Dem.* a 7.2. and 3.ax. b 4.2.

PROP. IX.



If a right line AB *be divided into equall parts* AC, CB, *and into unequall parts* AD, DB, *then are the ſquares of the unequall parts* AD, DB *together, double to the ſquare of the half line* AC, *and to the ſquare of the difference* CD.

I ſay that $ADq + DBq = 2\,ACq + 2\,CDq$. For $ADq + DBq$ [a] $= ACq + CDq + 2\,ACD + DBq$. a 4.2.
But $2\,ACD$ ([b] $2\,BCD$) $+ DBq$ [c] $= CBq\ (ACq) + CDq$. b hyp. c 7.2.
[d] Therefore $ADq + DBq = 2\,ACq + 2\,CDq$. *W. W. to be Dem.* d 2.ax.

This may be otherwiſe deliver'd and more eaſily demonſtrated thus; *The aggregate of the ſquares made of the ſumme and the difference of two right lines* A, E, *is equall to the double of the ſquares made from thoſe lines.*

For $Q{:}\ A + E$ [a] $= Aq + Eq + 2\,AE$. and $Q{:}\ A - E$ [b] $= Aq + Eq - 2\,AE$. Theſe added together make $2\,Aq + 2\,Eq$. *Which was to be Dem.* a 4.2. b ſch.7.2.

PROP.

I ſay that ACq+BCq = ABq + 2 BCD.

a 47. 1. b 7. 2. c 47. 1.

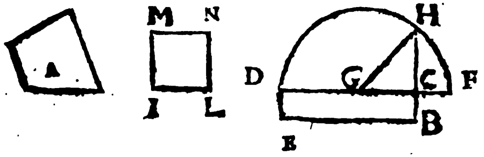
For theſe are equall $\left\{\begin{array}{l} ACq+BCq. \\ {}^{a}ADq+DCq+BCq. \\ {}^{b}ADq+BDq+2\,BCD. \\ {}^{c}ABq+2\,BCD. \end{array}\right.$

Coroll.

Hence, *The ſides of an acute-angled triangle* ABC *being known, you may find out the ſegment* DC *intercepted betwixt the perpendicular* AD *and the acute angle* ABC, *as alſo the perpendicular it ſelf* AD.

Let AB be 13, AC 15, BC 14. Take ABq (169) from ACq+BCq, that is, from 225 + 196 = 421. Then remains 252 for 2 BCD. wherefore BCD will be 126. divide this by BC 14, then will 9 be found out for DC. From whence it follows AD = √: 225 − 81 = 12.

PROP. XIV.



To find a ſquare ML *equall to a right-lined figure given* A.

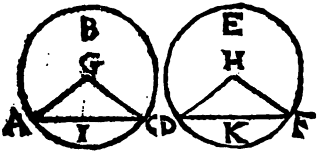
a 45. 1. b 10. 1. ᵃMake the rectangle DB = A, and produce the greater ſide thereof DC to F, ſo that CF = CB. ᵇbiſect DF in G, about which as the centre at the diſtance of GF deſcribe the circle FHD, and draw out CB till it touch the circumference in H. Then ſhall be CHq = *ML = A.

* 46. 1. c conſtr. d 5. 2. and 3. ax. e 47. 1. and 3. ax. For let GH be drawn. Then is A ᶜ= DB ᶜ= DCF ᵈ= GFq − GCq ᵉ= HCq ᶜ= ML. *W.W. to be Done.*

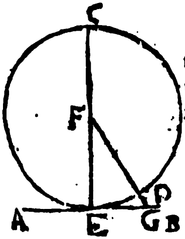
The End of the ſecond Book.

THE THIRD BOOK OF EUCLIDE'S ELEMENTS.

Definitions.

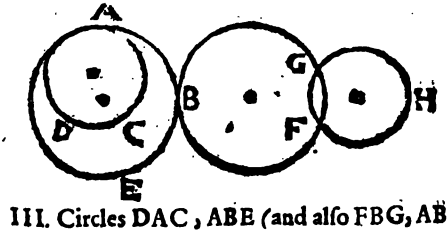


I. EQuall circles (GABC , HDEF) are ſuch whoſe diameters are equall; or, from whoſe centres right lines drawn GA, HD, are equall.



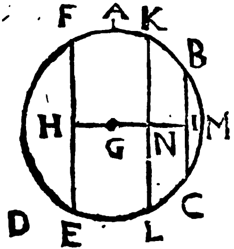
II. A right line AB is ſaid to touch a circle FED, when touching the ſame, and being produced, it cutteth it not.

The right line FG cuts the circle FED.

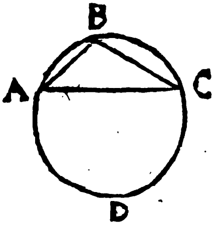


III. Circles DAC, ABE (and alſo FBG, ABE) are ſaid to touch one to the other, which touch, but cut not one the other.

The circle BFG cuts the circle FGH.



IV. In a circle GABD, right lines FE, KL are ſai to be equally diſtant fro the centre, when perpendiculars GH, G drawn from the centre G to them are equall. An that line BC is ſaid to b furtheſt diſtant from it, o whom the greater perpendicular GI falls.

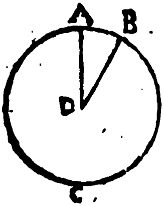


V. A ſegment of a circle (ABC) is a figure conteined under a right line AC, and a portion of the circumference of a circle ABC.

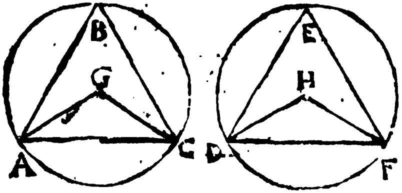
VI. An angle of a ſegment CAB, is that angl which is conteined under a right line CA and a arch of a circle AB.

VII. An angle ABC is ſaid to be in a ſegmen ABC, when in the circumference thereof ſome poin B is taken, and from it right lines AB, CB, drawn t the ends of the right line A C, which is the baſe o the ſegment; then the angle ABC conteined unde the adjoined lines AB, CB, is ſaid to be an angle i a ſegment.

VIII. But when the right lines AB, BC comprehending the angle ABC, do receive any periphery of the circle ADC, then the angle ABC is ſaid t ſtand upon that periphery.

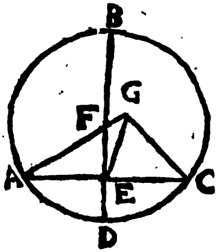


IX. A ſector of a circle (ADB) is when an angle ADB is ſet at the centre D of that circle; namely, that figure ADB, comprehended under the right lines AD, BD conteining the angle, and the part of the circumference received by them AB.



X. Like ſegments of a circle (ABC, DEF) are thoſe which conclude equall angles (ABC, DEF;) or, in whom the angles ABC, DEF are equall.

PROP. I.



To find the centre F *of a circle given* ABC.

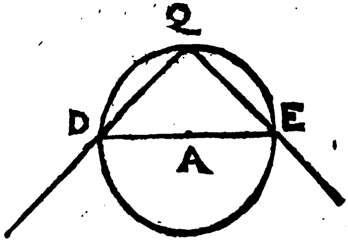
Draw a right line AC any-wiſe in the circle, which biſect in E, through E draw a perpendicular DB, and biſect the ſame in F; the point F ſhall be the center.

If you deny it, let G a point without the line DB be the centre (for it cannot be in the line BD, ſince that cannot be divided equally in any point but F;) let the lines GA, GC, GE be drawn. Now if G be the centre, a then is GA = GC, and AE = EC by conſtruction, and the ſide GE common. b Therefore are the angles GEA, GEC equall, and c conſequently right. d Therefore the angle GEC = FEC. e *Which is abſurd.*

a 15. *def.* 1.
b 8. 1.
c 10 *def.* 1.
d 12. *ax.*
e 9. *ax.*

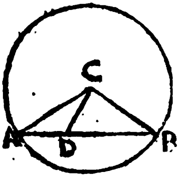
Coroll.

Hence, If a right line B D biſect any rig AC in a circle at right angles, the centre ſhal the line BD that cuts the other.



Andr. Tacq. *The centre of a circle is eaſily found out by a the top of a Square to the circumference thereof.* the right line D E that joins the points D, which the ſides of the Square Q D, Q E cut t cumference, be biſected in A, the point A ſh the centre. The demonſtration whereof deper on *Prop.* 31. of this Book.

P R O P. II.



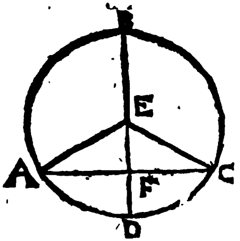
If in the circumference o cle CAB *any two points* *taken, the right line* AB *whi thoſe two points ſhall fall wi circle.*

Take in the right line point D; from the center CA, CD, CB. Becauſe C A[a] = C B, there
a 15. def. 1. the angle A[b] = B. But the angle CDB[c] ⊏ A
b 5. 1. c 16. 1. fore is C D B ⊏ B. therefore C B[d] ⊏ C I
d 19. 1. CB onely reaches the circumference, therefo comes not ſo far; wherefore the point D is wit circle. The ſame may be proved of any othe in the line AB. And therefore the whole line within the circle. *Which was to be Dem.*

Coroll.

Hence, If a right line touch a circle, ſo that it cut it not, it touches but in one point.

PROP. III.



If in a circle E A B C, *a right line* B D *drawn through the centre, biſect any other line* A G *not drawn through the centre, it ſhall alſo cut it at right angles: And if it cuts it at right angles, it ſhall alſo biſect the ſame.*

From the centre E let the lines E A, E C be drawn.

1. *Hyp.* Becauſe A F [a] = F C, and E A [b] = EC, and the ſide EF common; the angles EFA, EFC [c] ſhall be equall, and [d] conſequently right. *Which was to be Dem.*

a hyp.
b 15.def.1.
c 8.1.
d 10.def.1.

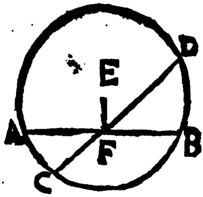
Hyp. 2. Becauſe E F A [e] = E F C, and the angle EAF [f] = ECF, and the ſide E F common; [g] therefore is AF = FC. Therefore AC is cut into two equall parts. *Which was to be Dem.*

e hyp and 12.ax.
f 5.1.
g 26.1.

Coroll.

Hence, In any equilateral or Iſoſceles triangle, if a line drawn from the verticall angle biſect the baſe, that line is perpendicular to it. And on the contrary, a perpendicular drawn from the verticall angle biſects the baſe.

PROP. IV.



If in a circle A C D *two right lines* AB, CD *cut through one another, yet neither of them paſſe through the centre* E, *then neither of thoſe lines are divided into equall parts.*

then both lines are bisected.

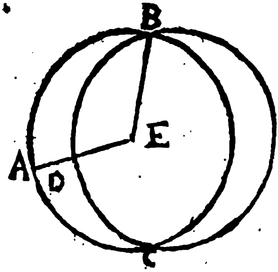
For if one line paſſe through

the centre , it appears that it cannot be biſected b the other;becauſe by Hypotheſis,the other does no paſſe through the centre.

If neither of them paſſe through the centre, the from the centre E draw E F: now if A B , C D wer both biſected in F , then a would the angles EFB EFD be both right,and conſequently equall. b Whic is abſurd.

a 3. 3.
b 9. ax.

PROP. V.

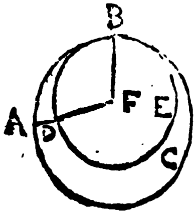


If two circles BAC BDC *cut one the othe they ſhall not have th ſame centre* E.

For otherwiſe th lines EB , EDA draw from E the commo centre , would D E b a = E B a = B A. *b Which is abſurd.*

a 15. def. 1.
b 9 ax.

PROP. VI.

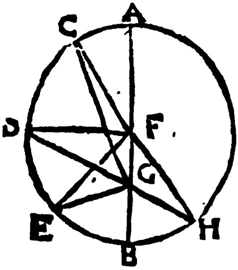


If two circles BAC , BDC *inwardly touch one the other* (i B) *they have not one and th ſame centre* F.

For otherwiſe the right line F B,FDA drawn fromthe centre F , would be F D a = F a = FA. *b Which is abſurd.*

a 15. def. 1.
b 9. ax.

PROP. VII.



If in A B *the diameter of a circle ſome point* G *be taken, which is not the centre of the circle, and from that point certain right lines* GC, GD, GE *fall on the circle, the greateſt line ſhall be that* (GA) *in which is the centre* F; *the leaſt, the remainder of the ſame line* (GB.) *And of all the other lines, the line* G C *neareſt to that which was drawn through the centre is alwayes greater then any line farther removed* G D; *and only two lines are equall* GE, GH, *which fall upon the circle from the ſame point, on each ſide of the leaſt* G B *or of the greateſt* GA.

From the centre F draw the right lines FC, FD, FE; and *a* make the angle $BFH = BFE$. a 23. 1.

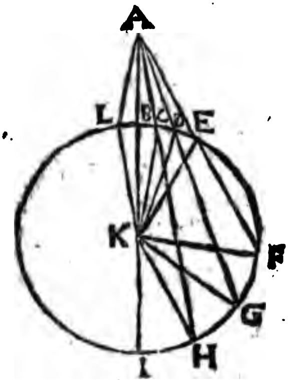
1. $GF + FC$ (that is GA) *a* $\sqsubset GC$. *Which was to be Dem.* a 20. 1.

2. The ſide FG is common, and FC *b* $=$ FD, and the angle GFC *c* $\sqsubset$ GFD; *d* wherefore the baſe GC $\sqsubset$ GD. b 15. def. 1. c 9. ax. d 24. 1.

3. FB (FE) *e* $\sqsupset GE+GF$. Therefore FG, which is common, being taken away from both, there remains BG $\sqsupset$ EG. e 20. f 5. ax.

4. The ſide FG is common, and $FE = FH$, and the angle BFH *g* $=$ BFE; *h* Therefore is $GE = GH$. But that no other line G D from the point G can be equall to GE, or GH, is already proved. *Which was to be Demonſtrated.* g conſt. h 4. 1.

PROP. VIII.



If ſome point A *be taken without a circle, and from that point be drawn certain right lines* AI, AH, AG, AF *to the circle, & of thoſe one* A I *be drawn through the centre* K, *and the others any-wiſe; of all thoſe lines that fall on the concave of the circumference, that is the greateſt* A I *which is drawn through the centre; and of the others, that which is neareſt* (AH) *to the line that paſſes through the centre is greater then that which is more diſtant* A G. *But of all thoſe lines that fall on the convexe part of the circle, the leaſt is that* AB *which is drawn from the point* A *to the diameter* I B; *and of the others, that* (A C) *which is neareſt to the leaſt, is leſſe then that which is farther diſtant* AD. *And from that point there can be onely two equall right lines* AC, AL *drawn, which ſhall fall on the circumference on each ſide of the leaſt line* A B *or of the greateſt* AI.

From the centre K draw the right lines KH, KG, K F, K C, K D, K E. and make the angle A K L = AKC.

a 20. 1. 1. AI (AK+KH) a $\sqsubset$ AH.

2. The ſide AK is common, and KH = KG, and
b 24. 1. the angle AKH $\sqsubset$ AKG; b therefore the baſe AH $\sqsubset$ AG.

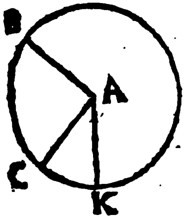
c 20. 1. 3. KA c $\sqsupset$ KC + CA. From hence take away
d 5. ax. AB $\sqsupset$ AC. KC, KB that are equall; then will remain AB d $\sqsupset$ AC.

4. AC + CK e $\sqsupset$ AD+DK. From thence take
e 21. 1. away CK, DK that are equall; then remains AC
f 5. ax. f $\sqsupset$ AD.

5. The ſide KA is common, and KL = KC, and the angle AKL g = AKC; h therefore KA = CA. But that no other line could be drawn equall to theſe, was proved above. Therefore, &c.

g *conſt.*
h 4. 1.

PROP. IX.

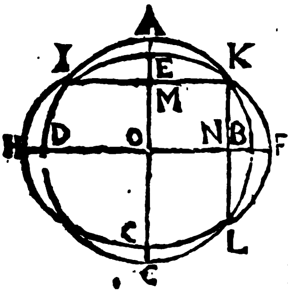


If in a circle BCK *a point* A *be taken, and from that point more then two equall right lines* AB, AC, AK, *drawn to the circumference, then is that point* A *the centre of the circle.*

For a from no point without the centre can more then two right lines equall be drawn to the circumference. Therefore A is the centre. *Which was to be Dem.*

a 7. 3.

PROP. X.

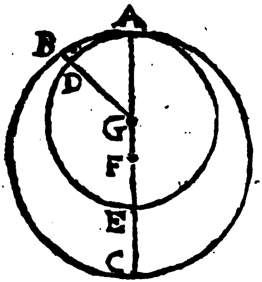


A circle IAKBL *cannot cut another circle* IEKFL *in more then two points.*

Let one circle, if it may be, cut the other in three points I, K, L. and IK, KL being join'd, let them be biſected in M and N. a Both circles have their centres in their perpendiculars MC, NH, and in the interſection of thoſe perpendiculars which is O. Therefore the circles that cut each other have the ſame centre. Which is falſe, by *Prop.* 5. 3.

a *cor.* 1. 3.

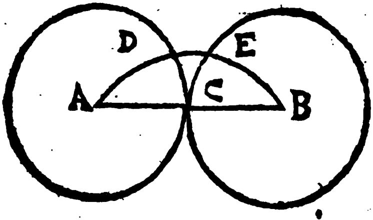
PROP. XI.



If two circles G A D E, F A B C *touch one the other inwardly, and their centres be taken* G, F; *a right line* F G *joyning their centres, and produced, ſhall cut the circumference in* A *the point of contact of the circles.*

If it can be, let the right line F G produced cut the circles in ſome other point then A. ſo that not FGA, but FGDB ſhall be a right line. Let
a 15. def 1. the line GA be drawn. Now, becauſe GD a= GA, &
b 7. 3. GB b $\sqsupset$ GA (ſince the right line F G B paſſes through F the centre of the greater circle) therefore
c 9. ax. is GB $\sqsupset$ GD. *c Which is abſurd.*

PROP. XII.

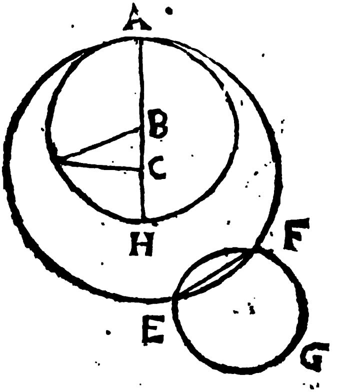


If two circles ACD, BCE *touch one the other outwardly, the right line* AB *which joins their centres* A, B, *ſhall paſſe through the point of contact* C.

If it may be, let ADEB be a right line cutting the circles not in the point of contact C, but in the points D, E; draw AC, CB. then is AD + EB (AC
a 20. 1. + CB) a $\sqsubset$ ADEB. *b Which is abſurd.*
b 9 ax.

PROP.

PROP. XIII.



A circle CAF *cannot touch a circle* BAH *in more points then one* A, *whether it be inwardly or outwardly.*

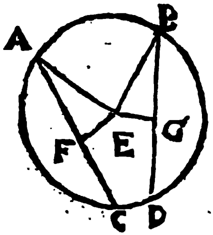
1. Let one circle (if it can be) touch another in two points A, H. [a] Then will the right line CB that joins the centres, if it be produced, fall as well in A, as H. Now because CH [b] = CA, and BH $\sqsubset$ CH, therefore is BA ([c] BH) $\sqsubset$ CA. [d] *Which is abſurd.*

a 11. 3.
b 15. def. 1.
c 15. def. 1.
d 9. ax.

2. If it be ſaid to touch outwardly in the points E and F, then draw the line EF, [e] which will be in both circles. Therefore thoſe circles cut one the other; Which is againſt the Hyp.

e 2. 3.

PROP. XIV.



In a circle EABC *equall right lines* AC, BD *are equally diſtant from the centre* E: *and right lines* AC, BD *which are equally diſtant from the centre, are equall among themſelves.*

From the centre E draw the perpendiculars EF, EG. [a] which will biſect the lines AC, BD. join EA, EB.

a 3. 3.

1. *Hyp.* AC = BD. therefore AF [b] = BG. But alſo

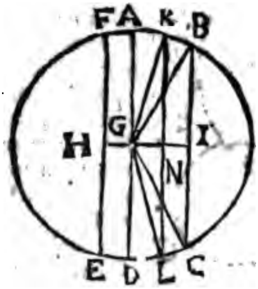
b 7. ax.

c 47. 1. and 3. ax. ſo EA = EB. therefore FEq *c* = EAq − AFq = BBq − BGq *c* = EGq. *d* Therefore FE = EG.

d ſchol. 48. 1. 2. *Hyp.* EF = EG. Therefore AFq *e* = EAq − EF = EBq − EGq = BGq. Therefore AF *d* = GB, an

e 6. ax. *e* conſequently AC = BD. *Which was to be Dem.*

P R O P. XV.



In a circle GAB(

the greateſt line is A

the diameter; and of a

other lines, that line FE

which is neareſt to th

centre G *is greater the*

any line BC *farther di*

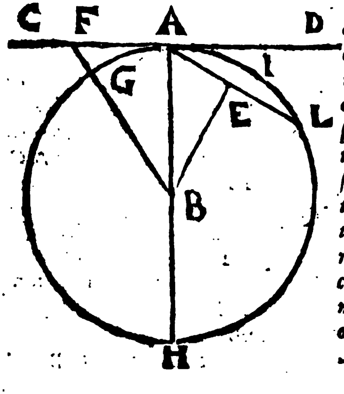
ſtant from it.

1. Draw GB, an GC. The diameter AI (*a* GB+GC) *b* ⊏ BC

a 15. def. 1. b 20. 1. 2. Let the diſtance G be ⊏ GH. Take GN = GH. Through the point I draw KL perpendicularly to GI: join GK, GL. Be cauſe GK = GB, and GL = GC, and the angle KGI

c 24. 1. ⊏ BGC; *c* therefore is KL (FE) ⊏ BC. *Which wa to be Dem.*

P R O P. XVI.



A line CI *drawn from th extreme point o the diameter* H *of a circle* BALH *perpendicular t the ſaid diameter ſhall fall withou the circle; and be tween the ſam right line and th circumference can not be drawn an other line* AL *And the angle o the*

the ſemicircle BAI, *is greater then any right-lined acute angle* BAL; *and the remaining angle without the circumference* DAI *is leſſe then any right-lined angle.*

1. From the centre B to any point F in the right line AC, draw the rigt line BF. The ſide BF ſubtending the right angle BAF is a greater then the ſide BA which is oppoſite to the acute angle BFA. Therefore whereas BA (BG) reaches to the circumference, BF ſhall reach further; and ſo the point F. and for the ſame reaſon is any other point of the line AC placed without the circle. a 19. 1.

2. Draw BE perpendicular to AL. The ſide BA oppoſite to the right angle BEA is b greater then the ſide BE which ſubtends the acute angle BAE; therefore the point E, and ſo the whole line EA, fals within the circle. b 19. 1.

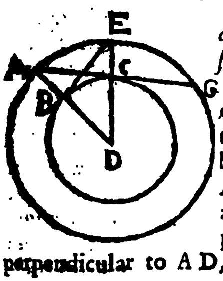
3. Hence it follows that any acute angle, to wit, EAD, is greater then the angle of contact DAI, and that any acute angle BAL is leſſe then the angle of a ſemicircle BAI. *Which was to be Dem.*

Coroll.

Hence, A right line drawn from the extremity of the diameter of a circle, and at right angles, is a tangent to the ſaid circle.

From this propoſition are gathered many paradox and wonderfull conſectaries, which you may meet with in the interpreters.

PROP. XVII.

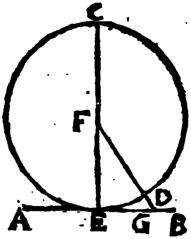


From a point given A *to draw a right line* AG *which ſhall touch a circle given* DBC.

From D the centre of the circle given let a line DA cutting the circumference in B, be drawn to the point given A; from the centre D deſcribe another circle through the point A; and from B draw a perpendicular to AD, which ſhall meet with the circle AE in the point E; and draw ED meeting with the circle BC in the point C. Then the line drawn from A to C ſhall touch the circle DBC.

a 15 def. 1. b 4. 1. For DB ^{a}= DC, and DE ^{a}= DA, and the angle D is common; ^{b} therefore the angle ACD = EBD and right. c cor. 16. 3. ^{c} Therefore AC touches the circle in C. *Which was to be Done.*

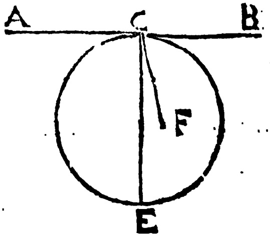
PROP. XVIII.



If any right line AB *touch a circle* FEDC; *and from the centre to the point of contact* E *a right line* FE *be drawn; that line* FE *ſhall be perpendicular to the tangent* AB.

If you deny it; let ſome other line FG be drawn from the centre F perpendicular to the tangent, and ^{a} cutting the circle in D. a 1. def. 3. Therefore, whereas the angle FGE is ſaid to be right, b cor. 17. 1. ^{b} thence is the angle FEG acute; c 19. 1. ^{c} ſo that FE (FD) $\sqsubset$ FG. d 9. ax. ^{d} *Which is abſurd.*

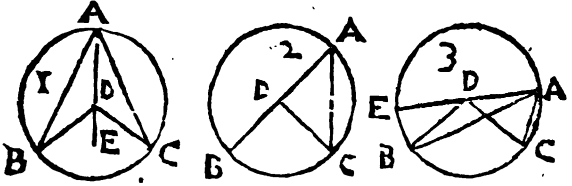
PROP. XIX.



If any right line AB *touch a circle, and from the point of contact* C *a right line* CE *be erected at right angles to the tangent, the centre of the circle ſhall be in the line* CE *ſo erected.*

If you deny it; let the centre be without the line CE in the point F; and from F to the point of contact let FC be drawn. Therefore the angle FCB is right, a 12. ax. and ^{a} conſequently equall to the angle ECB, which was right by Hypotheſis. b 9 ax. ^{b} *Which is abſurd.*

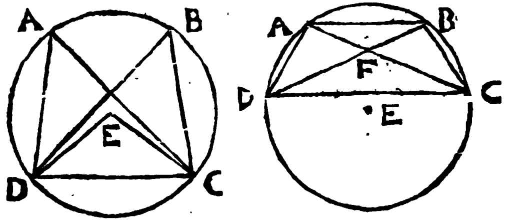
PROP. XX.



In a circle DABC, *the angle* BDC *at the centre is double of the angle* BAC *at the circumference, when the ſame arch of the circle* BC *is the baſe of the angles.*

Draw the diameter ADE. The outward angle BDE a = DAB + DBA b = 2 DAB. Likewiſe the angle EDC = 2 DAC. Therefore in the firſt caſe the whole angle BDC = 2 BAC. and in the third caſe the c remaining angle BDC = 2 BAC. *Which was to be Dem.* a 32. 1. b 5. 1. c 19. ax.

PROP. XXI.



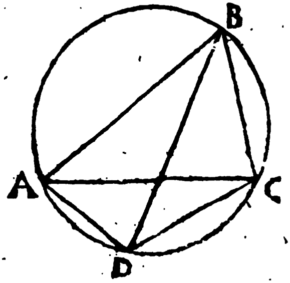
In a circle EDAC, *the angles* DAC *and* DBC *which are in the ſame ſegment, are equall one to the other.*

1. *Caſe.* If the ſegment DABC be greater then a ſemicircle, from the centre E draw ED, EC. Then is twice the angle A a = E a = 2 B. *W. W. to be Dem.*

2. *Caſe.* If the ſegment be leſſe then a ſemicircle, then is the ſum of the angles of the triangle ADF equall to the ſum of the angles of the triangle BCF. from each let AFD be taken away b equall to BFC, a 20. 3. b 15. 1.

e *by the 1. caſe.* BFC, and ADB [e] = ACB be likewiſe taken away; then remains DAC = DBC. *W.W. to be Dem.*

PROP. XXII.



The angles ADC, ABC *of a quadrilateral figure* ABCD *deſcribed in a circle, which are oppoſite one to the other, are equall to two right angles.*

Draw AC, BD. The angle ABC + BCA + BAC [a] = 2 right. But BDA [b] = BCA, and BDC [b] = BAC. [c] Therefore ABC + ADC = 2 right angles. *Which was to be Dem.*

a 32. 1.
b 21. 3.
c 2. *ax.*

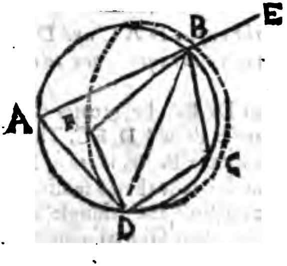
Coroll.

1. Hence, If one ſide * AB of a quadrilateral deſcribed in a circle be produced, the externall angle EBC is equall to the internall angle ADC, which is oppoſite to that ABC which is adjacent to EBC. as appears by 13. 1. and 3. *ax.*

* *See the following Diagr.*

2. A circle cannot be deſcribed about a Rhombus; becauſe it's oppoſite angles are greater or leſſe then two right angles.

Schol.

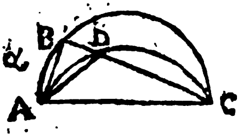


If in a quadrilateral ABCD *the angles* A *and* C, *which are oppoſite, be equall to two right, then a circle may be deſcribed about that quadrilateral.*

For a circle will paſſe through any 3 angles (as ſhall appear by 5. 4.) I ſay that ſhall paſſe through A the fourth alſo of ſuch

a quadrilaterall: For if you deny it, let the circle paſſe through F. Therefore the right lines BF, FD, BD being drawn, the angle C+F *a* = 2 right *b* = C+ a 22. 3.
A wherefore, A *c* is equall to F. *d Which is abſurd.* b hyp. c 3. ax. d 21. 1.

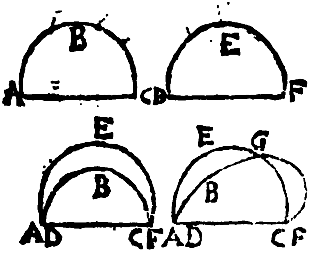
PROP. XXIII.



Two like and unequall ſegments of circles ABC, ADC, *cannot be ſet on the ſame right line* AC, *and the ſame ſide thereof.*

For if they are ſaid to be like, draw the line CB cutting the circumferences in D and B; join AB and AD. Becauſe the ſegments are ſuppoſed like, *a* therefore is the angle ADC = ABC. *b Which is abſurd.* a 10 def 3. b 16. 1.

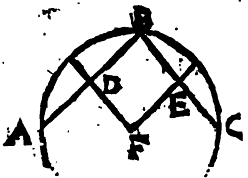
PROP. XXIV.



Like ſegments of circles ABC, DEF *upon equall right lines* AC, DF, *are equall one to the other.*

The baſe AC being laid on the baſe DF will agree with it, becauſe AC = DF. Therefore the ſegment ABC ſhall agree with the ſegment DEF (for otherwiſe it ſhall fall either within or without, and if ſo *a* then a 23. 3.
the ſegments are not like, which is contrary to the Hypotheſis, and at leaſt it ſhall fall partly within and partly without, and ſo cut in 3 points, *b* which is abſurd. *c* Therefore the ſegment ABC = DEF. b 10. 3. c 8. ax.
Which was to be Dem.

PROP. XXV.

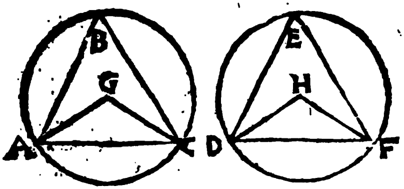


A ſegment of a circle ABC *being given, to deſcribe the whole circle whereof that is a ſegment.*

Let two right lines be drawn AB, BC, which biſect in the points D and E. From D and E draw the perpendiculars DF, EF meeting in the point F. I ſay this point ſhall be the centre of the circle.

a cor. 1. 3. For the centre ſhall be as well in a DF as EF, therefore it muſt be in the common point F. *Which was to be Done.*

PROP. XXVI.



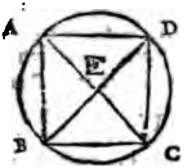
In equall circles GABC, HDEF, *equall angles ſtand upon equall parts of the circumference,* AC, DF; *whether thoſe angles be made at the centres,* G, H, *or at the circumferences,* B, E.

Becauſe the circles are equall, therefore is GA = HD, and GC = HF; alſo by Hypotheſis the angle G = H; a therefore AC = DF. Moreover the angle B b $= \frac{1}{2}$ G c $= \frac{1}{2}$ H b = E. d Therefore the ſegments ABC, DEF are like, and e conſequently equall. f whence the remaining ſegments alſo AC, DF are equall. *Which was to be Dem.*

a 4. 1.
b 20. 3.
c 1, p.
d 10. def. 3.
e 24. 3.
f 3 ax.

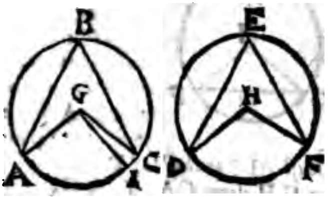
Schol.

Schol.



In a circle ABCD let an arch AB be equall to DC; then ſhall AD be parallel to BC. For the right line AC being drawn, the angle ACB a = CAD; wherefore by 27. 1. the ſaid ſides are parallel. a 26. 3.

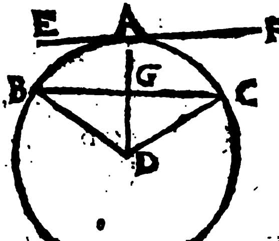
PROP. XXVII.



In equall circles GABC, HDEF, the angles ſtanding upon equall parts of the circumference AC, DF, are equall between themſelves, whether they be made at the centres G, H, or at the circumferences, B, E.

For if it be poſſible, let one of the angles AGC be ⊏ DHF, and make AGI = DFH. thence is the arch AI a = DF b = AC. c *Which is abſurd.* a 26. 3. b hyp. c 9. ax.

Schol.



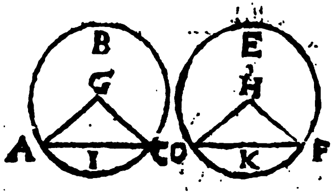
A right line EF, which being drawn from A the middle point of any periphery BC, toucheth the circle, is parallel to the right line BC ſubtending the ſaid periphery.

From the centre D draw a right line DA to the point of contact A, and join DB, DC.

The ſide DG is common, and DB = DC, and the angle BDA a = CDA, (becauſe the arches BA, a 27. 3.

b hyp. CA are *b* equall) therefore the angles at
c 4 1. DGB, DGC are *c* equall, and *d* conſequent
d 10.def.1. But the inward angles GAE, GAF are alſo
e hyp. f 28 1. *f* therefore BC,EF are parallel. *Which was to*

PROP. XXVIII.

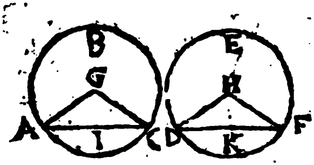


In equa
GABC,
equall rig
AC, DF
quall part
circumfer
greateſt
quall to t
eſt DEF, *and the leaſt* AIC *to the leaſt* DKF

From the centres G,H draw GA GC, & H
a hyp. Becauſe GA=HD, and GC=HF,and
b 28 1. DF, *b* therefore is the angle G=H; *c* wh
c 6. 3. arch AIC=DKF; *d* and ſo the remaining ar
d 3.ax. = DEF. *Which was to be Dem.*

But if the ſubtended line AC be ⊏ or
DF, then in like manner will the arch AC
⊐ then DF.

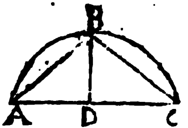
PROP. XXIX.



In equa
GABC,
equall ri
AC,DF *ſ*
quall p
ABC, DE
Draw t
GA,GC,and HD,HF. Becauſe GA=HD,
a hyp. = HF, and (becauſe the arches AC, D
b 27 3. quall) the angle G *b* = H. *c* therefore is
c 4. 1. AC = DF. *Which was to be Dem.*

This and the three precedent propoſitions
underſtood alſo of the ſame circle.

PROP. XXX.



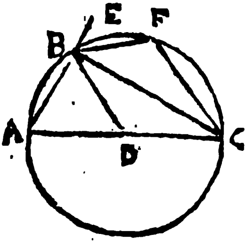
To cut a periphery given ABC *into two equall parts.*

Draw the right line AC, & biſect it in D; from D draw a perpendicular DB, meeting with the arch in B, it ſhall biſect the ſame.

For join AB, and CB. The ſide DB is common, and AD [a] = DC, and the angle ADB [b] = CDB. [c] therefore AB = BC; [d] whence the arch AB = BC. *Which was to be Done.*

a conſt.
b 12 ax.
c 4. 1.
d 28. 3.

PROP. XXXI.



In a circle the angle ABC, *which is in the ſemicircle, is a right angle; but the angle, which is in the greater ſegment* BAC *is leſſe then a right angle, and the angle which is in the leſſer ſegment* BFC *is greater then a right angle. Moreover, the angle of the greater ſegment is greater then a right angle, and the angle of the leſſer ſegment is leſſe then a right angle.*

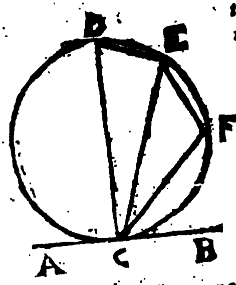
From the centre D draw DB. Becauſe DB = DA, therefore is the angle A [a] = DBA, and the angle DCB [a] = DBC, [b] therefore the angle +~~ABE~~ = A + ACB [c] = EBC, [d] ſo that ABC and EBC are right angles. *W. W. to be Dem.* [e] Therefore BAC is an acute angle. [f] *W. W. to be Dem.* And further, whereas BAC + BFC = 2 right, therefore BFC is an obtuſe angle. Laſtly the angle conteined under the right line CB, and the arch BAC is greater than the right angle ABC; but the angle made by the right line CB and the periphery of the leſſer ſegment BFC [g] is leſſe then the right angle ABC. *Which was to be Dem.*

a 5. 1.
b 2. ax.
c 32. 1.
d 10. def. 1. +AB
e cor. 17. 1.
f 22. 3.
; because the
angles in gre
fig: insc: in
are = to on
g 9. ax.

Schol.

In a right-angled triangle ABC, *if the hypotenuſe (or ſubtended line)* AC *be biſected in* D, *a circle drawn from the centre* D *through the point* A *ſhall alſo paſſe through the point* B: *As you may eaſily demonſtrate from this* prop. *and* 21. I.

PROP. XXXII.



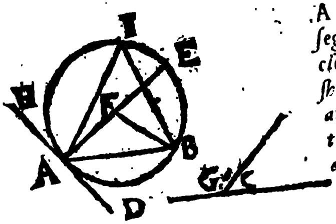
If a right line AB *touch a circle, and from the point of contact be drawn a right line* CE *cutting the circle, the angles* ECB, ECA *which it makes with the tangent line are equall to thoſe angles* EDC, EFC *which are made in the alternate ſegments of the circle.*

Let CD, the ſide of the angle EDC, be perpendicular to AB (a for it's to the ſame purpoſe) b therefore CD is the diameter. c therefore the angle CED in a ſemicircle is a right angle, d and therefore the angle D + DCE = to a right angle e = ECB + DCE. f Therefore the angle D = ECB. *Which was to be Dem.*

Now whereas the angle ECB+ECA g = 2 righ h = D+F, from both of theſe take away ECB an D, which are equall, k then remains ECA=F. *Whic was to be Dem.*

a 26 3.
b 19. 3.
c 31. 3.
d 32. 1.
e conſtr.
f 3 ax.
g 13. 1.
h 22 3.
k 3. ax.

PROP. XXXIII.

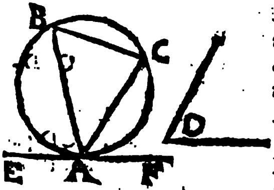


Vpon a right li
AB *to deſcribe*
ſegment of a c
cle AIEB *w*
ſhall contein
angle AIB *eq*
to a right-
angle given
a Make

a 23. 1.

angle BAD = C. Through the point A draw the line AE perpendicular to HD. At one end of the line given AB make an angle ABF = BAF, one ſide whereof let it cut the line AE in F; from the centre F through the point A, deſcribe a circle which ſhall paſſe through B (becauſe the angles FBA *b* = FAB, b conſtr.
and *c* therefore FB = FA,) AIB is the ſegment c 6. 1.
ſought. For becauſe HD is perpendicular to the diameter AE, it therefore *d* touches the circle HD which d cor 16. 3.
AB cuts. And therefore the angle AIB *e* = BAD *f* = e 32. 3.
C. *Which was to be done.* f conſtr.

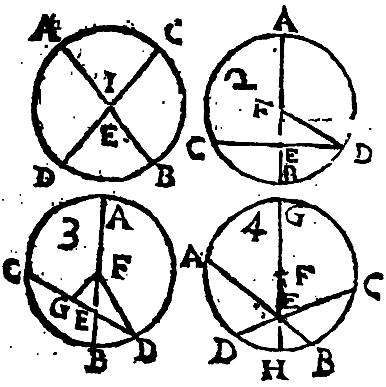
PROP. XXXIV.



From a circle given ABC *to cut off a ſegment* ABC *conteining an angle* B *equall to a right-lined angle given* D.

a Draw a right line EF which ſhall touch the circle given in A. a 17. 3.
b let AG be drawn alſo making an angle FAC b 23. 1.
= D. This line ſhall cut of ABC conteining an an- c 32. 3.
gle B *c* = CAF *d* = D. *Which was to be Done.* d conſtr.

PROP. XXXV.



If in a circle FBCA *two right lines* AB, DC *cut each other, the rectangle comprehended under the ſegments* AE, EB *of the*

ci.

one, ſhall be equall to the rectangle comprehended under the ſegments CE, ED *of the other.*

1.*Caſe.* If the right lines cut one the other in the centre, the thing is evident.

a 5. 2.
b ſch.4.2.
c 47. 1.
d hyp.
e 3.ax.

2.*Caſe.* If one line A B paſſe through the centre F, and biſect the other line CD, then draw FD. Now the rectangle AEB + FEq [a] = FBq [b] = FDq [c] = EDq + FEq [d] = CED + FEq. [e] Therefore the rectangle AEB = CED. *Which was to be Dem.*

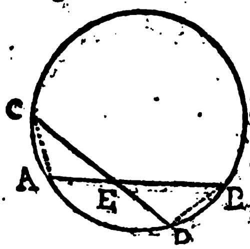
3.*Caſe.* If one of the lines AB be the diameter, and cut the other line CD unequally, biſect CD by FG a perpendicular from the centre.

f 5. 2.
g 47. 1.
h 5. 2.
k 47. 1.
l 3. ax.

All These are equall.
The rectangle AEB + FEq.
[f] FBq (FDq.)
[g] FGq + GDq.
FGq + [h] GEq + Rectang. CED,
[k] FEq + CED.

[l] Therefore the rectangle AEB = CED.

4.*Caſe.* If neither of ~~the right~~ lines AB, CD paſſe through the centre, then through the point of interſection E, draw the diameter G H. By that which hath been already demonſtrated, it appears that the rectangle AEB = GEH = CED. *W.W. to be Dem.*

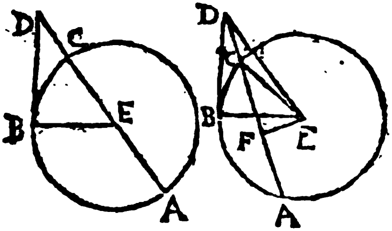


a 15. 1
b 21. 3.
c cor.32. 1.
d 4. 6.
e 16. 6.

More eaſily, and generally, thus; Join AC and BD, Then becauſe the angles [a] CEA, DEB; and [b] alſo C, B (upon the ſame arch AD) are equall, thence are the triangles CEA, BED [c] equiangular. [d] Wherefore CE, EA :: EB, ED. and [e] conſequently CE x ED = AE x EB. *Which was to be Dem.*

The citations out of the 6. Book, both here and in the following *prop.* have no dependance upon the ſame; ſo that it was free to uſe them.

PROP. XXXVI.



If any point D be taken without a circle EBC, and m that point two right lines DA, DB fall upon the rcle, whereof one DA cut the circle, the other DB uches it, the rectangle comprehended under the whole e DA that cuts the circle, and under DC that part hich is taken from the point given D to the convexe of e periphery, ſhall be equall to the ſquare made of the ngent line.

1. *Caſe.* If the ſecant AD paſſe through the centre, en join EB, this [a] will make a right angle with the ne DB, wherefore DBq + [d] EBq (ECq) [b] = EDq = AD x DC + ECq. Therefore AD x DC = Bq. *Which was to be Dem.*

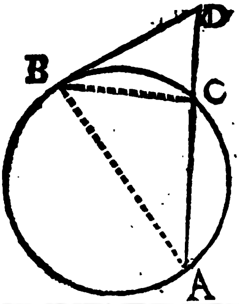
a 18. 3.
b 47. 1.
c 6. 2.
d 3. ax.

2. *Caſe.* But if AD paſſe not through the centre en draw EC, EB, ED, and EF perpendicular to D, [a] wherefore AC is biſected in F.

Becauſe BDq + EBq [b] = DEq [b] = EFq + Dq [c] = EFq + ADC + FCq [d] = ADC + CEq EBq.) [e] Therefore is BDq = ADC. *Which was to be* em.

a 3. 3.
b 47. 1.
c 6. 2.
d 47. 1.
e 3. ax.

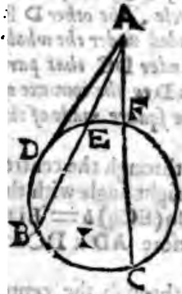
More



a 32. 3. More eaſily, and generally, thus; Draw AB and BC. Then becauſe the angles A, and DBC[a] are equall, and the angle D common to both, thence are the triangles BDC, A D B[b] equiangular. [c] Wherefore AD. DB :: DB. CD. and[d] conſequently $AD \times DC = DBq$. *Which was to be Dem.*

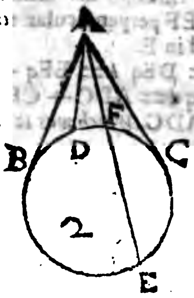
b 32. 1.
c 4. 6.
d 17. 6.

Coroll.



1. Hence, If from any point A taken without a circle, there be ſeverall lines AB, AC drawn which cut the circle, the rectangles comprehended under the whole lines AB, AC, and the outward parts AE, AF are equall between themſelves.

For if the tangent AD be drawn, then is $CAF = ADq$[a] $= BAE$. 36. 3.



2. It appears alſo from hence, that if two lines AB, AC drawn from the ſame point do touch a circle, thoſe two lines are equall one to the other.

For if AE be drawn cutting the circle, then is ABq[a] $= EAF$[b] $= ACq$.

a 36. 3.
b 36. 3.

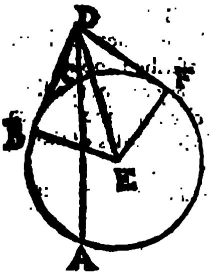
3. It is alſo evident that from a point A taken without a circle, there can be drawn but two lines AB, AC that ſhall touch the circle.

For if a third line AD be ſaid to touch the circle,
thence is AD $c=$ AB $c=$ AC. *d Which is abſurd.* c 2. cor.
d 8. 3.

4. And on the contrary, it is plain, that if two equall right lines AB, AC fall from any point A upon the convexe periphery of a circle, and that if one of theſe equall lines AB touch the circle, then the other AC touches the circle alſo.

For if poſſible, let not AC, but another line AD,
touch the circle; therefore is AD $e=$ AC $f=$ AB. e 2. cor.
f hyp.
g Which is abſurd. g 8. 3.

PROP. XXXVII.



If without a circle EBF *any point* D *be taken, and from that point two right lines* DA, DB *fall on the circle, whereof one line* DA *cuts the circle, the other* DB *falls upon it; and if alſo the rectangle comprehended under the whole line that cuts the circle, and under that part of it* DC *which is taken betwixt the point* D *and the convexe periphery, be equall to that ſquare which is made of the line* DB *falling on the circle, I ſay that that line* DB *ſo falling ſhall touch the circle given.*

From the point D *a* let a tangent DF be drawn, a 17. 3.
and from the centre E draw ED, EB, EF. Now be-
cauſe DBq $b=$ ADC $c=$ DFq, therefore is DB b hyp.
$d=$ DF: But EB = EF, and the ſide ED common; c 36. 3.
e therefore the angle EBD = EFD. but EFD is a d 1. ax. & ſch. 41. 1.
right angle, and *f* therefore EBD is right alſo. and e 8. 1.
g therefore DB touches the circle. *W. W. to be Dem.* f 12. ax.
g cor. 16. 3.

Coroll.

From hence it follows that the *h* angle EDB = h 8. 1.
EDF.

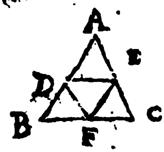
The End of the third Book.

THE

THE FOURTH BOOK
OF
EUCLIDE'S ELEMENT

Definitions.

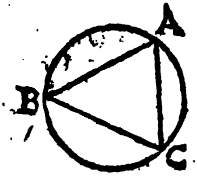
I. A Right-lined figure is ſaid to be i ſcribed in a right-lined figure, wh every one of the angles of the i ſcribed figure touch every one of t ſides of the figure wherein it is i ſcribed.

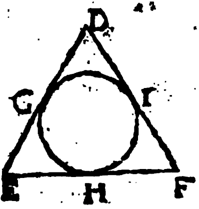


So the triangle DEF *is inſcrib in the triangle* ABC.

II. In like manner a figu is ſaid to be deſcribed about a gure, when every one of the ſid of the figure circumſcribed tou every one of the angles of the gure about which it is circumſcribed.

So the triangle ABC *is deſcribed about the tria* DEF.



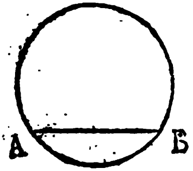


III. A right-lined figure is ſaid to be inſcribed i a circle, when all the angles of that figure which i inſcribed do touch the circumference of the circle.

IV. A right-lined figure is ſaid to be deſcribe about a circle, when all the ſides of the figu which is circumſcribed touch the periphery of the circle.

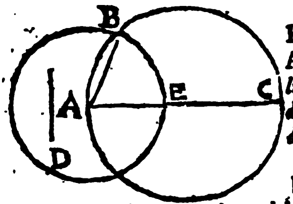
V. After the like manner a circle is ſaid to be inſcribed in a right-lined figure, when the periphery of the circle touches all the ſides of the figure in which it is inſcribed.

VI. A circle is ſaid to be deſcribed about a figure when the periphery of the circle touches all the angles of the figure, which it circumſcribes.



VII. A right line is ſaid to be coapted or applyed in a circle when the extremes thereof fall upon the circumference; *as the right line* AB.

PROP. I. *Probl.* 1.

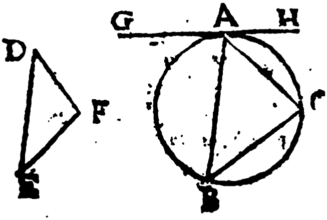


In a circle given ABC *to apply a right line* AB *equall to a right line given* D, *which doth not exceed* AC *the diameter of the circle.*

From the centre A by the ſpace AE = D a deſcribe a circle meeting with the circle given in B. draw AB. Then is AB b = AE c = D. *W.W. to be Done.*

a 3 *poſt. and* 3. 1.
b 15 *def.* 1.
c *conſtr.*

PROP. II. *Probl.* 2.



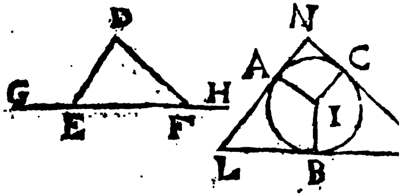
In a circle given ABC *to deſcribe a triangle* ABC, *equiangular to a triangle given* DEF.

Let the right

a 17. 3. right line GH [a] touch the circle given in A
b 23. 1. the angle HAC = E, [b] and the angle GA
then join BC; and the thing is done.

c 32. 3. For the angle B [c] = HAC [d] = E, and t
d conſtr. C [c] = GAB [d] = F; [e] whence alſo the ang
e 32. 1. = D. Therefore the triangle BAC inſcrib
circle is equiangular to DEF. *Which was to be*

PROP. III.



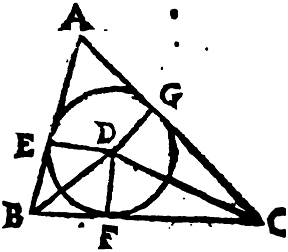
About a circle given IABC *to deſcribe a*
LNM *equiangular to a triangle given* DEF.

Produce the ſide EF on both ſides; at the
a 23. 1. [a] make an angle AIB = DEG, and an an
= DFH. Then in the points A, B, C let th
b 17. 3. lines LN, LM, NM [b] touch the circle, and t
is done.

For it's evident that the right lines LN, L
will meet and make a triangle, [c] becauſe th
c [illegible] LAI, LBI are right; ſo that the [d] right line
d 11. 3. duced will make the angles LAB, LBA, leſſ
right angles.

Since, therefore the angle AIB + L [e] =
e ſchol. 32. 1. angles [f] = DEG + DEF, and AIB [g] =
f 13. 1. [h] therefore is the angle L = DEF. By the l
g conſtr. of argument the angle M = DEF. [k] therefore
h 3. ax. angle N = D. And therefore the triangle L
k 32. 1. ſcribed about the circle is equiangular to E
triangle given. *Which was to be done.*

PROP. IV.



In a triangle given ABC, *to deſcribe a circle* EFG.

a Biſect the angles B and C with the right lines BD, CD meeting in the point D, b & draw the perpendiculars DE, DF, DG. A circle deſcribed from the centre D through E, will paſſe through G and F, and touch the three ſides of the triangle. a 9. 1. b 12. 1.

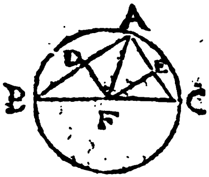
For the angle $DBE \overset{c}{=} DBF$; and the angle $DEB \overset{d}{=} DFB$; and the ſide DB common. e therefore $DE = DF$. By the like argument $DG = DF$. The circle therefore deſcribed from the centre D paſſes through the 3 points E, F, G. and whereas the angles at E, F, G are right, therefore it touches all the ſides of the triangle. *Which was to be done.* c *conſtr.* d 12 *ax.* e 26. 1.

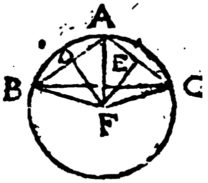
Schol.

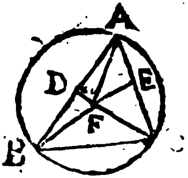
Hence, *The ſides of a triangle being known, their ſegments which are made by the touchings of the circle inſcribed ſhall be found,* Thus; *Pet. Herig.*

Let AB be 12, AC 18, BC 16. then is $AB + BC = 28$. Out of which ſubduct $18 = AC = AE + FC$, then remains $10 = BE + BF$. Therefore BE, or $BF = 5$; and conſequently FC, or $CG, = 11$. Wherefore GA, or AE, $= 7$.

PROP. V.







About a triangle given ABC *to deſcribe a c* FABC.

a 10. & 11. 1. a Biſect any two ſides BA, CA with perpend lars DF, EF meeting in the point F. I ſay this ſ be the centre of the circle.

For, let the right lines FA, FB, FC be dra Now becauſe AD b = DB and the ſide DF co mon, and the angles FDA c = FDB, therefor FB d = FA. After the ſame manner is FC = Therefore a circle deſcribed from the centre F ſ paſſe through the angles of the triangle given (o B, A, C. *Which was to be done.*

b conſtr.
c conſtr. and 12. ax.
d 4. 1.

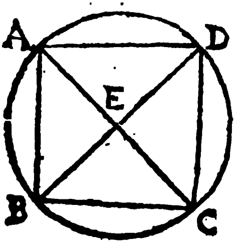
Coroll.

e 31. 3. e Hence, if a triangle be acute-angled, the ce ſhall fall within the triangle; if right-angled, in ſide oppoſite to the right angle, & if obtuſe-ang without the triangle.

Schol.

By the ſame method may a circle be deſcrib that ſhall paſſe through 3 points given, not bein the ſame ſtrait line.

PROP. VI.

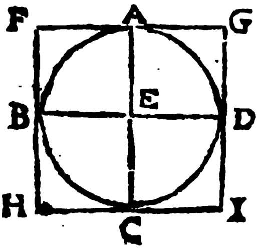


In a circle given EABCD *to inſcribe a ſquare* ABCD.

[a] Draw the diameters AC, BD cutting each other at right angles in the centre E. Join the extremes of theſe diameters with the right lines AB, BC, CD, DA. *And the thing is done.* a 11. 1.

Now becauſe the 4 angles at E are right, the [b] arches and [c] ſubtended lines AB, BC, CD, DA are b 26. 3.
equall; therefore is the figure ABCD equilateral, c 29. 3.
and all the angles in ſemicircles, and ſo [d] right. d 31. 3.
[e] Therefore ABCD is a ſquare inſcribed in a circle e 29. 1.def. 1.
given. *Which was to be done.*

PROP. VII.



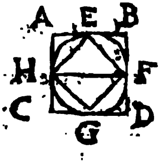
About a circle given EABCD *to deſcribe a ſquare* FHIG.

Draw the diameters AC, BD cutting one the other at right angles; through the extremes of theſe diameters [a] draw tangents meeting in F, H, I, G then I ſay it's done. a 17. 3.

For becauſe [b] the angles A and C are right, [c] therefore is FG parallel to HI. After the ſame manner is FH parallel to GI, and therefore FHIG is a Pgr. and alſo right-angled. It is equilaterall becauſe FG [d] = HI [d] = DB [e] = CA [d] = FH [d] = GI. Wherefore FHIG is a [f] ſquare deſcribed about the circle given. *Which was to be done.*

b 18. 3.
c 28. 1.
d 34. 1.
e 15. def. 1.
f 29. def. 1.

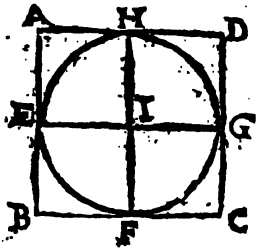
Schol.



A ſquare ABCD deſcribed about a circle is double of the ſquare EFGH inſcribed in the ſame circle.

For the rectangle HB = 2 HEF, and HD = 2 HGF by the 41. 1.

PROP. VIII.



In a ſquare given ABCD *to inſcribe a circle* IEFGH.

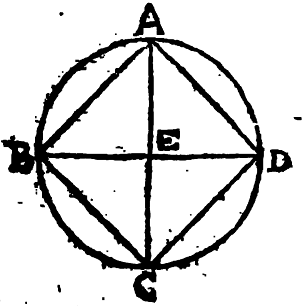
Biſect the ſides of the ſquare in the points H, E, F, G cutting one the other in I. a circle drawn from the centre I through H ſhall be inſcribed in the ſquare.

For becauſe AH and BF are [a] equall and [b] parallel, [c] therefore is AB parallel to HF parallel to DC. After the ſame manner is AD parallel to EG, parallel to BC; therefore IA, ID, IB, IC are parallelograms. Therefore AH [d] = AE [e] = HI = EI = FI = IG. The circle therefore deſcribed from the centre I through H ſhall paſſe through H, E, F, G and touch the ſides of the ſquare being the angles H, E, F, G are right. *Which was to be done.*

[a] 7. ax.
[b] hyp.
[c] 33. 1.
[d] 7. ax.
[e] 34. 1.

PROP.

PROP. IX.

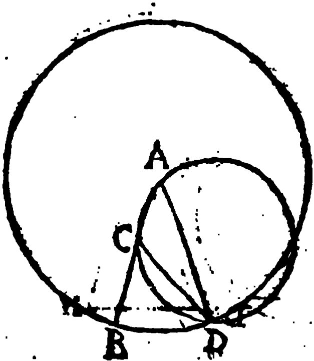


About a ſquare given ABCD *to deſcribe a circle* EABCD.

Draw the diameters AC, BD cutting one the other in E. From the centre E through A deſcribe a circle, then I ſay that circle is deſcribed about the ſquare.

For the angles ABD and BAC are a half of right angles, b therefore EA = EB. After the ſame manner is EA = ED = EC. The circle therefore deſcribed from the centre E paſſes through A, B, C, D the angles of the ſquare given. *Which was to be done.* a 4. cor. 32. 1. b 6. 1.

PROP. X.



To make an Iſoſceles triangle BD, *having each angle at the baſe* B *and* DB *double to the remaining angle* A.

Take any right line AB, and divide it in C, a ſo that AB x BC may be equall to ACq. From the centre A through B, deſcribe the circle ABD; and in this circle b apply BD = AC, and join AD, I ſay ABD is the triangle required. a 11. 2. b 1. 4.

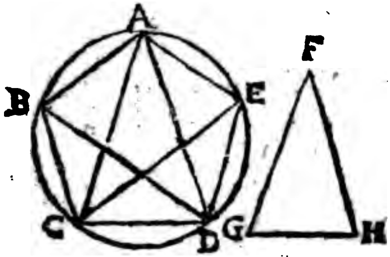
For, draw DC, and through the points C, D, A.
c 5. 4. *c* draw a circle. Now becauſe $AB \times BC = ACq$, *d* it
d 37. 3. is evident that BD touches the circle ACD which
CD cutteth; e 32. 3. *e* therefore is the angle $BDC = A$, and
f 2.ax. therefore the angle $BDC + CDA \overset{f}{=} A + CDA$
g 32. 1. h 5. 1. $\overset{g}{=} BCD$. But $BDC + CDA = BDA \overset{h}{=} CBD$.
k 1.ax. *k* therefore the angle $BCD = CBD$; and therefore
l 6. 1 m constr. $DC \overset{l}{=} DB \overset{m}{=} AC$, *n* wherefore the angle CDA
n 5 1. $= A = BDC$. therefore $ADB = 2A = ABD$.
Which was to be done.

This conſtruction is Analytically found out thus;
Take the thing for done, and let the right line DC
a 3. 6 biſect the angle BDA; *a* therefore DA, DB :: CA,
b constr. CB. alſo becauſe the angle $CDA \overset{b}{=} \frac{1}{2} ADB \overset{c}{=}$
c hyp. d 6. 1. A, *c* therefore $CA = DC$. and becauſe the angle
e 32. 1. $DCB \overset{e}{=} A + CDA = 2A \overset{c}{=} B$, *d* thence will
f 2.ax. be $DB = DC$. *f* from whence alſo $DB = CA$. and
g 17. 6 ſo DA (BA.) CA :: CA, CB. *g* whence $BA \times CB =$
CAq.

Coroll.

b 32. 1. Whereas all the angles A, B, D *b* make up two right angles, it's evident that A is $\frac{1}{5}$ of two right angles.

P R O P. XI.



In a circle given ABCDE *to deſcribe a Pentagone figure* ABCDE *equilateral and equiangular.*

a 10. 4. *a* Deſcribe an Iſoſceles triangle FGH, having each
b 2. 4. angle at the baſe double to the other; *b* inſcribe a triangle

triangle CAD equiangular to the ſaid triangle FGH.[c] Biſect the angles at the baſe ACD & ADC with the right lines DB, CE meeting with the circumference in B and E. join the right lines CB, BA, AE, ED. *Then I ſay it is done.* c 9. 1.

For it is evident by conſtruction that the angles CAD, CDB, BDA, DCE, ECA are equall; wherefore the [d] arches and [e] ſubtended lines DC, CB, BA, AE, DE are equall. Therefore the Pentagone is equilateral, and equiangular [f] becauſe the angles of it BAE, AED, &c. ſtand on equall [g] arches BCDE, ABCD, &c.

d 26. 1.
e 29. 3.
f 27. 3.
g 2.ax.

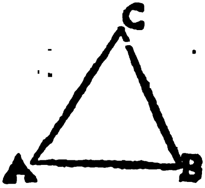
A more eaſy practiſe of this probleme ſhall be deliver'd at 10. 13.

Coroll.

Hence, Each angle of an equilateral and equiangular Pentagone is equall to $\frac{3}{5}$ of two right angles, or $\frac{6}{5}$ of one right angle.

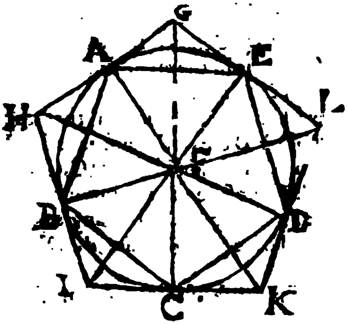
Schol.

Generally all figures of odd number of ſides are inſcribed in circles by the help of Iſoſceles triangles, whoſe angles at the baſe are multiplices of thoſe at the top: and figures of even number of ſides are inſcribed in a circle by the help of Iſoſceles triangles, whoſe angles at the baſe are multiplices ſeſquialter of thoſe at the top. Petr. Herig.



As in the Iſoſceles triangle CAB if the angle A = 3 C = B. then will AB be the ſide of a Heptagone. If A = 4 C; then is AB the ſide of an Enneagone. But if A = $\frac{1}{2}$ C then is AB the ſide of a^{2} ſquare. And if A = 2 $\frac{1}{2}$ C AB will ſubtend the ſixt part of a circumference and likewiſe if A = 3 $\frac{1}{2}$ then will AB be the ſide of an Octagone.

PROP. XII.



About a circle given FABCDE,*to deſcribe an e*
teral and an equiangular pentagone HIKLG.

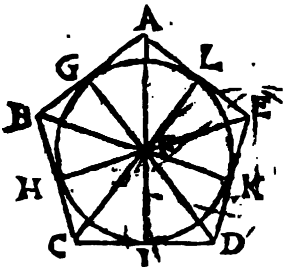
a 11. 4. a Inſcribe a pentagone ABCDE in the circl
ven; and from the centre draw the right line
FB, FC, FD, FE; and to thoſe lines draw ſo
perpendiculars GAH, HBI, ICK, KDL, LEG
ing in the points H, I, K, L, G. then I ſay it is
b cor. 16 13. For becauſe GA, GE from the ſame point G b
c 2 cor. 36 3. the circle, c therefore is GA = GE, and d the
d 8. 1. the angle GFA = GFE, therefore the angle
= 2 GFA. After the ſame manner is the angle
= FHB, and conſequently the angle AFB
e 27. 3. AFH. e But the angle AFE = AFB, f therefor
f 7. ax. angle GFA = AFH. But alſo the angle FAH
g 12. ax. FAG; and the ſide FA is common, h therefor
h 26. 1. = AG = GE. = EL, &c. k Therefore HG
k 2. ax. LK, KI, IH the ſides of the pentagone are equa
angles alſo are, becauſe double of the equall a
AGF, AHF. therefore, &c.

Coroll.

After the ſame manner, if any equilateral
quiangled figure be deſcribed in a circle, and
extreme points of the ſemi-diameters drawn
the centre at angles, be drawn perpendicular li
the ſaid diameters, I ſay that theſe perpendi

ſhall make another figure of as many equall ſides and equall angles, deſcribed about the circle.

PROP. XIII.



In an equilateral and equiangular pentagone given ABCDE *to inſcribe a circle* FGHK.

a Biſect two angles of the pentagone A and B with the right lines AF, BF meeting in the point F. From F draw the perpendiculars FG, FH, FI, FK, FL. Then a circle deſcribed from the centre F through G will touch all the ſides of the pentagone. a 9.1.

Draw FC, FD, FE. Becauſe BA b = BC and the ſide BF common, and the angle FBA c = FBC, d therefore is AF = FC and the angle FAB = FCB, but the angle FAB e = $\frac{1}{2}$ BAE = $\frac{1}{2}$ BCD. Therefore the angle FCB = $\frac{1}{2}$ BCD. After the ſame manner are all the whole angles C, D, E biſected. Now whereas the angle FGB f = FHB, and the ang. FBH = FBG & the ſide FB is common, g therefore is FG = FH. In like manner are all the right lines FH, FI, FK, FL, FG equal. Therefore a circle deſcribed from the centre F through G paſſes through the points H, I, K, L and h touches the ſides of the pentagone becauſe the angles at thoſe points are right. *Which was to be done.* b hyp. c conſtr. d 4.1. e hyp. f 12.ax. g 26.1. h cor.16.3.

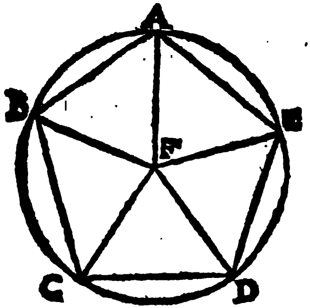
Coroll.

Hence, If any two neareſt angles of an equilateral and equiangular figure, and from that point in which the lines meet that biſect the angles be drawn right lines to the remaining angles of the figure, all the angles of the figure ſhall be biſected.

Schol.

By the ſame method ſhall a circle be inſcribed in any equilateral and equiangular figure.

PROP. XIV.



About a pentagone given ABCDE *equilateral and equiangular to deſcribe a circle* FABCDE.

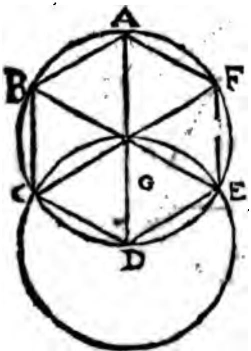
Biſect any two angles of the pentagone with the right lines AF, BF meeting in the point F; the circle deſcribed from the centre F through A ſhall be deſcribed about the pentagone.

a per 13. 4. b 6. 1. For let FC, FD, FE be drawn. [a] Then the angles C, D, E are biſected; [b] & therefore FA, FB, FC, FD, FE are equall; therefore the circle deſcribed from the centre F paſſes through A, B, C, D, E all the angles of the pentagone. *Which was to be done.*

Schol.

By the ſame art is a circle deſcribed about any figure which is equilateral and equiangular.

PROP. XV.



In a circle given GABCDEF *to inſcribe an Hexagone (or ſix-ſided figure) equilateral .and equiangular* ABCDEF.

Draw the diameter AD; from the centre D through the centre G deſcribe a circle cutting the circle given in the points C and E. Draw the diameters CF, EB; and join AB, BC, CD, DE, EF, FA. *Then I ſay it's done.*

For the angle CGD [a] $=$ $\frac{1}{3}$ of 2 right [a] $=$ DGE [a] 32. 1.
[b] $=$ AGF [b] $=$ AGB. [c] Therefore BGC $=$ [b] 15. 1.
$\frac{1}{3}$ of 2 right $=$ FGE; therefore the [d] arches and [e] sub- [c] cor. 13. 1.
tenſes AB, BC, CD, DE, EF are equall. Therefore [d] 26. 3.
the Hexagone is equilateral; but it is equiangled al- [e] 29. 3.
ſo, [f] becauſe all the angles of it ſtand upon equall [f] 27. 3.
arches.

Coroll.

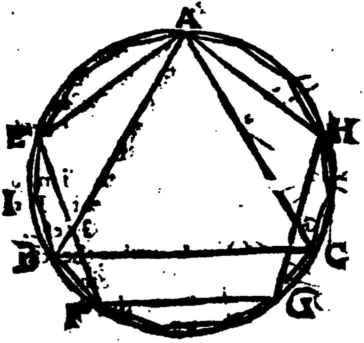
1. Hence, The ſide of an Hexagone inſcribed in a circle is equall to the ſemidiameter.

2. Hereby an equilateral triangle ACE may very eaſily be deſcribed in a circle given.

Schol.

To make a true Hexagon upon a right line given CD. *Andr. Tacq.*
[a] Make an equilateral triangle CGD upon the [a] 1. 1.
line given CD; from the centre G through C and D
deſcribe a circle. That circle ſhall contein the
Hexagone made upon the given line CD.

PROP. XVI.



In a circle given AEBC *to inſcribe a quindecagone (or fifteen-ſided figure) equilateral and equiangular.*

a 11.4. b 2.4. [a] Inſcribe an equilateral pentagone AEFGH in the circle given, and [b] alſo an equilateral triangle ABC, then I ſay BF is the ſide of the quindecagone required.

c conſtr. For the arch AB [c] is $\frac{1}{3}$ or $\frac{5}{15}$ of that periphery whereof AF is $\frac{2}{5}$ or $\frac{6}{15}$. therefore the remaining part BF is $\frac{1}{15}$ of the periphery; and therefore the quindecagone, whoſe ſide is BF, is equilateral; but it is equiangu-

d 27.3. lar alſo [d] becauſe all the angles inſiſt on equall arches of a circle, whereof every one $\frac{13}{15}$ of the whole circumference. Therefore, &c.

Schol.

A circle is geometrically divided into parts
- 4, 8, 16, &c. by 6, 4, and 9, 1.
- 3, 6, 12, &c. by 15, 4, and 9, 1.
- 5, 10, 20, &c. by 11, 4, and 9, 1.
- 15, 30, 60, &c. by 16, 4, and 9, 1.

Any other way of dividing the circumference into any parts given is as yet unknown; wherefore in the conſtruction of ordinate figures, we are forced to have recourſe to mechanick artifices, concerning which you may conſult the writers of practical Geo*metry.*

THE

THE FIFTH BOOK OF EUCLIDE'S ELEMENTS.

Definitions.

I. A Part, is a magnitude of a magnitude, a leſſe of a greater, when the leſſe meaſureth the greater.

II. Multiplex is a greater magnitude in reſpect of a leſſer, when the leſſer meaſureth the greater.

III. Ratio (or rate) is the mutuall habitude or reſpect of two magnitudes of the ſame kind each to other, according to quantity.

In every ratio that quantity which is referred to another quantity is called the antecedent of the ratio, and that to which the other is referrd is called the conſequent of the ratio. as in the ratio of 6 to 4, 6 is the antecedent and 4 the conſequent.

Note: *The quantity of any ratio is known by dividing the antecedent by the conſequent; as the ratio of 12 to 5 is expreſſed by* $\frac{12}{5}$; *or the quantity of the ratio of* A *to* B *is* $\frac{A}{B}$. *Wherefore often for brevity ſake we denote the quantities of rations thus;* $\frac{A}{B} \sqsubset$, or $=$ or $\sqsupset \frac{C}{D}$. *that is, the ratio of* A *to* B *is greater, equall, or leſſe then the ratio of* C *to* D. *And this note muſt be diligently obſerved in the underſtanding of the following Book.*

Concerning the diverſe ſpecies of ratio's, you may pleaſe to conſult interpreters.

IV. Proportion is a ſimilitude of ratio's.

That which is here termed proportion, *is more rightly called* proportionality *or* analogy; *for propor-*

tion commonly denotes no more then the ratio betwixt two magnitudes.

V. Thoſe numbers are ſaid to have a ratio betwixt them, which being multiplyed may exceed one the other.

E,12.	A,4. B,6.	G,24.
F,30.	C,10. D,15.	H,60.

VI. Magnitudes are ſaid to be in the ſame ratio, the firſt A to the ſecond B, and the third C to the fourth D, when the equimultiplices E and F of the firſt A, and the third C compared with the equimultiplices G, H of the ſecond B and the fourth D, according to any multiplication whatſoever, either both together E, F are leſſe then GH both together, or equall taken together, or exceed one the other together, if thoſe be taken E, G and F, H, which anſwer one to the other.

The note hereof is :: ; *as* A.B :: C.D. *That is, as* A *is to* B, *ſo is* C *to* D. *which ſignifyes that* A *to* B, *and* C *to* D *are in the ſame ratio. We ſometimes thus expreſſe it;* $\frac{A}{B} = \frac{C}{D}$. *that is* A.B :: C.D.

VII. Magnitudes that have the ſame ratio (A.B :: C.D. are called proportional.

E,30.	A, 6. B, 4.	G, 28.
F,60.	C,12, D,9.	H,6.3.

VIII. When of equimultiplices, E the multiplex of the firſt magnitude A exceeds the G the multiplex of the ſecond B, but F the multiplex of the third C exceeds not H the multiplex of the fourth D, then the firſt A to the ſecond B has a greater ratio then the third C to the fourth D.

If $\frac{A}{B} \sqsubset \frac{C}{D}$, *it is not neceſſary from this definition that* E *ſhould alwayes exceed* G, *when* F *is leſſe then* H; *but it is granted that this may be.*

IX. Proportionality conſiſt in three termes at leaſt. *Whereof the ſecond ſupplyes the place of two.*

X. When 3 magnitudes A, B, C are proportional, the firſt A ſhall have a duplicate ratio to the 3 C of that it hath to the ſecond B: But when four magnitudes A, B, C, D are proportional, the firſt A ſhall have a triplicate ratio to the fourth D of what it had to the ſecond B; and ſo alwayes in order one more, as the proportion ſhall be extended.

Duplicate ratio is thus expreſſed $\frac{A}{C} = \frac{A}{B}$ *twice, that is, the ratio of* A *to* C *is double of the ratio of* A *to* B. *Triple ratio is thus expreſſed;* $\frac{A}{D} = \frac{A}{B}$ *thrice. That is, the ratio of* A *to* D *is triple of the ratio of* A *to* B.

∺ *denotes continued proportionals; as* A, B, C, D; *or* 2, 6, 18, 64. *are* ∺

XI. Magnitudes of a like ratio, are antecedents to antecedents, and conſequents to conſequents; *As if* A. B :: C.D. A *and* C; *and* B *and* D *are homologous or magnitudes of a like ratio.*

XII. Alternate proportion is the comparing of antecedent to antecedent, and conſequent to conſequent. *As if* A. B :: C.D. *therefore alternately, or by permutation,* A.C :: B.D. *by the* 16. *of* 5.

In this definition, and the 5 *following, names are given to the ſixe wayes of arguing which are often uſed by Mathematicians: the force of which inferences depends on the propoſitions of this book, which are named in their explications.*

XIII. Inverſe ratio is when the conſequent is taken as the antecedent, and ſo compared to the antecedent as the conſequent; *as* A.B :: CD. *therefore generally* B. A :: D.C. *by cor.*4. 5.

XIV. Compounded ratio is when the antecedent and conſequent taken both as one are compared to the conſequent it ſelf. *As* A.B :: C. D. *therefore by compoſition* A + B.B :: C + D.D *by* 18. 5.

XV. Divided ratio is when the exceſſe wherein the antecedent exceedeth the conſequent, is compared to the conſequent. *As* A.B :: C.D. *therefore by diviſion* A - B.B :: C - D.D. *by* 17. 5.

XVI. Converſe ratio is when the antecedent is compared to the exceſſe wherein the antecedent exceeds the conſequent. *As* A.B :: C.D. *therefore by converſe ratio.* A. A - B :: C. C - D. *by the coroll. of the* 19. *of the* 5.

XVII. Proportion of equality is where there are taken more magnitudes then two in one order, and alſo as many magnitudes in another order, comparing two to two being in the ſame ratio; it commeth to paſſe that as in the firſt order of magnitudes, the firſt is to the laſt, ſo in the ſecond order of magnitudes is the firſt to the laſt. Or otherwiſe: it is a compariſon of the extremes together, the mean magnitudes being taken away.

XVIII. Ordinate proportionality is, when, as the antecedent is to the conſequent, ſo is the antecedent to the conſequent, and as the conſequent is to any other, ſo is the conſequent to any other. *As* A.B :: D.E. *alſo* B.C :: E.F. *it ſhall be true alſo* A.C :: D.F *by the* 22. *of the* 5.

XIX. Inordinate proportion is, when, three magnitudes being put, and others alſo, which are equall to theſe in multitude, as in the firſt magnitudes the antecedent is to the conſequent, ſo in the ſecond magnitudes is the antecedent to the conſequent: and as in the firſt magnitudes the conſequent is to any other, ſo in the ſecond magnitudes any other thing to the antecedent. *As* A.B :: F.G. *alſo* B.C :: E.F, *it ſhall be true in inordinate proportion:* A.C :: E.G. *by the* 23. *of the* 5.

XX. Any number of magnitudes being put; the proportion of the firſt to the laſt is compounded out of the proportions of the firſt to the ſecond, the ſecond to the third, and the third to the fourth, and ſo forwards till the proportion ariſe.

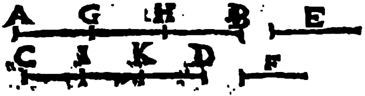
Let there be any number of magnitudes A, B, C, D. by this definition $\frac{A}{D} = \frac{A}{B} + \frac{B}{C} + \frac{C}{D}$.

Axiome.

Axiome.

Magnitudes equimultiplices to the ſame multi-
x, are alſo equimultiplices betwixt themſelves.

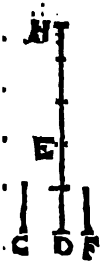
PROP. I.



If there be a number of magnitudes how manyſoever,
B, CD *equimultiplices to a like number of magni-*
es E, F *each to other; how multipex one magnitude*
is to one E, *ſo multiplices are all the magnitudes*
+CD *to all the other magnitudes* E+F.

Let AG, GH, HB the parts of the quantity AB,
equall to E, and alſo let CI, IK, KD the parts of
quantity CD be equall to F. The number of
ſe are put equall to thoſe. Now whereas AG+CI
= E+F; and GH+IK = E+F; and HB+ a 2 ax.
= E+F, it is evident that AB+CD doth ſo
en contain E+F as one AB conteins E. *Which was*
e done.

PROP. II.

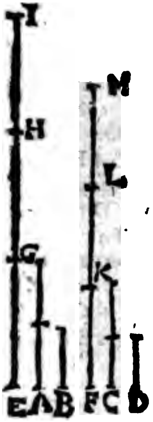


If the firſt AB *be equimultiplex to the ſecond* C, *as the third* DE *is to the fourth* F, *and if the fifth* BG *be equimultiplex to the ſecond* C *as the ſixth* EH *is to the fourth* F; *then ſhall the firſt compounded with the fifth* (AG) *be equimultiplex to the ſecond* C, *as the third compounded with the ſixth* (DH) *is to the fourth* F.

The number of parts in AB equall each to C is put equall to the numbers of parts in DE, whereof each part is equall to F. Likewiſe the
nber of parts in BG is put equall to the number
arts in EH. Therefore the number of parts in
+BG is equall to the number of parts in DE+
EH.

e 2.ax. EH.e That is, the whole line AG is as equimultiplex of C, as the whole line DH is of F. *Which was to be Dem.*

PROP. III.

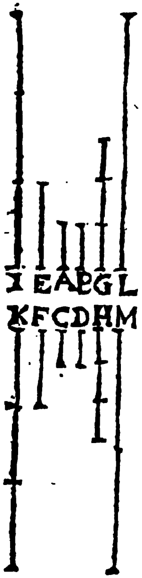


If the first A *be equimultiplex of the second* B, *and the third* C *of the fourth* D, *and there be taken* EI, FM *equimultiplices of the first and third, then will each of the magnitudes taken be alike equimultiplex of both, the one* EI *to the second* B, *the other* FM *to the fourth* D.

Let EG, GH, HI the parts of the multiplex EI be equall to A, also let FK, KL, LM the parts of the multiplex FM be equall to F, a hyp. a the number of these is equall to the number of those. Moreover A (that is) E G or G H or H I is put as equimultiplex of B, as C, b 2.5. or FK, &c. of D. b Therefore EG + GH is equimultiplex of the second B, as FK + KL is of the fourth c 2.5. D. c By the same way of argument is EI (EH+HI) as multiplex of B, as FM (FL+LN) is of D. *Which was to be done.*

PROP.

PROP. IV.



If the firſt A *have the ſame ratio to the ſecond* B, *as the third* C *to the fourth* D; *then alſo* E *and* F *the equimultiplices of the firſt* A *and the third* C, *ſhall have the ſame ratio to* G *and* H *the equimultiplices of the ſecond* B *and the fourth* D, *according to any multiplication, if ſo taken as they anſwer each to other* (E. G :: F. H.)

Take I and K the equimultiplices of E and F; and alſo L and M the equimultiplices of G & H. *a* Then is I as multiplex of A, as K of C; *a* and alſo L is as multiplex of B, as M of D. Therefore whereas it is A. B *b* :: C. D; according to the ſixt definition, if I be ⊏, =, ⊐ L, then conſequently after the ſame manner is K ⊏, =, ⊐ M. Therefore when I and K are taken as multiplices of E and F, as L and M of G, & H, then will it be by the 7 definition E. G :: F. H. *Which was to be Dem.*

a 3.5.
b hyp.

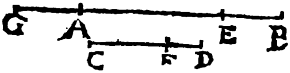
Coroll.

From hence is wont to be demonſtrated the proofe of inverſe ratio.

For becauſe A. B :: C. D, therefore if E ⊏, =, ⊐ G, then is *c* likewiſe F ⊏ = ⊐ H. therefore it is evident that if G ⊏, =, ⊐ E, then is H ⊏, =, ⊐ F; *d* therefore B. A :: D. C. *Which was to be Dem.*

c 1.def.5.
d 6.def.5.

PROP. V.

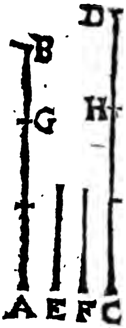


If a magnitude AB *be as multiplex of a magnitude* CD, *as a part taken from the one* AE *of a part taken from the other* CF; *the reſidue of the one* EB *ſhall be as multiplex of the reſidue of the other* FD *as the whole* AB *is of the whole* CD.

Take any other GA, which ſhall be as multiplex to FD the reſidue, as AB is of the whole CD, or as the part taken away AE is of the part taken away CF. *a* Therefore the whole GA + AE is as multiplex of the whole CF + FD, as the one AE of the one CF, that is as AB of CD. therefore GE *b* = AB; and *c* ſo AE that was common being taken away, there remains GA = EB.

a 1. 5.
b 6. *ax.*
c 3. *ax.*

PROP. VI.



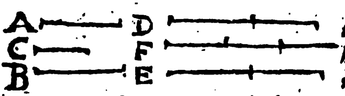
If two magnitudes AB, CD *be equimultiplices of two magnitudes* E, F; *and ſome magnitudes* AG *and* CH *equimultiplices of the ſame* E, F, *be taken away*; *then the reſidues* GB, HD *are either equall to theſe magnitudes* E, F, *or elſe equimultiplices of them.*

For becauſe the number of parts in AB, whereof each is equall to E, is put equall to the number of parts in CD, whereof each is equall to F. and alſo the number of parts in AG equall to the number of parts in CH; If from one you take AG, and from the other CH, *a* then remains the number of parts in the remainder GB equall to the number of parts in HD. therefore if GB be once E, then is HD once C. if GB be many times E, then is HD ſo of C. *Which was to be Dem.*

a 3. *ax.*

PROP.

PROP. VII.



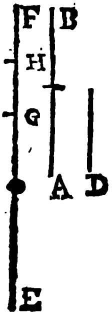
Equall magnitudes A *and* B *have to the ſame magnitude* C *the ſame proportion or ratio. And one and the ſame magnitude* C *hath the ſame ratio to equall magnitudes* A *and* B.

Take D and E equimultiplices of the equall magnitudes A and B, and F any-wiſe multiplex of C; a 6 *ax.*
then is D $^{a}=$ E. Wherefore if D $\sqsubset, =, \sqsupset$ F, b 6 *def.* 5.
then alſo E will be $\sqsubset, =, \sqsupset$ F. ^{b} therefore A. C :: c *cor.* 4. 5.
B.C. & ^{c} by inverſion C. A :: ^{c} C.B. *W.W. to be Dem.*

Schol.

If in ſtead of the multiplex F, two equimultiplices be taken, it ſhall be the ſame way proved that equall magnitudes have the ſame ratio to other magnitudes that are equall between themſelves.

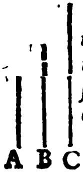
PROP. VIII.



Of unequall magnitudes AB, AC; *the greater* AB *hath a greater ratio to the ſame third line* D, *then the leſſer* AC; *and the ſame third line* D *hath a greater ratio to the leſſer* AC, *then to the greater* AB.

Take EF, EG equimultiplices of the ſaid AB, AC, ſo that EH being multiplex of D be greater then EG, but leſſer then EF. (which will eaſily happen, if both EG and GF be taken greater then D.) It is manifeſt from 8 *def.* 5. that $\frac{AB}{D} \sqsubset \frac{AC}{D}$, and $\frac{D}{AB} \sqsupset \frac{D}{AC}$. *Which was to be Dem.*

PROP. IX.



Magnitudes which to one and the ſame magnitude have the ſame ratio, are equall the one to the other. And if a magnitude have the ſame ratio to other magnitudes, thoſe magnitudes are equall one to the other.

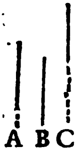
1. *Hyp.* If A. C :: B. C; I ſay that $A = B$.

a 8. 5. For let A be greater or leſſe then C. [a] then is $\frac{A}{C} \sqsubset$ or $\sqsupset \frac{B}{C}$. *Which is contrary to the Hypotheſis.*

2. *Hyp.* If C.B :: C.A. I ſay that $A = B$. For let

b 8. 5. A be $\sqsubset$ B, [b] then $\frac{C}{B} \sqsubset \frac{C}{A}$. *Which is againſt the Hypotheſis.*

PROP. X.



Of magnitudes having ratio to the ſame magnitude, that which has the greater ratio, is the greater magnitude: and that magnitude to which the ſame carryes a greater ratio, is the leſſer magnitude.

1. *Hyp.* If $\frac{A}{C} \sqsubset \frac{B}{C}$. I ſay that $A \sqsubset B$.

a 7. 5. For if it be ſaid that $A = B$, [a] then A. C :: B. C. *which is contrary to the Hyp.* If $A \sqsupset B$, [b] then is

b 8. 5. $\frac{A}{C} \sqsupset \frac{B}{C}$. *which is alſo againſt the Hyp.*

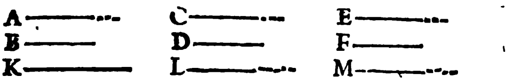
2. *Hyp.* If $\frac{C}{B} \sqsubset \frac{C}{A}$, I ſay that $B \sqsupset A$. for if you ſay $B = A$, it's againſt the Hypotheſis, for it will

c 7. 5. [c] follow that C.B :: C.A. If you ſay $B \sqsubset A$, [d] then

d 8. 5. is $\frac{C}{A} \sqsubset \frac{C}{B}$. *Which is alſo againſt the Hyp.*

PROP.

PROP. XI.



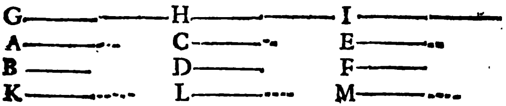
Proportions which are one and the ſame to any third, are alſo the ſame one to another.

Let A. B :: E. F, and C. D :: E. F. I ſay that A. B :: C. D. Take G, H, I, the equimultiplices of A, C, E; and K, L, M the equimultiplices of B, D, F. Now *a* becauſe A. B :: E. F; if G ⊏, =, ⊐ K, *b* then after the ſame manner I ⊏, =, ⊐ M. And likewiſe *a* becauſe E. F :: C. D. if I ⊏, =, ⊐ M, *b* then is H likewiſe ⊏, =, ⊐ L. *c* Wherefore A. B :: C. D, *Which was to be Dem.* a *hyp.* b 6 *def.* 5. c 6. *def.* 5.

Schol.

Proportions that are one and the ſame to the ſame proportions, are the ſame betwixt themſelves.

PROP. XII.



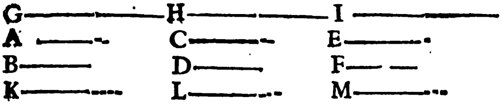
If any number of magnitudes A, B; C, D; E *and* F *be proportionall; as one of the Antecedents* A *is to one of the Conſequents* B, *ſo are all the antecedents* A, C, E *to all the conſequents* B, D, F.

Take the equimultiplices of the antecedents G, H, I, and of the conſequents K, L, M. Becauſe that as multiplex as one G is of one A, *a* ſo multiplices are all G, H, I, of all A, C, E; & likewiſe as multiplex as one K is of one B, ſo multiplices are all K, L, M, of all B, D, F. moreover becauſe A. B *b* :: C. D *b* :: E. F. if G be ⊏, =, or ⊐ K, then will H likewiſe be ⊏, =, ⊐ L, and I ⊏, =, ⊐ M. and ſo if G ⊏, =, ⊐ K. in like manner will G + H + I be ⊏, =, ⊐ K + L + M. *c* wherefore A. B :: A + C + E. B + D + F. *Which was to be Dem.* a 1. 5. b *hyp.* c 6 *def.* 5.

Coroll.

From hence, if like proportionals be added to like proportionals, the whoſes ſhall be proportionall.

PROP. XIII.



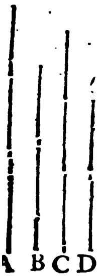
If the firſt A *have the ſame ratio to the ſecond* B, *that the third* C *hath to the fourth* D; *and if the third* C *have a greater proportion to the fourth* D, *then the fift* E *to the ſixt* F; *then alſo ſhall the firſt* A *have a greater proportion to the ſecond* B, *then the fift* E *to the ſixt* F.

Take G, H, I equimultiplices of A, C, E, and K, L, M, equimultiplices of B, D, F. Now becauſe
a 6. def. 5. that A. B :: C. D, if H $\sqsubset$ L, *a* then is G $\sqsubset$ K. but
b 8. def. 5. becauſe $\frac{C}{D} \sqsubset \frac{E}{F}$, *b* it may be that H $\sqsubset$ L, and
c 8. def. 5. yet I not $\sqsubset$ M. *c* Therefore $\frac{A}{B} \sqsubset \frac{E}{F}$. *Which was to be Dem.*

Schol.

But if $\frac{C}{D} \sqsupset \frac{E}{F}$, then alſo is $\frac{A}{B} \sqsupset \frac{E}{F}$. Alſo, if $\frac{A}{B} \sqsubset \frac{C}{D} \sqsubset \frac{E}{F}$, then is $\frac{A}{B} \sqsubset \frac{E}{F}$. And if $\frac{A}{B} \sqsupset \frac{C}{D} \sqsupset \frac{E}{F}$, then is $\frac{A}{B} \sqsupset \frac{E}{F}$.

PROP. XIV.



If the firſt A *have the ſame ratio to the ſecond* B, *that the third* C *hath to the fourth* D; *and if the firſt* A *be greater then the third* C; *then ſhall the ſecond* B *be greater then the fourth* D. *But if the firſt* A *be equall to the third* C, *then the ſecond* B *ſhall be equall to the fourth* D. *but if* A *be leſſer, then is* B *alſo leſſer.*

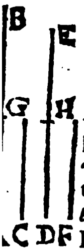
Let A $\sqsubset$ C. *a* then $\frac{A}{B} \sqsubset \frac{C}{B}$. *b* but $\frac{A}{B} = \frac{C}{D}$. therefore $\frac{C}{D} \sqsubset \frac{C}{B}$. *c* therefore $\sqsubset$ D. By the like way of argument, if A $\sqsupset$ C, then is B $\sqsupset$ D. But if A be put equall to C, then . B :: *e* A. B *f* :: C. D. *g* therefore B = D. *Which* *as to be Dem.*

a 8.5.
b hyp.
c 13.5.
d 10.5.
e 7.5.
f hyp.
g 11.5.&9.5.

Schol.

By an argument *a fortiori*, if $\frac{A}{B} \sqsupset \frac{C}{D}$, and A $\sqsubset$, then is B $\sqsubset$ D. Likewiſe if A = B, then is C = D. and if A $\sqsubset$, or $\sqsupset$ B, then alſo is C $\sqsubset$ or $\sqsupset$ D.

PROP. XV.

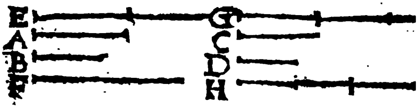


Parts C *and* F *are in the ſame ratio, with their like multiplices* A B *and* D E, *if taken correſpondently.* (A B. D E :: C. F.)

Let A G, G B parts of the multiplex A B be equall to C; and let D H, H E parts of the multiplex D E be equall to F. *a* The number of theſe parts is equall to the number of thoſe. Therefore whereas *b* A G. C :: D H. F, and G B. C :: H E. F, therefore is *c* A G + G B (A B.) H + H E (D E) :: C. F. *Which was to be Dem.*

a hyp.
b 7.5.
c 12.5.

PROP. XVI.

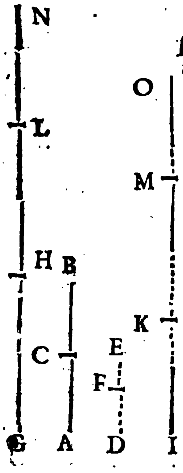


If four magnitudes A, B, C, D *be proportion*
alſo ſhall be alternately proportionall (A. C :: E
Take E and F equimultiplices of A and B
alſo G and H equimultiplices of C and D. Th
a 15. 5. E. F [a] :: A. B [b] :: C. D [a] :: G. H. Wherefore i
b hyp. =, ⊐ G, [c] then likewiſe is F ⊏, =,
c 11. 5. & 14. 5. d 6. def. 5. [d] Therefore A. C :: B. D. *Which was to be D*

Schol.

Alternate ratio has place onely then w
quantities are of the ſame kind. For heterog
quantities are not compared together.

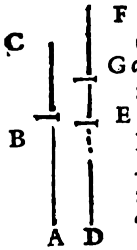
PROP. XVII.



If magnitudes compounded
portionall (A B. C B :: D
they ſhall be proportionall a
divided. (A C. C B :: D F
Take G H, H L, I K,
order the equimultiplices
C B, D F, F E; and alſ
M O, the equimultiplices
a 1. 5. F E. The whole G L
multiplex of the whole A
b constr. one G H of one A C, [b] t
d 1. 5. I K of D F, [c] or as the wh
of the whole D E. Al
e 2. 5. (H L + L N) is as [d]
plex of C B, as K O (
M O) is of F E. Therefore,
as by Hyp. A B. B C :: D
if G L be ⊏, =, ⊐ H N
likewiſe [e] will I M ⊏,

K O. Take from theſe H L, K M that are equall; and if the remainder GH be ⊏, =, ⊐ L N, *f* then will I K ⊏, =, ⊐ M O. *g* whence A C. C B :: D F. F E. *Which was to be Dem.* e 6. def. 5. f 5. ax. g 6. def. 5.

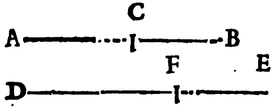
PROP. XVIII.



If magnitudes divided be proportionall (AB.BC :: DE. EF.) *the ſame alſo being compounded ſhall be proportionall* (AC.CB :: DF. FE.)

For if it can be, let AB. CB :: DF. FG ⊐ FE. *a* Then by diviſion will AB. BC :: DG. GF. *b* that is, DG. GF :: DE. EF. and being DG ⊏ DE, *c* therefore is GF ⊏ EF. *d Which is Abſurd.* The like abſurdity will follow if it be ſaid AB. CB :: DE. GF ⊏ FE. a 17. 5. b hyp. & 11. 5. c 14. 5. d 9. ax.

PROP. XIX.



If the whole AB *be to the whole* DE *as the part taken away* AC *is to the part taken away* DF, *then ſhall the reſidue* CB *be to the reſidue* FE *as the whole* AB *is to the whole* DE.

Becauſe *a* AB. DE :: AC. DF, *b* therefore by permutation AB. AC :: DE. DF. *c* and thence by diviſion AC. CB :: DF. FE. *b* wherefore again by permutation AC. DF :: CB. FE. *d* that is, AB. DE :: CB. FE. *W. W. to be Dem.* a hyp. b 10. 5. c 17. 5. d hyp. & 11. 5.

Coroll.

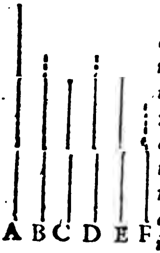
Hence, If like proportionals be ſubſtracted from like proportionals, the reſidues ſhall be proportionall.

2. *Hence is converſe ratio demonſtrated.*

Let AB.CB :: DE.FE. I ſay that AB. AC :: DE. DF. For by *a* permutation AB.DE :: CB.FE, *b* therefore AB. DE :: AC.DF. whence again by permutation AB. AC :: DE. DF. *W. W. to be Dem.* a 16. 5. b 19. 5.

PROP.

PROP. XX.



If there be three magnitudes A, B, C, *and others* D, E, F *equall to thoſe in number, which being taken two & two in each order are in the ſame ratio*, (A.B :: D.E; *and* B.C :: E.F,) *and if of equality the firſt* A *be greater then the third* C; *then ſhal the fourth* D *be greater then the ſixth* F. *But if the firſt* A *be equall to the third* C, *then the fourth* D *is ſo to the ſixth* F; & *if* A *be leſſe then* C, *ſo* D *is leſſe then* F.

a hyp. b cor 4. 5. c hyp & 8. 5. d ſchol. 13. 5. e 10. 5.

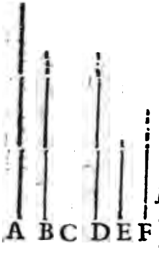
1. *Hyp.* Let $A \sqsubset C$. Becauſe [a] E F :: B.C. by [b] inverſion ſhall be F.E :: C.B. [c] But $\frac{C}{B} \sqsupset \frac{A}{B}$, [d] therefore $\frac{F}{E} \sqsupset \frac{A}{B}$ or $\frac{D}{E}$, [e] therefore $D \sqsubset F$. *W.W. to be Dem.*

2. *Hyp.* By the ſame way of argument, if $A \sqsupset C$, it will appear that $D \sqsupset F$.

f 7. 5. g 11. 5. and 9. 5.

3. *Hyp.* If $A = C$. Becauſe F. E :: C.B :: [f] A.B :: D.E. [g] therefore is $D = F$. *W.W. to be Dem.*

PROP. XXI.



If there be three magnitudes A, B, C, *and others alſo* D, E, F *equall to them in number, which taken two and two are in the ſame ratio; and their proportion inordinate* (A. B :: E. F. *and* B.C :: D.E.) & *if of equality, the firſt* A *be greater then the third* C; *then is the fourth* D *greater then the ſixth* F: *but if the firſt be equall to the third, then is the fourth equall to the ſixth; if leſſe, ſo is the other likewiſe.*

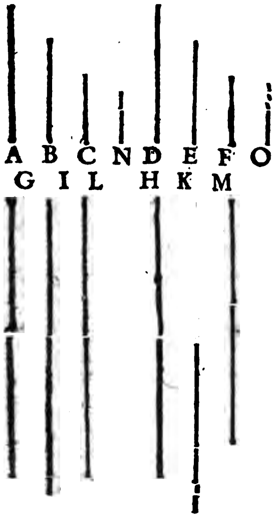
a hyp. b 8, 5. c ſchol. 13. 5. d 10. 5.

1. *Hyp.* If $A \sqsubset C$; then becanſe [a] D. E :: B. C, therefore inverſely E.D :: C.B. but $\frac{C}{B}$ [b] $\sqsupset \frac{A}{B}$; [c] therefore $\frac{E}{D} \sqsupset \frac{A}{B}$, that is $\frac{E}{F}$. [d] therefore $D \sqsubset F$.

2.*Hyp.* By the like argument, if A ⊐ C, then is D ⊐ F.

3.*Hyp.* If A = C; then becauſe E. D :: *e* C. B :: *e* A.B :: *f* E.F, *g* therefore is D = F. *W.W. to be Dem.* e 7.5. f hyp. g 9.5.

PROP. XXII.



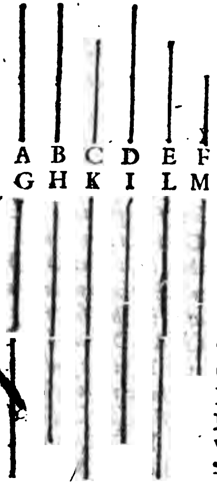
If there be any number of magnitudes A, B, C, *and others equall to them in number* D, E, F, *which taken two and two are in the ſame ratio* (A.B :: D.E. *and* B.C. :: E.F.) *they ſhall be in the ſame ratio alſo by equality,* (A.C :: D.F.)

Take G, H equimultiplices of A, D; & I, K of B, E; and alſo L, M of E. F.

Becauſe *a* A.B :: D.E, *b* therefore G. I :: H.K. and in like manner I.L :: K.M. therefore, if G ⊏, =, ⊐, L, *c* then is H ⊏, =, ⊐ M, *d* therefore A.C :: D.F. By the ſame way of demonſtration if further C.N :: F.O, then by equality A.N :: D.O. *W. W. to be Dem.* a hyp. b 4.5. c 20.5. d 6.def.5.

PROP.

PROP. XXIII.



If there be three magnitudes A, B, C, and others D, E, F, equall to them in number, which taken two and two are in the ſame ratio, and their proportionality inordinate (A.B :: E.F. *and* B.C :: D.E.) *they ſhall be in the ſame ratio alſo by equality.*

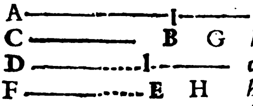
Take G, H, I equimultiplices of A, B, D; and alſo K, L, M equimultiplices of C, E, F.
a 15. 5. b hyp. Then G. H :: [a] A. B :: [b] E. F [a] :: L.M. Moreover becauſe [b] B. C :: D.E, thence is [c] H.K :: I. L; therefore G, H, K, and I,
c 4. 5. L, M are according to 21. 5. Wherefore if G be ⊏, =, ⊐ K, then is likewiſe I ⊏,
d 6. def. 5. =, ⊐ M. and ſo [d] conſequently A.C :: D.F. *W.W. to be Dem.*

If there be more magnitudes then three, this way of demonſtration holds good in them alſo.

Coroll.

* 22. and 23. 5. & 20. def. 1. From hence * it follow's that ratio's compounded of the ſame ratio's, are among themſelves the ſame; as alſo that the ſame parts of the ſame ratio's, are among themſelves the ſame.

PROP. XXIV.

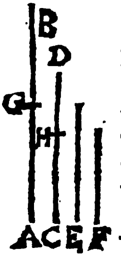


If the firſt magnitude AB *have the ſame ratio to the ſecond* C, *which the third* DE *hath to the fourth* F; *& if the fifth* BG *have the ſame ratio to the ſecond* C, *which the ſixth* EH *hath to the fourth* F, *then ſhall the firſt compounded with the fifth* (AG) *have the ſame ratio to the ſecond* C, *which the third compounded with the ſixth* (DH) *hath to the fourth* F.

a hyp. For becauſe [a] AB. C :: DE. F, and by the Hyp.

and inverſion C. BG :: F. EH; therefore by [b] equality AB. BG :: DE. EH. whence by compounding, AG. BG :: DH. EH. Alſo [c] BG. C :: EH. F. Therefore again by [b] equality A G. C :: D H. F. *W.W.to be Dem.* b 22.5. c hyp.

PROP. XXV.



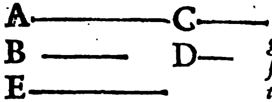
If four magnitudes be proportionall (AB. CD :: E.F) *the greateſt* AB *and the leaſt* F *ſhall be greater then the reſt* CD, *and* E.

Make AG $=$ E, and CH $=$ F. Because AB. CD :: [a] E. F :: [b] AG. CH, [c] thence is AB.CD :: GB.HD. [d] but AB $\sqsubset$ CD. [e] therefore GB $\sqsubset$ HD. But AG $+$ F $=$ E $+$ CH, therefore AG $+$ F $+$ GB $\sqsubset$ E $+$ CH $+$ HD, that is, AB $+$ F $\sqsubset$ E $+$ CD. *W.W.to be Dem.*

a hyp.
b 7.5.
c 19.5.
d hyp.
e ſchol.14.5

Theſe propoſitions which follow are not Euclid's, *but taken out of other Authors, and here ſubjoyned becauſe of their frequent uſe.*

PROP. XXVI.

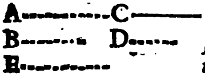


If the firſt have a greater proportion to the ſecond; then the third to the fourth, then contrarywiſe, by converſion, the ſecond ſhall have a leſſe proportion to the firſt, then the fourth to the third.

Let $\frac{A}{B} \sqsubset \frac{C}{D}$. I ſay that $\frac{B}{A} \sqsupset \frac{D}{C}$. For conceive $\frac{C}{D} = \frac{E}{B}$; [a] therefore $\frac{A}{B} \sqsubset \frac{E}{B}$. [b] whence A $\sqsubset$ E. [c] therefore $\frac{B}{A} \sqsupset \frac{B}{E}$, [d] or $\frac{D}{C}$. *W.W. to be Dem.*

a 13.5.
b 10.5.
c 8 5.
d cor. 4.5

PROP. XXVII.

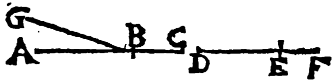


If the firſt have a greater proportion to the ſecond, then the third to the fourth; then alternatly the firſt ſhall have a greater proportion to the third, then the ſecond to the fourth.

Let $\frac{A}{B} \sqsubset \frac{C}{D}$. then I ſay $\frac{A}{C} \sqsubset \frac{B}{D}$. For conceive

a 10.5. b 1.5. c 16.5. $\frac{E}{B} = \frac{C}{D}$. [a] therefore A $\sqsubset$ E, [b] and therefore $\frac{A}{C} \sqsubset \frac{E}{C}$ [c] or $\frac{B}{D}$. *W.W. to be Dem.*

PROP. XXVIII.

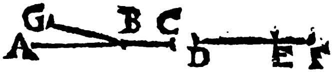


If the firſt have a greater proportion to the ſecond then the third to the fourth, then the firſt compounded with the ſecond ſhall have a greater proportion to the ſecond, then the third compounded with the fourth to the fourth.

Let $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. I ſay that $\frac{AC}{BC} \sqsubset \frac{DE}{EF}$. For conceive $\frac{GB}{BC} = \frac{DE}{EF}$. [a] therefore is AB $\sqsubset$ GB. adde BC to each part, then [b] will AC $\sqsubset$ GC. [c] therefore $\frac{AC}{BC} \sqsubset \frac{GC}{BC}$ [d] that is $\frac{DF}{FE}$. *W.W. to be Dem.*

a 10.5. b 4 ax. c 8.5. d 18.5.

PROP. XXIX.

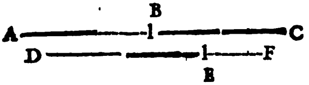


If the firſt compounded with the ſecond have a greater proportion to the ſecond, then the third compounded with the fourth hath to the fourth; then by diviſion the firſt ſhall have a greater proportion to the ſecond, then the third to the fourth.

Let $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. then I ſay $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. For conceive $\frac{GC}{BC} = \frac{DE}{EF}$. [a] therefore AC $\sqsubset$ GC, Take away BC, that is common; then [b] remains AB $\sqsubset$ GB. [c] Therefore $\frac{AB}{BC} \sqsubset \frac{GB}{BC}$ [d] or $\frac{DE}{EF}$. *W.W. to be Dem.*

a 10.5. b 5.ax. c 8.4. d 17.5.

PROP. XXX.

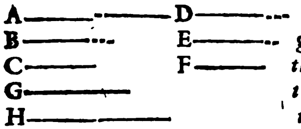


If the firſt compounded with the ſecond have a greater proportion to the ſecond, then the third compounded with the fourth hath to the fourth, then by converſe ratio ſhal the firſt compounded with the ſecond have a leſſer ratio to the firſt, then the third compounded with the fourth ſhall have to the third.

Let $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Then I ſay that $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. For becauſe that $\frac{AC}{BC}$ *a* $\sqsubset \frac{DF}{EF}$ *b* therefore by diviſion $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$ by converſion *c* therefore $\frac{BC}{AB} \sqsupset \frac{EF}{DE}$ and *d* therefore by compoſition $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. *W. W. to be Dem.*

a hyp.
b 29. 5.
c 26. 5.
d 28. 5.

PROP. XXXI.



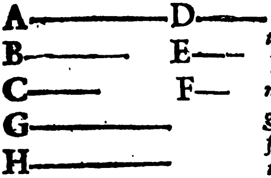
If there be three magnitudes A, B, C, *& others alſo* D, E, F *equall to them in number; & if there be a greater proportion of the firſt of the former to the ſecond, then there is of the firſt of the laſt to their ſecond* $\left(\frac{A}{B} \sqsubset \frac{D}{E}\right)$ *and and there be alſo a greater proportion of the ſecond of the firſt magnitudes to the third, then there is of the ſecond of the laſt magnitudes to their third* $\left(\frac{B}{C} \sqsubset \frac{E}{F}\right)$ *Then by equality alſo ſhall the ratio of the firſt of the former magnitudes to the third be greater then the ratio of the firſt of the latter magnitudes to the third* $\left(\frac{A}{C} \sqsubset \frac{D}{F}\right)$

Conceive $\frac{G}{C} = \frac{E}{F}$. *a* therefore is $B \sqsubset G$, & *b* therefore $\frac{A}{G} \sqsubset \frac{A}{B}$. Again conceive $\frac{H}{G} = \frac{D}{F}$ *c* therefore $\frac{H}{G} \sqsupset \frac{A}{B}$, therefore much more $\frac{H}{G} \sqsupset \frac{A}{G}$. *d* wherefore $A \sqsubset H$, *e* & conſequently $\frac{A}{C} \sqsubset \frac{H}{C}$ *f* or $\frac{D}{F}$.

a 10 5.
b 8. 5.
c 13. 5.
d 10. 5.
e 8. 5.
f 12. 5.

PROP.

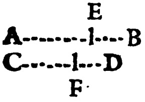
PROP. XXXII.



If there be three magnitudes A, B, C, and others D, E, F, equall to them in number; and there be a greater proportion of the firſt of the former magnitudes to the ſecond, then there is of the ſecond of the latter to the third $\left(\frac{A}{B} \sqsubset \frac{E}{F}.\right)$ *and alſo the ratio of the ſecond of the former to the third be greater then the ratio of the firſt of the latter to the ſecond* $\left(\frac{B}{C} \sqsubset \frac{D}{E}.\right)$ *then by equality alſo ſhall the proportion of the firſt of the former to the third, be greater then that of the firſt of the latter to the third* $\left(\frac{A}{C} \sqsubset \frac{D}{E}.\right)$

The demonſtration of this propoſition is altogether like that of the precedent.

PROP. XXXIII.

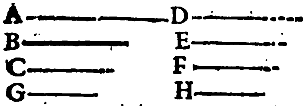


If the proportion of the whole A B *to the whole* C D *be greater then the proportion of the part taken away* A E *to the part taken away* C F; *then ſhall alſo the ratio of the remainder* E B *to the remainder* F D *be greater then that of the whole* A B *to the whole* C D.

a hyp.
b 27. 5.
c 30. 5.

Becauſe that $\frac{AB}{CD}$ *a* $\sqsubset \frac{AE}{EF},$ *b* therefore by permutation $\frac{AB}{AE} \sqsubset \frac{CD}{CF},$ *c* therefore by converſe ratio $\frac{AB}{EB} \sqsupset \frac{CD}{FD},$ and by permutation again $\frac{AB}{CD} \sqsupset \frac{EB}{FD}.$ *W. W. to be Dem.*

PROP. XXXIV.



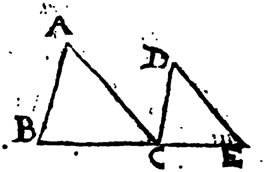
If there be any number of magnitudes, and others alſo equall to them in number; and the proportion of the firſt of the former to the firſt of the latter be greater then that of the ſecond to the ſecond, and that greater then the proportion of the third to the third, and ſo forward: all the former magnitudes together ſhall have a greater ratio to all the latter together, then all the former, leaving out the firſt, ſhall have to all the latter, leaving out the firſt; but leſſe then that of the firſt of the former to the firſt of the latter; and laſtly greater then that of the laſt of the former to the laſt of the latter.

You may pleaſe to conſult Interpreters for the demonſtration hereof, we having for brevities ſake omitted it, and becauſe 'tis of no uſe in theſe Elements.

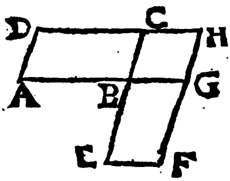
The End of the fifth Book.

THE SIXTH BOOK
OF
EUCLIDE'S ELEMEN

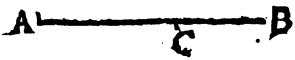
Definitions.



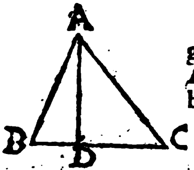
I. LIke right-lined figures (ABC, D
are ſuch whoſe ſeverall angles
quall one to the other, and
their ſides about the equall a
proportional.
The angle B = DCE, *and* AB. BC :: DC.
Alſo the angle A = D, *and* BA. AC :: CD.DE
ly the angle ACB = E, *and* BC. CA :: CE. E



II. Reciprocal figur
(BD, BF) when in eit
gure are the terms a
dents and conſequents
tio's (that is, AB. E
EB. BC.)



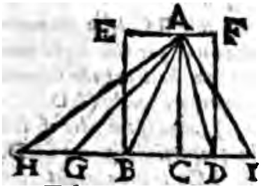
III. A righ
AB is ſaid to b
according to
and extreme proportion, when as the whole
to the greater ſegment AC, ſo is the greater ſeg
AC to the leſſe CB (AB, AC :: AC. CB.)



I V. The altitude of any figure ABC, is a perpendicular line AD drawn from the top A to the baſe BC.

V. A ratio is ſaid to be compounded of two ratio's, when the quantities of the ratio's being multiplyed the one into the other, do produce any ratio. *As the ratio of* A *to* C *is compounded of the ratio's of* A *to* B *and* B *to* C. *For* $\frac{A}{B} + \frac{B}{C}$ [a] $= \frac{A}{C}$ [b] $= \frac{AB}{BC}$. a 10. def. 5. b 15. 5.

P R O P. I.

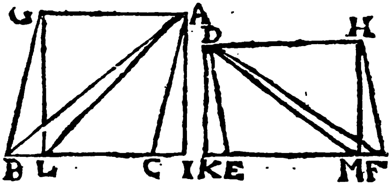


Triangles A B C, A C D, *and parallelograms* BCAE, CDFA, *which have the ſame height, are in proportion one to the other, as their baſes,* BC, CD *are.*

[a] Take as many as you pleaſe, BG, GH, equall to BC, and alſo DI = CD. and join AG, AH, AI. a 3. 1.

[b] The triangles ACB, ABG, AGH are equall, and [b] alſo the triangle ACD = ADI. Therefore the triangle ACH is as multiplex of the triangle ACB, as the baſe HC is of the baſe BC; and the triangle ACI as multiplex of the triangle ACD, as the baſe CI is of CD. But if HC $\sqsubset$, =, $\sqsupset$ CI, [c] then is likewiſe the triangle AHC $\sqsubset$, =, $\sqsupset$ ACI; and [d] therefore BC. CD :: the triangle ABC. ACD :: [e] Pgr. CE: CF. *W. W. to be Dem.* b 38. 1. c ſch. 38 1. d 6 def. 5. e 41. 1. and 15. 5.

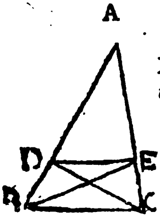
Scholium.



Hence, *Triangles* ABC , DEF,*and Pgrs.* AGBC, DEFH,*whoſe baſes* BC, EF *are equall,are in ſuch proportion as their altitudes,* AI, DK *are.*

a 3. 1. a Take IL = CB,and KM = EF ; and join LA,
b 7. 5. LG, MD, MH. then is it evident that the triang.
c 1. 6 ABC. DEF :: b ALI. DKM :: c AI. DK :: d pgr.
d 41. 1.and 15. 5. AGBC. DEFH. *W.W.to be Dem.*

PROP. II.



If to one ſide B C *of a triangle* ABC *be drawn a parallel right line* DE , *the ſame ſhall cut the ſides of the triangle proportionally* (AD. BD :: AE. EC.) *And if the ſides of the triangle be proportionally cut* (AD. BD :: AE. EC) *then a right line* DE *joined at the ſections* D;E , *ſhall be parallel to the remaining ſide of the triangle* BC. Draw CD and BE.

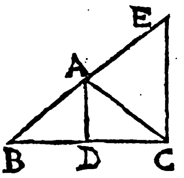
1. *Hyp.* Becauſe the Triangle DEB a = DEC,
a 37. 1. b therefore ſhall be the triangle ADE. DBE :: ADE.
b 7. 5. ECD. But the triang. ADE. DBE :: c AD.DB, and
c 2. 6. the triangle ADE. DEC : : AE. EC,d therefore AD
d 11. 5. DB :: AE. EC.

2. *Hyp.*Becauſe AD.DB :: AE.EC.e that is the tri-
e 1. 6. angle ADE. DBE :: ADE. ECD;f therefore is the
f 9. 5. triangle DBE= ECD; and g therefore DE,BC are
g 39. 1 parallels. *W. W. to be Dem.*

Scholium.

If there be drawne many parallelles to one ſide of any triangle, then all the ſegments of the ſides ſhall be proportionall; as is eaſily deducible from the precedent.

PROP. III.



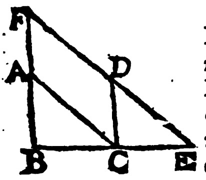
If an angle BAC *of a triangle* BAC *be biſected, and the right line* AD*, that biſects the angle, cut the baſe alſo; then ſhall the ſegments of the baſe have the ſame ratio that the other ſides of the triangle have,* (BD. DC :: AB. AC.) *And if the ſegments of the baſe have the ſame ratio, that the other ſides of the triangle have* (BD. DC :: AB. AC) *then a right line* AD *drawn from the top* A *to the ſection* D, *ſhall biſect that angle* BAC *of the triangle.*

Produce BA, and make AE = AC, and join CE.

1. *Hyp.* Becauſe AE = AC, therefore is the angle
ACE [a] = E [b] = $\frac{1}{2}$ BAC [c] = DAC; [d] therefore a 5. 1. b 32. 1.
DA, CE are parallels. [e] Wherefore BA. AE c hyp.
(AC) :: BD. DC. d 27. 1. e 2. 6.

2. *Hyp.* Becauſe BA.AC (AE) :: BD.DC, [f] there- f 2. 6.
fore are DA, CE parallels; and [g] therefore is the an- g 29. 1. h 5. 1.
gle BAD = E; and the angle DAC [g] = ACE [h] = k 1. ax.
E. [k] therefore the angle BAD = DAC. Wherefore
the angle BAC is biſected. *W.W. to be Dem.*

PROP. IV.



Of equiangular triangles ABC, DCE, *the ſides are proportionall which are about the equall angles,* B, DCE, (AB. BC :: DC. CE, &c.) *and the ſides* AB, DC, *&* C *which are ſubtended under the equall angles* ACB, E, &c. *are homologous, or of like ratio.*

Set the ſide BC in a direct line to the ſide CE,
and produce BA and ED till they [a] meet. a 32. 1.

b *hyp.* Because the angle F *b* = [illegible]
c 28. 1. CD are parallel: Alſo becauſe the angle B[illegible] = CED, *c* therefore are CA, EF parallel. Therefore
d 34. 1. the figure CAFD is a Pgr. *d* therefore AF = CD, and AC = *d* FD. Whence it is evident that AB. AF
e 2. 6.
f 16. 5. (CD) :: *e* BC. CE. *f* by permutation therefore AB. BC :: CD. CE. alſo BC. CE :: FD (AC.) DE. *f* and thence by permutation BC. AC :: CE. DE.
g 22. 5. *g* Wherefore alſo by equality AB. AC :: CD. DE. Therefore, &c.

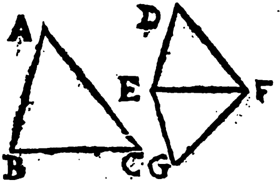
Coroll.

Hence AB. DC :: BG. CE :: AC. DE.

Schol.

Hence, If in a triangle FBE there be drawn AC a parallel to one ſide FE, the triangle ABC ſhall be like to the whole FBE.

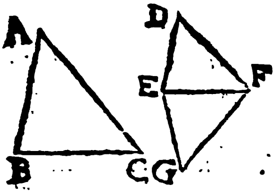
PROP. V.



If two triangles ABC, DEF *have their ſides proportionall* (AB. BC :: DE. EF, *and* AC. BC :: DF. EF, *&* *alſo* AB. AC :: DE. DF) *thoſe triangles are equiangular, and thoſe angles equall under which are ſubtended the homologous ſides.*

At the ſide EF *a* make the angle FEG = B, and
a 23. 1.
b 32. 1. the angle EFG = C; *b* whence the angle G = A.
c 4. 6. Therefore GE. EF *c* :: AB. BC :: *d* DE. EF. *e* and
d *hyp.* therefore GE = DE. Likewiſe GF. FE *c* :: AC. CB :
e 11. 5. *and* 9. 5. *d* DF. FE. *e* therefore GF = DF. Therefore the triangles DEF, GEF are mutually equilaterall
f 8. 1. *f* Therefore the angle D = G = A, and the ang FED *f* = FEG = B, and *g* conſequently the ang
g 32. 1. DFE = C. Therefore, &c.

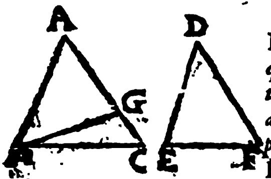
PROP. VI.



If two triangles ABC,DEF *have one angle* B *equall to one angle* DEF, *and the ſides about the equal angles* B,DEF *proportionall* (AB. BC :: DE.EF) *then thoſe triangles* ABC, DEF *are equiangular, and have thoſe angles equall, under which are ſubtended the homologous ſides.*

At the ſide EF make the angle FEG = B; and a 32. 1.
the angle EFG = C; *a* then will the angle G = A. b 4. 6.
Therefore GE. EF :: *b* AB. BC :: *c* DE. EF, *d* and c hyp.
therefore DE = GE. But the angle DEF *e* = B *f* = d 9. 5.
GEF; therefore the angle D *g* = G = A, *h* and con- e hyp.
ſequently the angle EFD = C. *W. W. to be Dem.* f conſtr.
g 4. 1.
h 32. 1.

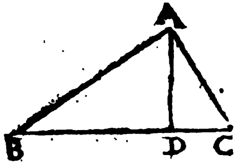
PROP. VII.



If two triangles ABC, DEF *have one angle* A *equall to one angle* D, *and the ſides about the other angles* ABC, E, *proportional* (AB.BC :: DE. EF) *and if they have both of the remaining angles* C,F *either leſſe or not leſſe then a right angle; then ſhall the triangles* ABC, DEF *be equiangular, and have thoſe angles equall about which the proportionall ſides are.*

For, if it can be, let the angle ABC ⊏ E, and
make the angle ABG = E. Therefore, whereas the
angle A *a* = D, *b* thence is the angle AGB = F. a hyp.
Therefore AB. BG *c* :: DE. EF :: *d* AB. BC. *e* there- b 32. 1.
fore BG = BC. *f* therefore the angle BGC = BCG. c 4. 6.
f Therefore BGC or C is leſſe then a right angle, & d hyp.
g conſequently AGB or F is greater then a right: e 9. 5.
Therefore the angles C and F are not of the ſame f 5. 1.
ſpecies or kind, which is againſt the Hypotheſis. g cor. 17. 1.
h cor. 13. 1.

PROP. VIII.



If in a right-angled triangle ABC, *from the right-angle* BAC *there be drawn* AD *a perpendicular to the baſe* BC; *then the triangles about the perpendicular* (ADB, ADC) *are like both to the whole triangle* ABC, *and alſo one to the other.*

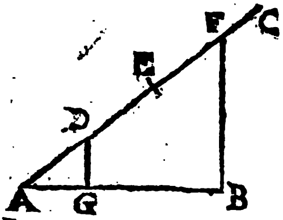
a hyp. b 12.ax. c 32.1 & 4.6. d 21.6. For becauſe BAC, ADB are [a] right angles, [b] and ſo equall, and B common; the triangles BAC, ADB [c] are like. By the ſame argum. BAC, ADC are like [d] whence alſo ADB, ADC will be like. *W.W. to be Dem.*

Coroll.

e 1.def.6. Hence, 1. BD. DA [e] :: DA. DC.

2. BC. AC :: AC. DC. and CB. BA :: BA. BD.

PROP. IX.



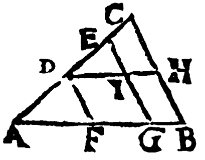
From a right line given AB *to cut off any part required, as* $\frac{1}{3}$ (AG.)

a 3.1. *From the point* A *draw an infinite line* AC *any-wiſe, in which* [a] *take any three equall parts* AD, DE, EF. *join* FB, *to which from* D [b] *draw the parallel* DG; and the thing is done. b 31.1.

c 2.6. d 18.5. For GB. AG :: [c] FD. AD; whence by [d] compoſition AB. AG :: AF. AD. therefore, whereas AD = $\frac{1}{3}$ of AF, therefore is AG = $\frac{1}{3}$ of AB. *W.W. to be Done.*

PROP.

PROP. X.

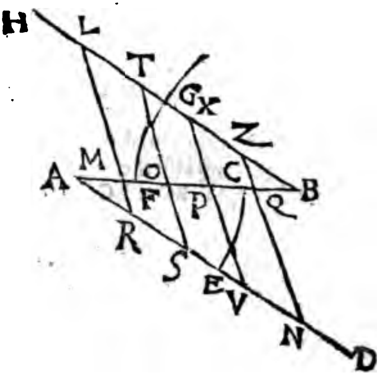


To divide a right line given AB *not divided* (in F and G) *as another line given* AC *was cut* (in D and E.)

Let a right line BC join the extremities of the line divided, and of the line not divided; and to that line from the points E, D [a] draw the parallels E G, D F meeting with the right line that is to be cut, in G & F; Then the thing is Done. a 31. 1.

For let DH be [a] drawn parallel to AB. Then AB. DE :: [b] AF. FG. and DE. EC [b] :: DI. IH :: [c] FG. GB. *W. W. to be Done.* b 2. 6. c 34. 1. and 7. 5.

Schol.



Hence is learnt to cut a right line given AB *into as many equall parts as you pleaſe (ſuppoſe 5;)* which will be more eaſily performed thus.

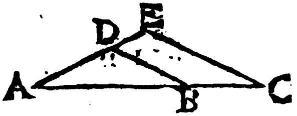
Draw an infinite line AD, and another BH parallel to it and infinite alſo. Of thoſe take equall parts, A R, R S, S V, V N; and B Z, Z X, X T, T L;

in each line leſſe parts by one, then are required in AB; then let the right lines LR, TS, XV, ZN be drawn; theſe lines ſo drawn ſhall cut the right line given AB into five parts.

a 33. 1.
b conſtr.
c 2.6.

For RL, ST, VX, NZ are a parallels; therefore, whereas AR, RS, SV, VN are b equall; c thence AM, MO, OP, PQ are equall alſo. Likewiſe, becauſe that BZ = ZX, therefore is BQ = PQ and therefore AB is cut into five parts. *W. W. to be Done.*

PROP. XI.

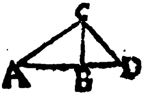


Two right lines being given, AB, AD, *to find out a third in proportion to them* (DE.)

Join BD, and from AB being produced take BC = AD. Through C draw CE parallel to BD; with which let AD produced meet in E. then is DE the proportionall required.

a 2.6.

For AB. BC (AD) a :: AD. DE. *W. W. to be Done.*

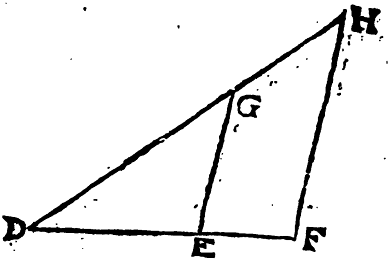


b cor. 9.

Or thus: make the angle ABC right, and alſo the angle ACD right. then b AB. BC :: BC. BD.

PROP.

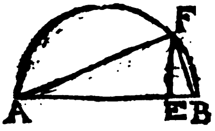
PROP. XII.



Three right lines being given DE, EF, DG, *to find out a fourth proportionall* GH.

Join EG, and thorough F draw FH parallel to
EG; with which let DG produced to H meet. Then a 2. 6.
it is evident that DE. EF a :: DG. GH. *W. W. to be Done.*

PROP. XIII.



Two right lines being given AE, EB, *to find out a mean proportionall* EF.

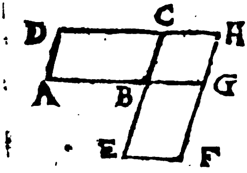
Upon the whole line AB as a diameter deſcribe a ſemicircle AFB, & from E erect a perpendicular EF meeting with the periphery in F. then AE.EF
:: EF. EB. For let AF & FB be drawn; a then from a 31. 3.
the right angle of the right-angled triangle AFB
is drawn a right line FE perpendicular to the baſe. b cor. 8. 6.
b Therefore AE.FE :: FE.EB. *W.W. to be Done.*

Co-

Coroll.

Hence, A right line drawn in a circle from any point of the diameter perpendicularly, and extended to the circumference, is a mean proportion all betwixt the two ſegments of the diameter.

PROP. XIV.



Equall Parallelograms having one angle ABC *equall to one* EBG, *have the ſides* BD, BF *which are about the equall angles reciprocall* (AB. BG :: EB. BC;) *and thoſe Parallelograms* BD, BF *which have one angle* ABC *equall to one* EBG, *and the ſides which are about the equall angles reciprocall, are equall.*

For let the ſides AB, BG about the equall angles make one right line; [a] wherefore EB, BC ſhall doe the ſame. Let FG, DC be produced till they meet.

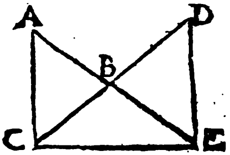
1.*Hyp*. AB.BG [b] :: BD. BH :: [c] BF. BH :: [d] BE. BC. [e] therefore, &c.

2.*Hyp*. BD. BH :: [f] AB. BG :: [g] BE. BC :: [h] BF. BH. [k] Therefore the Pgr. BD = BF. *Which was to be Dem.*

[a] ſch.15.1.
[b] 1. 6.
[c] 7.5.
[d] 1. 6.
[e] 11.5.
[f] 1. 6.
[g] hyp.
[h] 1.6.
[k] 11, & 9.5.

PROP.

PROP. XV.



Equall triangles having one angle ABC *equall to one* DBE, *their ſides which are about the equall angles are reciprocall* (AB. BE :: DB. BC.) *And thoſe triangles that have one angle* ABC *equall to one* DBE, *and have alſo the ſides that are about the equall angles reciprocall* (AB. BE :: DB. BC.) *are equall.*

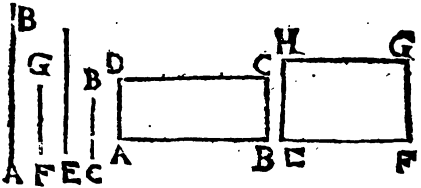
Let the ſides CB, BC, which are about the equall
angles be ſet in a ſtrait line; a therefore ABE is a a *ſchol.* 15. 1.
right line. Let CE be drawn.

1. *Hyp.* AB. BE :: b the triangle ABC. CBE c :: the b 1. 6.
triangle DBE. CBE :: d DB. BC. e therefore, &c. c 7. 5.

2. *Hyp.* The triangle ABC. CBE :: f AB. BE :: d 1. 6.
g DB. BC h :: the triangle DBC. CBE. k Therefore e 11. 5.
the triangle ABC = DBE. *W. W. to be Dem.* f 1. 6.

g *hyp.*
h 1. 6.
k 11, & 9. 5.

PROP. XVI.



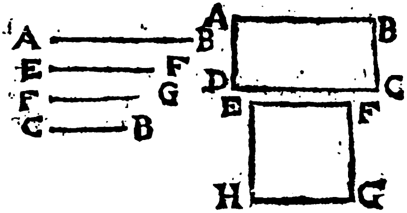
If four right lines be proportionall (AB. FG :: EF. CB) *the rectangle* AC *comprehended under the extremes* AD, CB, *is equall to the rectangle* EG *comprehended under the means* FG, EF. *And if the rectangle* AC *comprehended under the extremes* AB, CB *be equall to the rectangle* EG *comprehended under the means* FG, EF, *then are the four right lines proportionall.* (AB. FG :: EF. CB.)

b 12.ax. 1.*Hyp*. The angles B and F are right, and a conſequently equall,& by hypotheſis AB. FG :: EF. CB.
b 14. 6. b therefore the rectangle AC = EG.
c hyp. 2. *Hyp*. The rectangle AC c = EG, and the angle
d 14. 6. B=F; d therefore AB. FG :: EF. CB. *W. W. to be Dem.*

Coroll.

Hence, it is eaſy to apply a rectangle given EG
e 12. 6. to a right line given AB; (*viz.*) by making AB. EF :: FG. BC.

PROP. XVII.



If three right lines be proportionall (AB. EF :: EF.CB.) *the rectangle* AC *made under the extreames* AB, CB *is equall to the ſquare* EG *made of the middle* EF. *And if the rectangle* AC *comprehended under the extremes* AB, CB *be equal to the ſquare* EG *made of the middle* E F, *then the three lines are proportional*, (AB: EF:: EF. CB).

Take FG=EF.

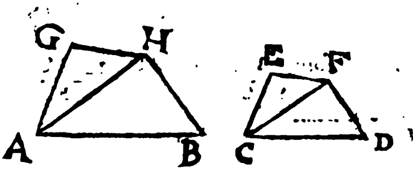
a hyp. 1.*Hyp*. AB. EF :: a EF (FG.) CB. therefore the
b 16. 6. rectangle AC b =EG c = EFq.
c 29. def. 1.
d hyp. 2 *Hyp*. The rectangle AC d = to the ſquare E-
e 16. 6. G=EFq. e therefore AB. EF :: FG (EF.) BC. *W. W. to be Dem.*

Coroll.

Let AxB = Cq. therefore A.C :: C. B.

PROP.

PROP. XVIII.



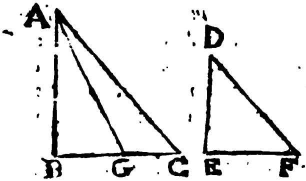
From a right line given AB *to describe a right-lined figure* AGHB *like and alike ſituate to a right-lined figure given* CEFD.

Reſolve the right-lined figure given into triangles; *a* Make the angle ABH = D, *a* & the angle BAH = DCF, *a* and the angle AHG = CFE, *d* and the angle HAG = FCE. then AGHB ſhall be the right-lined figure ſought. *a* 23. 1.

For the angle B *b* = D, and the angle BAH *b* = DCF. *c* wherefore the angle AHB = CFD. *b* alſo the angle HAG = FCE, and the angle AHG *b* = CFE. *c* wherefore the angle G = E, and the whole angle GAB *d* = ECD, and the whole angle GHB *d* = EFD. The polygones therefore are mutually equiangular. Moreover becauſe the triangles are equiangular, therefore AB.BH *e* :: CD.DF; and AG. GH *e* :: CE. EF. Likewiſe AG.AH :: *e* CE.CF. and AH. AB :: CF. CD. *f* From whence by equality AG. AB :: CF. CD. After the ſame manner GH. HB :: EF. FD. Therefore the Polygones ABHG, CDFE are like and alike ſituate. *W.W. to be Done.*

b conſtr.
c 32. 1.
d 2. ax.
e 4. 6.
f 22. 5.
g 6. def. 5.

PROP. XIX.



Like triangles ABC, DEF *are in duplicate ratio of their homologous ſides,* BC, EF.

a Let there be made BC. EF :: EF. BG, and let AG be drawn. Becauſe that AB. *a* 11. 6.

DE

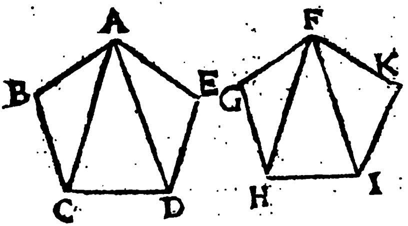
b cor.4.6. DE [b] :: BC. EF [c] :: EF. BG, and the angle B = E.
c conſtr. [d] therefore is the triangle ABG = DEF. But the
d 15.6. triangle ABC. ABG :: [e] BC. BG, and [f] $\frac{BC}{BG} = \frac{BC}{EF}$
e 1.6. twice; therefore $\frac{ABC}{ABG}$ that is $\frac{ABC}{DEF}$ [g] $= \frac{BC}{EF}$ twice.
f 10.def.5.
g 11. 5.

W.W. to be Dem.

Coroll.

Hence, If three right lines (BC, EF, BG) be proportionall, then as the firſt is to the third, ſo is a triangle made upon the firſt BC, to a triangle like and alike deſcribed upon the ſecond EF: or ſo is a triangle deſcribed upon the ſecond EF to a triangle like and alike deſcribed upon the third.

PROP. XX.



Like polygones ABCDE, FGHIK *are divided into equall triangles* ABC, FGH, *and* ACD, FHI, *and* ADE, FIK; *both equall in number and homologous to the wholes* (ABC. FGH :: ABCDE. FGHIK :: ACD. FHI :: ADE. FIK.) *And the polygones* ABCDE, FGHIK *have a double ratio one to the other of what one homologous ſide* BC *hath to the other homologous ſide* GH.

a hyp. 1. For the angle B [a] = G, and AB. BC [a] :: FG.
b 6.6. GH. [b] therefore the triangles ABC, FGH

are equiangular. After the ſame manner are the triangles AED, FKI like. Whereas therefore the angle BCA b = GHF, and the angle ADE b = FIK, and the whole angles BCD, GHI, and the whole angles CDE, HIK are c equall, there remains the angle ACD d = FHI, and the angle ADC = FIH; e from whence alſo the angle CAD = HFI. therefore the triangles ACD, FHI are like: Therefore, &c. b 6. 6. c hyp. d 3 ax. e 32. 1.

2. Becauſe that the triangles BCA, GHF are like, f therefore is $\frac{BCA}{GHF} = \frac{BC}{GH}$ twice. For the ſame reaſon is $\frac{CAD}{HFI} = \frac{CD}{HI}$ twice; laſtly $\frac{DEA}{IKF} = \frac{DE}{IK}$ twice. now whereas that BC.GH g :: CD. HI g :: DE. IK. h therefore is the triangle BCA. GHF :: CAD. HFI :: DEA. IKF k :: the polygone ABCDE. FGHIK :: $\frac{BC}{GH}$ twice. f 19. 6. g hyp. & 16. 5. h ſch 23 5. k 12. 5.

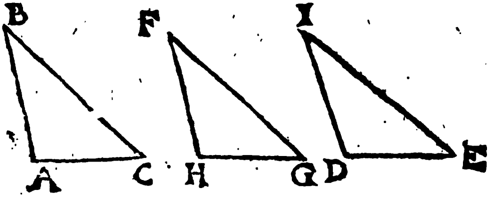
Coroll.

1. Hence, If there be three right lines proportionall, then as the firſt is to the third, ſo is a polygone made upon the firſt to a polygone made on the ſecond like and alike deſcribed; or ſo is a polygone deſcribed upon the ſecond to a polygone made on the third like and alike deſcribed.

By which is found out a method of inlarging or diminiſhing any right-lined figure in a ratio given: As if you would make a pentagone quintuple of that pentagone whereof CD is the ſide, then betwixt AB and 5 AB find out a mean proportional, * upon this raiſe a pentagone like to that given, and it ſhall be quintuple of the pentagone given. * 18 6.

2. Hence alſo, If the homologous ſides of like figures be known, then will the proportion of the figures be evident, *viz.* by finding out a third proportional.

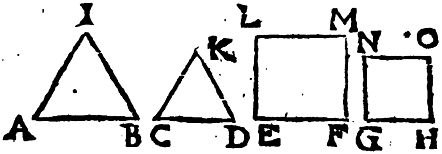
PROP. XXI.



Right-lined figures ABC, DIE *which are like to the ſame right-lined figure* HFG, *are alſo like one to the other.*

a 1.def.6. For the angle A a = H a = D; and the angle C a = G a = E; and the angle B a = F a = I. Alſo a AB.AC :: HF.HG :: DI.DE. & a AC.CB :: HG. GF :: DE. EI. And AB. BC :: HF, FG :: DI. IE. Therefore a ABC, DIE are like. *W. W. to be Dem.*

PROP. XXII.



If four right lines be proportionall (AB. CD :: EF. GH) *the right-lined figures alſo deſcribed upon them being like and in like ſort ſituate, ſhall be proportionall* (ABI.CDK :: EM. GO.) *And if the right-lined figures deſcribed upon the lines, like and alike ſituate, be proportionall* (ABI. CDK :: EM. GO) *then the right lines alſo ſhall be proportionall* (AB. CD :: EF. GH.)

a 19. 6. 1. *Hyp.* $\frac{ABI}{CDK}$ a $= \frac{AB}{CD}$ twice a $= \frac{EF}{GH}$ twice a $= \frac{EM}{GO}$.

b therefore ABI.CDK :: EM. GO.

b hyp.
c 10. 6.
d cor.23.5.

2. *Hyp.* $\frac{AB}{CD}$ twice a $= \frac{ABI}{CDK}$ b $= \frac{EM}{GO}$ c $= \frac{EF}{GH}$ twice.

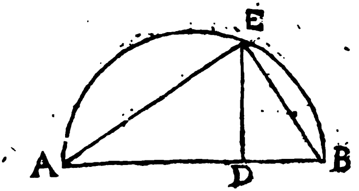
Therefore AB. CD :: EF. GH. *W. W. to be Dem.*

Schol.

Schol.

Hence is deduced the manner and reaſon of multiplying ſurd quantities. Ex.g. Let $\surd 5$ be to be multiplyed into $\surd 3$. I ſay that the product will be $\surd 15$. For by the definition of multiplication it ought to be, as 1. $\surd 3$ :: $\surd 5$. to the product. Therefore by *this* q.1. q. $\surd 3$ :: q. $\surd 5$.q. of the product. That is 1.3.:: 5. to the ſquare of the product. therefore the ſquare of the product is 15. Wherefore $\surd 15$ is the product of $\surd 3$ into $\surd 5$. *W.W.to be Dem.*

THEOREME.



If a right line AB *be cut any-wiſe in* D, *the rectangle comprehended under the parts* AD, DB *is a mean proportionall betwixt their ſquares. Likewiſe the rectangle comprehended under the whole* AB *and one part* AD, or DB, *is a mean proportionall betwixt the ſquare of the whole* AB *and the ſquare of the ſaid part* AD, or DB. Pitr. Herig.

Upon the diameter AB deſcribe a ſemicircle; from D erect a perpendicular DE meeting with the periphery in E. join AE, BE.

It's evident that AD.DE *a* :: DE.DB. *b* therefore ADq. DEq :: DEq. DBq. *c* that is, ADq. ADB :: ADB. DBq. *W.W.to be Dem.*

a cor.8.6.
b 22.6.
c 17.6.

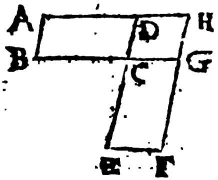
Moreover BA. AE :: *d* AE.AD. *e* therefore BAq. AEq :: AEq. ADq. *f* that is BAq. BAD :: BAD. ADq. After the ſame manner ABq. ABD :: ABD. BDq. *W.W.to be Dem.*

d cor.8.6.
e 22.6.
f 17.6.

Or thus: ſuppoſe Z=A+E. It is manifeſt that Aq.AE :: *a* A.E :: *a* AE.Eq. alſo Zq.ZA :: *a* Z. A :: ZA.Aq.and Zq.ZE :: *a* ZE :: Z.E.Eq.

a 1.6.

PROP. XXIII.



Equiangular parallelograms AC, CF, *have the ratio one to the other, which is compounded of their ſides.* $\left(\frac{AC}{CF} = \frac{BC}{CG} + \frac{DC}{CE}\right)$

Let the ſides about the equall angles C be a ſet in a direct line, and let the Pgr. CH be compleated, Then is the ratio of $\frac{AC}{CF} {}^{b}= \frac{AC}{CH} + \frac{CH}{CF} {}^{c}= \frac{BC}{CG} + \frac{DC}{CE}$ *W.W. to be Dem.*

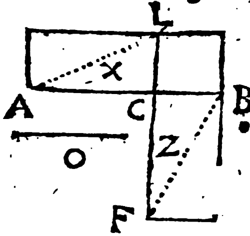
a *ſch.* 15.
b 20 *def.* 5.
c 1. 6.

Coroll.

Andr. Tacq. 15. 5.

Hence, *and from* 34. 1. *it appears* 1. *That triangles which have one angle equall* (*at* C) *have a ratio compounded of the ratio's of the right lines,* (AC *to* CB, *and* LC *to* CF,) *containing the equall angle.*

a 35. 1.



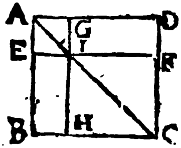
2. *That all rectangles, &* a *conſequently all parallelograms, have their ratio one to the other compounded of the ratio's of baſe to baſe; and altitude to altitude.* After the like manner you may argue in triangles.

3. *From hence is apparent how to give the proportion of triangles and paralelograms.* Let there be two pgrs. X, and Z, whoſe baſes are AC, CB, and altitudes CL, CF. Make CL. CF :: CB. O. * then will it be X, Z :: AC. O.

* 14 6 and 1. 6

PROP.

PROP. XXIV.

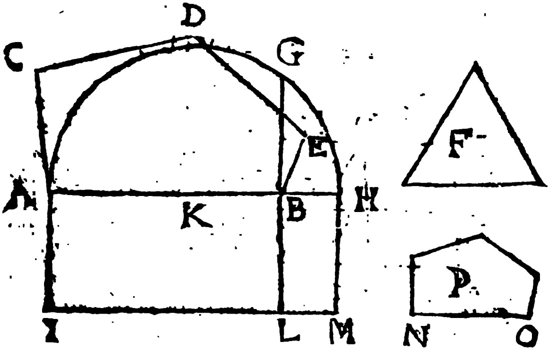


In every paralielogram ABCD, *the parallelograms* EG, HF *which are about the diameter* AC *are like to the whole, and alſo one to the other.*

For the Pgrs. EG, HF have each of them one angle common with the whole; [a] therefore they are equiangular to the whole, & alſo one to the other. Alſo both the triangles ABC, AEI, IHC and the triangles ADC, AGI, IFC are equiangular mutually; [b] therefore AE. EI :: AB. BC, and [b] AE. AI :: AB. AC, and [b] AI. AG :: AC. AD. [c] Therefore by equality, AE. AG :: AB. AD. [d] Therefore the Pgrs. EG, BD are like. After the ſame manner are HF, BD like alſo. Therefore, &c.

a 29. 1.
b 4. 6.
c 22. 5.
d 1. def. 6.

PROP. XXV.



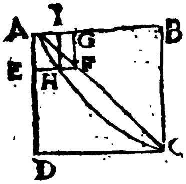
Unto a right-lined figure given ABECD *to deſcribe another figure* P *like and alike ſituate, which alſo ſhall be equall to another right-lined figure given* F.

[a] Make the rectangle AL = ABEDC; [b] alſo upon BL make the rectangle BM = F; Betwixt AB and BH [c] find out a mean proportionall NO; Upon NO [d] make the polygone B like to the right-lined figure given ABEDC. I ſay the polygone P ſo made ſhall be equall to F that was given.

a 45. 1.
b 44. 1.
c 13. 6.
d 18. 6.

e cor. 20.6. For ABEDC (AL.) P :: [e] AB. BH :: [f] AL. BM.
f 1.6 Therefore P [g] = BM [h] = F. *W.W. to be Done.*
g 14 5.
h conſtr.

PROP. XXVI.

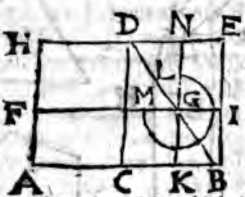


If from a Parallelogram ABCD, be taken away another Parallelogram AGFE, *like unto the whole, and in like ſort ſet, having alſo an angle common with it* EAG; *then is that parallelogram about the ſame diagonall* AC *with the whole.*

If you denye AG to be the common diagonall, then let AHC be it, cutting EF in H, and let HI be drawn parallel to AE. Then are Pgrs EI, DB [a] like, [b] therefore AE.EH :: AD.DC :: [c] AE.EF. & [d] conſequently EH = EF. [f] *Which is Abſurd.*

a 24.6.
b 1.def.6.
c hyp.
d 9.5.
f 9.ax.

PROP. XXVII.

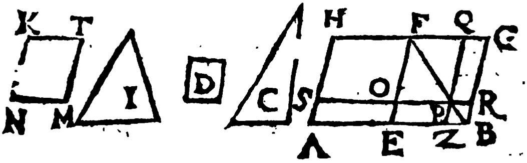


Of all parallelograms AD, AG *applyed to the ſame right line* AB, *and wanting in figure by the parallelograms* CE, KI *like and alike ſet to the Pgr.* AD *which is deſcri- taken away bed upon the greateſt is that* AD *which is applyed to the half being like to the defect* KI.

For becauſe that GE [a] = GC, and KI added in common. [b] thence is KE = CI [c] = AM. adde CG in common, [d] then is AG = to the Gnomon MBL. But the Gnomon MBL [e] $\sqsupset$ CE (AD.) Therefore AG $\sqsubset$ AD. *W.W. to be Dem.*

a 43.1.
b 2 ax.
c 36.1.
d 2.ax.
e 9.ax.

PROP. XXVIII.

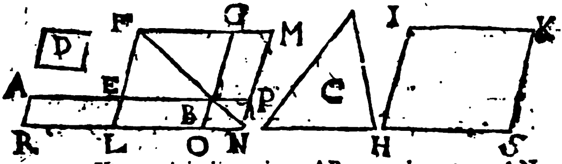


Vpon a right line given AB, *to apply a parallelogram* **AP** *equal to a right-lined figure given* C, *deficient by a parallelogram* ZR *which is like to another parallelogram given* D. * *Now it is requiſite that the right-lined figure given* C, *whereunto the Pgr. to be applyed* A P *muſt be equall , be not greater then the Pgr.* A F *which is applyed upon the half line , the defects being like , namely the defect of the Pgr.* **AF**, *which is applyed to the half line, and the defect of the Pgr.* D *to be applyed whoſe defect is to be like to the Pgr. given.* * 27. 6.

Biſect AB in E; upon EB *a* make the Pgr. EG like to the Pgr. D ; and *b* let $EG = C + I$. *c* Make the Pgr. $NT = I$, and like to the Pgr. given D, or EG; Draw the diameter FB ; Make $FO = KN$, and $FQ = KT$; thorough O and Q draw the parallels SR, QZ. Then is the Pgr. A P that which was ſought. a 18. 6. b ſch. 45. 1. c 25. 6.

For the Pgrs D, E G, OQ, NT, ZR are all *d* like one to the other, and the Pgr. $EG =$ *e* $NT + C =$ *e* $OQ + C$. *f* wherefore C = to the Gnomon O B Q *g* $= AO + PG =$ *h* $AO + EP = AP$. *Which was to be Done.* d conſtr. & 24 6. e conſtr. f 3. ax. g 2. ax. h 43. 1.

PROP. XXIX.

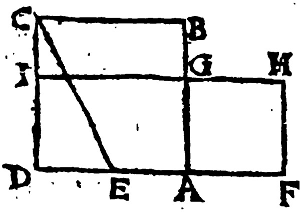


Upon a right line given AB *to apply a pgr.* AN *equall to a right lined figure given* C, *exceeding by a pgr.* OP, *which ſhall be like to another pgr. given* D.

Biſect AB in E. Upon EB a make a Pgr. EG like
a 18.6. to D, which was given. And b let the pgr. HK = EG
b 25.6. + C, and like to D given, or EG. Make FEL =
c 3.1. c IH; and c FGM = HK. Thorough L, M draw the parallels MN and RN and AR parallel to NM. produce ABP, GBO. Draw the diameter FBN. Then is AN the parallelogram required.

d conſtr. For the pgrs. D, HK, LM, EG are d like. e therefore
e 24.6. the pgr. OP is like to the pgr. LM, or D. Alſo LM
f conſtr. f = HK f = EG + C. g Therefore C = to the
g 3 ax. Gnomon ENG. But AL h = LB k = BM. l there-
h 36.1. fore C = AN. *W. W. to be Done.*
k 43.1.
l 2. & 1. ax.

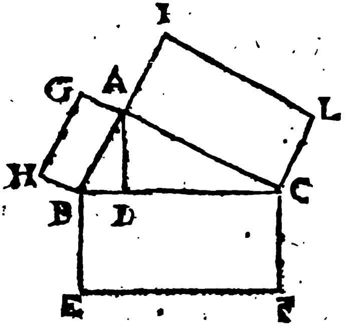
PROP. XXX.



To cut a finite right line given AB *according to extreme and mean ratio* (AB. AG :: AG. GB.)

a 11.2. a Cut AB in G, in ſuch wiſe that AB x BG = AGq.
b 17.6. b Then BA. AG :: AG. GB. *Which was to be Done.*

PROP. XXXI.



In right-angled triangles BAC, *any figure* BF *deſcribed upon the ſide* BC *ſubtending the right-angle* BAC, *is equall to the figures* BG, AL, *deſcribed upon the ſides* BA, AC, *containing the right-angle, like and alike ſituate to the former* BF.

From the right angle BAC let down a perpendi-
cular AD. Becauſe that DC.CA :: a CA.CB, b there- a cor. 8. 6. b cor. 20. 6.
fore AL. BF :: DC. CB. Alſo, becauſe DB. BA ::
a BA.BC, b therefore BG. BF :: DB.BC, c therefore c 24. 5.
AL+BG. BF :: DC+DB (BC.) BC. d Therefore d ſchol. 14. 5.
AL+BG = BF. *W.W. to be Dem.*

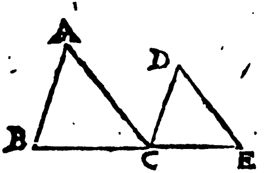
Or thus: BG.BF :: e BAq. BCq. And e AL. BF :: e 22. 6.
ACq. BCq. f therefore BG+AL.BF :: BAq+ACq.
BCq. g Therefore whereas BAq + ACq = h BCq. f 24. 5. g ſch. 14. 5.
g thence is BG + AL = BF. *W.W. to be Dem.* h 47. 1.

Coroll.

From this propoſition you may learn how to add or ſubſtract any like figures, by the ſame method that is uſed in adding and ſubſtracting of ſquares, in *Schol.* 47. 1.

PROP.

PROP. XXXII.

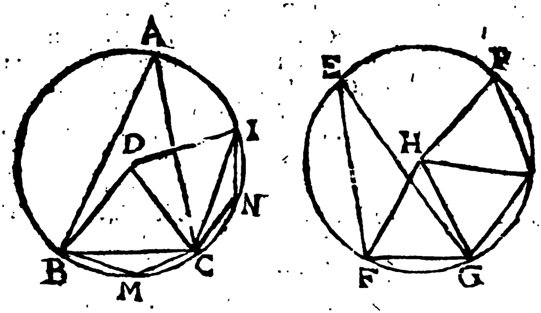


If two triangles **ABC, DCE** *having two ſides proportional to two* (**AB. AC :: DC. DE.**) *be ſo compounded or ſet together at one angle* **ACD** *that, their homologous ſides be alſo parallel* (**AB** *to* **DC,** *and* **AC** *to* **DE,**) *then the remaining ſides of thoſe triangles*(**BC, CE**) *ſhall be found placed in one ſtrait line.*

a 29. 1.
b hyp.
c 6. 6.
d 2. ax.
e 32. 1.
f 1. ax.
g 14. 1.

For the angle A *a* = ACD *a* = D. and AB. AC *b* :: DC. DE. *c* therefore the angle B = DCE. Therefore the angle B + A *d* = ACE. But the angle B + A + ACB *e* = 2 right. *f* therefore the angle ACE + ACB = 2 right. *g* therefore BCE is a right line. *W.W. to be Dem.*

PROP. XXXIII.



In equall circles **DBCA, HFGP**, *the angles* **BDC, FHG** *have the ſame ratio with their peripheries* **BC, FG** *on which they inſiſt; whether the angles be ſet at the centers* (*as* **BDC, FHG**) *or at the circumferences,* **A, E:** *And in like ſort are the Sectors* **BDC, FHG,** *becauſe deſcribed upon the centers.*

Draw the right lines BC, FG. Make CI=CB, &
GL=FG=LP. and join DI, HL, HP.

The arch BC *a*=CI. *a* alſo the arch FG, GL, LP are a 28. 3.
equall; *b* therefore the angle BDC=CDI. *b* and the b 27. 3.
angle FGH=GHL=LHP. Therefore the arch BI
is as multiplex of the arch BC, as the angle BDI is
of the angle BDC. And in like manner is the arch FP
as multiplex of the arch FG, as the angle FHP is of
the ang. FHG. But if the arch BI ⊏, =, ⊐ FP, *c* then c 27. 3.
likewiſe is the angle BDI ⊏, =, ⊐ FHP. Therefore d 6. def. 5.
is the arch BC. FG :: *d* the angle BDC. FHG *e* :: e 15. 5.
$\underbrace{BDC}_{2}$. $\underbrace{FHG}_{2}$ *f* :: A. E. *W.W. to be Dem.* f 20. 3.

Moreover, the angle BMC *g* =CNI; *h* and there- g 27. 3. h 24. 3.
fore the ſegment BCM = CIN. *k* Alſo the triangle k 4. 1.
BDC=CDI; *l* wherefore the Sector BDCM=CD- l 2. ax.
IN. After the ſame manner are the ſectors FHG,
GHL, LHP equall one to the other. Therefore ſince
accordingly as the arch BI ⊏, =, ⊐ FGP, ſo is m 6. def. 5.
likewiſe the ſector BDI ⊏, =, ⊐ FHP; *m* thence
ſhall be the ſector BDC. FHG :: the arch BC. FG.
W.W. to be Dem.

Coroll.

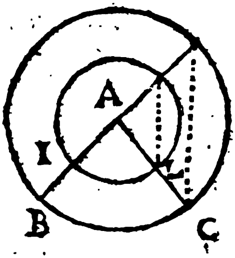
1. Hence, *As ſector is to ſector, ſo is angle to angle.* 11. 5.

2. *The angle* BDC *in the center is to four right angles, as the arch* BC, *on which it inſiſts, to the whole circumference.*

For as the angle BDC is to a right angle, ſo is the arch BC to a quadrant. Therefore BDC is ſo to four right angles as the arch BC is to four quadrants, that is, the whole circumference. Alſo, the angle A. 2 right :: the arch BC. periphery.

3. Hence, *The arches* IL, BC *of unequall circles which ſubtend equall angles, whether at the centers, as* IAL *and* BAC, *or at the periphery, are like.*

For IL. periph :: angle IAL (BAC.) 4 right. Alſo Arch BC. Periph :: angle BAC. 4 right. Therefore IL. periph :: BC periph. And conſequently the arches IL and BC are like. Whence



4. *Two ſemidiameters* AB, AC *cut off like arches* IL, BC *from concentricall peripheries.*

The End of the ſixth Book of Euclide's *Elements.*

THE SEVENTH BOOK OF EUCLIDE'S ELEMENTS.

Definitions.

I. VNity is that, by which every thing that is, is called One.

II. Number is a multitude compoſed of unities.

III. One number is a Part of another, the leſſer of the greater, when the leſſer meaſureth the greater.

Every part is denominated from that number, by which it meaſures the number whereof it is a part; as 4 is called the third part of 12, *becauſe it meaſures* 12 *by* 3.

IV. But the leſſer number is termed Parts, when it meaſureth not the greater.

All parts whatſoever are denominated from thoſe two numbers, by which the greateſt common meaſure of the two numbers meaſures each of them; as 10 *is ſaid to be* $\frac{2}{3}$ *of the number* 15; *becauſe the greateſt common meaſure, which is* 5, *meaſures* 10 *by* 2, *and* 15 *by* 3.

V. A number is Multiplex (or Manifold) a greater in compariſon of a leſſer, when the leſſer meaſureth the greater.

VI. An Even number is that which may be divided into two equall parts.

VII. But an Odde number is that which cannot be divided into two equall parts; or, that which differeth from an even number by an unitie.

VIII. A number Evenly Even is that which an even number meaſureth by an even number.

IX. But a number Evenly Odde is that which an even number meaſureth by an odde number.

X. A number Oddly Odd is that which an odd number meaſureth by an odd number.

XI. A Prime (or firſt) number is that, which is meaſured only by an unitie.

XII. Numbers Prime the one to the other, are ſuch as onely an unitie doth meaſure, being their common meaſure.

XIII. A Compoſed number is that which ſome certain number meaſureth.

XIV Numbers Compoſed the one to the other, are they, which ſome number, being a common meaſure to them both, doth meaſure.

In this, and the preceding definition, unitie is not a number.

XV. One number is ſaid to Multiply another, when the number multiplied is ſo often added to it ſelf, as there are unities in the number multiplying, and another number is produced.

Hence in every multiplication a unitie is to the multiplier, as the multiplied is to the product.

Obſ. *That many times, when any numbers are to be multiplied* (*as* A *into* B) *the conjunction of the letters denotes the Product*: *So* $AB = A \times B$, *and* $CDE = C \times D \times E$.

XVI. When two numbers multiplying themſelves produce another, the number produced is called a Plane number; and the numbers which multiplied one another, are called the Sides of that number: *So* $2\ (C) \times 3\ (D) = 6 = CD$ *is a plane number.*

XVII. But when three numbers multiplying one another produce any number, the number produced is termed a Solid number; and the numbers multiplying one another, are the ſides thereof: *So* $2\ (C) \times 3\ (D) \times 5\ (E) = 30 = CDE$ *is a ſolid number.*

XVIII. A Square number is that which is equally equall; or, which is conteined under two equall numbers. *Let* A *be the ſide of a ſquare; the ſquare is thus noted*, AA, *or* Aq.

XIX. A Cube is that number which is equally equall

quall equally; or, which is conteined under three equall numbers. *Let* A *be the ſide of a Cube; the Cube is thus noted,* AAA, *or* Ac.

In this definition, and the three foregoing, unitie is a number.

XX. Numbers are proportionall, when the firſt is as multiplex of the ſecond as the third is of the fourth; or, the ſame part; or, when a part of the firſt number meaſures the ſecond, and the ſame part of the third meaſures the fourth, equally: and on the contrary. *So* A.B :: C. D. *that is,* 3.9 :: 5.15.

XXI. Like Plane, and ſolid numbers are they, which have their ſides proportionall: *Namely, not all the ſides, but ſome.*

XXII. A Perfect number is that which is equall to its own parts.

As 6. & 28. But a number that is leſſe then it's parts is called an Abounding *number; and a greater a* Diminutive: *ſo 12 is an abounding, 15 a diminutive number.*

XXIII. One number is ſaid to meaſure another, by that number, which, when it multiplies, or is multiplied by it, it produceth.

In Diviſion, a unitie is to the quotient as the diviſor is to the dividend. Note, *that a number placed under another with a line between them, ſignifies diviſion:* So $\frac{A}{B}$ = A divided by B, & $\frac{CA}{B}$ = C x A divided by B.

Two numbers are called Termes or Roots of Proportion, leſſer then which cannot be found in the ſame proportion.

Poſtulates, or Petitions.

1. THat numbers equall or manifold to any number may be taken at pleaſure.

2. That a greater number may be taken then any number whatſoever.

3. That Addition, Subſtraction, Multiplication, Di-

Diviſion, and the Extractions of roots or ſides of ſquare and cube numbers, be alſo granted as poſſible.

Axiomes.

1. WHatſoever agrees with one of many equall numbers, agrees likewiſe with the reſt.

2. Thoſe parts that are the ſame to the ſame part, or parts, are the ſame amongſt themſelves.

3. Numbers that are the ſame parts of equall numbers, or of the ſame number, are equall amongſt themſelves.

4. Thoſe numbers, of whom the ſame number, or equall numbers, are the ſame parts, are equall amongſt themſelves.

5. Unitie meaſures every number by the unities that are in it; that is, by the ſame number.

6. Every number meaſures it ſelf by a unitie.

7. If one number, multiplying another, produce a third, the multiplier ſhall meaſure the product by the multiplied; and the multiplied ſhall meaſure the ſame by the multiplier.

Hence, No prime number is either a plane, ſolid, ſquare, or cube number.

8. If one number meaſure another, that number by which it meaſureth ſhall meaſure the ſame by the unities that are in the number meaſuring, that is, by the number it ſelf that meaſures.

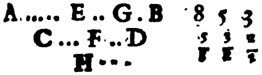
9. If a number meaſuring another, multiply that by which it meaſureth, or be multiplied by it, it produceth the number which it meaſureth.

10. How many numbers ſo ever any number meaſureth, it likewiſe meaſures the number compoſed of them.

11. If a number meaſure any number, it alſo meaſureth every number which the ſaid number meaſureth.

12. A number that meaſures the whole & a part taken away, doth alſo meaſure the reſidue.

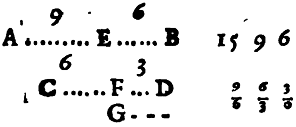
PROPOSITION I.



If two unequall numbers AB, CD, *being given, the leſſer* CD *be continually taken from the greater* AB *(and the reſidue* EB *from* CD,&c.) *by an alternate ſubſtraction, and the number remaining do never meaſure the precedent, till the unitie* GB *be taken; then are the numbers which were given* AB, CD, *prime the one to the other.*

If you deny it, let AB, CD have a common meaſure, namely the number H. Therefore H meaſuring CD, doth a alſo meaſure AE; and conſequently the a 11.ax 7.
remainder EB; a therefore it likewiſe meaſures CF, and b ſo the remainder FD; a wherefore it alſo mea- b 12.ax.7.
ſures EG. But it meaſured the whole EB, and b therefore it muſt meaſure that which remaineth GB, a unitie, it ſelf being a number. c *Which is Abſurd.* c 9.ax.1.

PROP. II.



Two numbers AB, CD *being given, not prime the one to the other, to find out their greateſt common meaſure* FD.

Take the leſſer number CD from the greater AB as often as you can. If nothing remains, a it is manifeſt that CD is the greateſt common meaſure. But if a 6.ax.7.
there remains ſomething (as EB) then take it out of CD, and the reſidue FD out of EB, and ſo forward till ſome number (FD) meaſure the ſaid EB (b for this will be, before you come to a unitie.) FD b 1.7.
ſhall be the greateſt common meaſure.

For FD c meaſures EB, and d therefore alſo CF; c conſtr. d 11 ax.7.
and c conſequently the whole CD; d therefore likewiſe AE; and ſo meaſures the whole AB. Wherefore e 12.ax.7.

it is evident that FD is a common meaſure. If you deny it to be the greateſt, let there be a greater (G;)
d 11. ax. 7. then whereas G meaſureth CD, it [d] muſt likewiſe
e 12. ax. 7. meaſure AE, [e] & the reſidue EB, [d] as alſo CF, [e] and
g ſuppoſ. by conſequence the reſidue FD, [g] the greater the
h 9. ax. 1. leſſe. [h] *Which is abſurd.*

Coroll.

Hence, A number that meaſures two numbers, does alſo meaſure their greateſt common meaſure.

PROP. III.

A............ 12
B 8
D 4
C 6
E .. 2
F - - -

Three numbers being given, A, B, C, not prime to one another, to find out their greateſt common meaſure E.

Find out D the greateſt common meaſure of the two numbers A, B. If D meaſures C the third, it is clear that D is the greateſt common meaſure of all the three numbers. If D does not meaſure C, at leaſt D and C will be compoſed the one to the other, by the Coroll. of the Prop. preceding. Therefore let E be the greateſt common meaſure of the ſaid numbers D and C, and it appears to be the number required.

a conſtr. For E [a] meaſures C and D, and D meaſures A
b 11. ax. 7. and B; therefore [b] E meaſures each of the numbers A, B, C: neither ſhall any greater (F) meaſure them;
c cor. 1. 7. for if you affirm that, [c] then F meaſuring A and B, does likewiſe meaſure D their greateſt common meaſure; and in like manner, F meaſuring D and C, does alſo meaſure E [c] their greateſt common mea-
d ſuppoſ. ſure, [d] the greater the leſſe. [e] *Which is abſurd.*

e 9. ax. 1.

Coroll.

Hence, A number that meaſures three numbers, does alſo meaſure their greateſt common meaſure.

PROP. IV.

A......6
B.......7
B.....................18
B......A 9

Every leſſe number A is of every greater B either a part or parts.

If A and B be prime to one another, [a] A ſhall be as many parts of the number B, as there are unities in A (as $6 = \frac{6}{7} 7$.) But if A meaſures B, it is [b] plain that A is a part of B (as $6 = \frac{1}{3} 18$.) Laſtly, if A and B be otherwiſe compoſed to one another, [c] the greateſt common meaſure ſhall determine how many parts A does contain of B; as $6 = \frac{2}{3} 9$.

a 4.def.7.
b 3.def.7.
c 4.def.7.

PROP. V.

A......6 D....4
6 6 4 4
B......G......C 12. E....H....F 8.

If a number A *be a part of a number* BC, *and another number* D *the ſame part of another number* EF; *then both the numbers together* (A + D) *ſhall be the ſame part of both the numbers together* (BC + EF,) *which one number* **A** *is of one number* **BC**.

For if BC be reſolved into it's parts BG, GC, equall to A; and EF alſo into its parts EH, HF equall to D; [a] the number of parts in BC ſhall be equall to the number of parts in EF. Therefore ſince A + D [b] = BG + EH = GC + HF, thence A + D ſhall be as often in BC + EF, as A is in BC. *Which was to be Demonſtrated.*

a hyp.
b conſt. and 2.ax.1.

Or thus. Let $a = \frac{x}{2}$, and $b = \frac{y}{2}$. then $2a = x$, [c] and $2b = y$. wherefore $2a + 2b = x + y$. therefore $a + b = \frac{x + y}{2}$. *W. W. to be Dem.*

c 2.ax.1.

PROP. VI.

3 3 4 4
A...G...B 6 D.... H.... E 8
C......... 9 F............ 12.

If a number AB *be parts of a number* C, *and another number* DE *the ſame parts of another number* F; *then both numbers together* AB+DE *ſhall be of both numbers together* C+F *the ſame parts, that one number* AB *is of one number* C.

Divide AB into its parts AG, GB; and DE into its parts DH, HE. The multitude of parts in both AB, DE is equall by ſuppoſition; Wherefore ſince a hyp. AG *a* is the ſame part of the number C, that DH is b 5.7. of the number F; therefore AG+DH *b* ſhall be the ſame part of the compounded number C+F, that one number AG is of one number C. *b* In like manner GB+HE is the ſame part of the ſaid C+F, that c 2.ax.7. one number GB is of one number C. *c* Therefore AB+DE is the ſame parts of C+F, that AB is of C. *W.W. to be Dem.*

Or thus. Let $a = \frac{2}{3}x$, and $b = \frac{2}{3}y$, and $x+y = g$. then, becauſe $3a = 2x$, and $3b = 2y$, is $3a+3b = 2x+2y = 2g$. therefore $a+b = \frac{2}{3}g = \frac{2}{3} : x+y$.

PROP. VII.

5 3
A..... E ... B 8
6 10 6
G...... C.......... F...... D 16

If a number AB *be the ſame part of a number* CD, *that a part taken away* AE *is of a part taken away* CF; *then ſhall the reſidue* EB *be the ſame part of the reſidue* FD *that the whole* AB *is of the whole* CD.

a 1.poſt.7. *a* Let EB be the ſame part of the number GC that b 5.7. AB is of CD, or AE of CF, *b* therefore AE+EB is the ſame part of CF+GC that AE is of CF, or AB c 6 ax.1. of CD. *c* therefore GF=CD. Take away CF com- d 3 ax.1. mon to both, & *d* there remains GC=FD. *e* Where- e 2.ax.7. fore EB is the ſame part of the reſidue FD (GC) that the whole AB is of the whole CB. *Which was to be Dem.*

Or thus. Let $a+b=x$; and $c+d=y$; and $x=3y$, in like manner as $a=3c$; I ſay $b=3d$. For $3c+3d$ f $=3y=x$ g $=a+b$. take away from both $3c$ g $=a$, and *b* there remains $3d=b$. *W.W. to be Dem.* f 1. 2. g hyp.

PROP. VIII.

6 2 4 2 2
A ...... H .. G .... E .. L .. B 16
18 6
C .... F ...... D 24

If a number AB *be the ſame parts of a number* CD, *that a part taken away* AE *is of a part taken away* CF; *the reſidue alſo* EB *ſhall be of the reſidue* FD *the ſame parts, that the whole* AB *is of the whole* CD.

Divide AB into AG, GB, parts of the number CD; alſo AE into AH, HE, parts of the number CF; and take GL = AH = HE. *a* wherefore HG = EL. And becauſe *b* AG = GB, *c* therefore HG = LB. Now whereas the whole AG is the ſame part of the whole CD that the part taken away AH is of the part taken away CF, *d* the reſidue HG or EL ſhall be the ſame part alſo of the reſidue FD that AG is of CD. In like manner, becauſe GB is the ſame part of the whole CD, that HE or GL are of CF, *d* therefore the reſidue LB ſhall be the ſame part of the reſidue FD that GB is of the whole CD. Therefore EL + LB (EB) is the ſame parts of the reſidue FD, that the whole AB is of the whole CD. *W.W. to be Dem.* a 3. ax. 1. b conſtr. c 3 ax. 1. d 7 7.

Or thus more eaſily. Let $a+b=x$, and $c+d=y$. Alſo $y=\frac{2}{3}x$ as well as $c=\frac{2}{3}a$; or, *e* which is the ſame, $3y=2x$; and $3c=2a$. I ſay $d=\frac{2}{3}b$. For $3c+3d$ f $=3y=2x$ f $=2a+2b$. g Therefore $3c+3d=2a+2b$. take away from each $3c$ *h* $=2a$, and *k* there remains $3d=2b$. *l* therefore $d=\frac{2}{3}b$. *Which was to be Dem.* e 9 ax. 7. f 1. 2. g 1 ax. 1. h hyp k 3. ax. 1. l 8 ax. 7.

PROP. IX.

A 4
4 4
B .... G .... C 8
5 D 5
5 5
E ..... H ..... F 10

If a number A be a part of a number BC, and another number D the ſame part of another number EF; then alternately what part or parts the firſt A is of the third D, the ſame part or parts ſhall the ſecond BC be of the fourth EF.

A is ſuppoſed $\sqsupset$ D. therefore let BG, GC, and EH, HF, parts of the numbers BC, EF be equall; BG and GC to A; and EH, HF to D. The multitude of parts is put equall in both. But it is clear that BG is
a 1. ax. 7. and 4. 7. [a] the ſame part or parts of EH, that GC is of HF;
b 5. & 6. 7. [b] wherefore BC (BG+GC) is the ſame part or parts of EF (EH + HF) that FG alone (A) is of EH alone (D.) *Which was to be Dem.*

Or thus. Let $a = \frac{b}{3}$, and $c = \frac{d}{3}$; or $3a = b$,
* 15. 5. and $\frac{3c}{a} = d$, then is $\frac{c}{a} \overset{*}{=} \frac{3c}{3a} = \frac{d}{b}$.

PROP. X.

A .. G .. B 4
C 6
5 5
D ..... H ..... E 10
F 15

If a number AB be parts of a number C, and another number DE the ſame parts of another number F; then alternately, what parts or part the firſt AB is of the third DE, the ſame parts or part ſhall the ſecond C be of the fourth F.

AB is taken $\sqsupset$ DE, and C $\sqsupset$ F. Let AG, GB, and DH, HE be parts of the numbers C and F; *viz.* as many in AB, as in DE. It is manifeſt that AG is
a 9. 7. the ſame part of C, that DH is of F. [a] whence alter-
b 5. and 9. 7. nately AG is of DH, and likewiſe GB of HE, & [b] ſo conjointly AB of DE the ſame part, or parts, that C is of F. *Which was to be Dem.*

Or thus. Let $a = \frac{2b}{3}$, & $c = \frac{2d}{3}$; or $3a = 2b$, & $3c = 2d$. Then is $\frac{c}{a} = \frac{3c}{3a} = \frac{2d}{2b} = \frac{d}{b}$.

PROP.

PROP. XI.

4 3
A .... E ... B 7
8 6
C .... F ...... D 14

If a part taken away AE *be to a part taken away* CF, *as the whole* AB *is to the whole* CD, *the reſidue alſo* EB *ſhall be to the reſidue* FD, *as the whole* AB *is to the whole* CD.

Firſt, let AB be ⊐ CD; [a] then AB is either a part a 4. 7.
or parts of the number CD; and likewiſe AE is [b] the b 20. *def* 7.
ſame part or parts of CF; [c] therefore the reſidue EB c 7, *or* 8. 7.
is the ſame part or parts of the reſidue FD that the whole AB is of the whole CD. [b] & ſo AB. CD :: EB. FD. But if AB be ⊏ CD, then according to what is already ſhewn, will CD. AB :: FD. EB. therefore by inverſion AB. CD :: EB. FD.

PROP. XII.

A, 4. C, 2. E, 3.
B, 8. D, 4. F, 6.

If there be divers numbers, how many ſoever, proportionall (A. B :: C. D :: E. F;) *then as one of the antecedents* A *is to one of the conſequents* B, *ſo ſhall all the antecedents* (A + C + E) *be to all the conſequents* (B + D + F.)

Firſt, let A, C, E, be ⊐ B, D, F; then (becauſe of the ſame proportions) [a] ſhall A be the ſame part or parts a 20 *def* 7.
of B that C is of D; [b] and likewiſe conjointly A + C b 5, & 6. 7.
ſhall be the ſame part or parts of B + D that A alone is of B alone. In the like manner A + C + E is the ſame part or parts of B + D + F that A is of B. [c] Therefore A + C + E. B + D + F :: A. B. But if c 20. *def* 7.
A, C, E, be put greater then B, D, F, the ſame may be ſhewn by inverſion.

PROP. XIII.

A, 3. C, 4.
B, 5. D, 12.

If there be four numbers proportionall (A. B :: C. D.) *then alternately they ſhall alſo be proportionall,* (A.C :: B.D.)

First let A and C be ⊐ B and D, and A ⊐ C. By reaſon of the ſame proportion a ſhall A be the ſame part or parts of B, that C is of D. b Therefore alternately A is the ſame part or parts of C that B is of D. and ſo A.C :: B.D But if A be ⊏ C, and A and C ſuppoſed ⊐ B and D, the matter will be the ſame by inverting the proportions.

a 10. def. 7.
b 9. and 10. 7.

PROP. XIV.

A, 9. D, 6.
B, 6. E, 4.
C, 3. F, 2.

If there be numbers how many ſoever, A, B, C, *and as many more equall to them in multitude, which may be compared two and two in the ſame proportion* (A.B :: D.E. *and* B.C :: E.F;) *they ſhall alſo, of equality, be in the ſame proportion* (A.C :: D.F.)

For becauſe A.B :: D.E. a therefore alternately is A.D :: B. E :: a C. F. a and ſo again by changing A.C :: D.F. *W.W. to be Dem.*

a 13. 7.

PROP. XV.

1. D..
B...3. E......6.

If a unite meaſure any number B, *and another number* D *do equally meaſure ſome other number* E; *alternately alſo ſhall a unite meaſure the third number* D, *as often as the ſecond* B *doth the fourth* E.

For ſeeing 1 is the ſame part of B, that D is of E; a therefore alternately ſhall 1 be the ſame part of D, that B is of E. *W.W. to be Dem.*

a 9. 7.

PROP. XVI.

4. A, 3. *If two numbers A, B, multi-*
3. B, 4. *plying themſelves the one into*
, 12, BA, 12. *the other, produce any numbers*
AB, BA; *the numbers produced*
and BA *ſhall be equall the one to the other.*

For becauſe AB = A x B, a therefore ſhall 1 a 15. def. 7.
as often in A, as B in AB, b and by conſequence b 15. 7.
ernately 1 ſhall be as often in B as A in AB. But
that BA = B x A, a therefore ſhall 1 be as often
B, as A in BA. therefore as often as 1 is in AB, ſo
en is 1 in BA. and c ſo AB = BA. *W.W. to be Dem.* c 4. ax. 7.

PROP. XVII.

A, 3. *If a number* A *multiplying*
2. C. 4. *two numbers* B, C, *produce other*
, 6. AC, 12. *numbers* AB, AC; *the numbers*
produc'd of them ſhall be in
ſame proportion that the numbers multiplyed are.
B. AC :: B. C.)

For being AB = A x B, a therefore ſhall 1 be as a 15. def. 7.
en in A, as B in AB. a Likewiſe becauſe AC = A
C, therefore ſhall 1 be as often in A, as C in AC.
l ſo alſo B as often in AB as C in AC. b wherefore b 20. def. 7.
AB :: C. AC. c and therefore alſo alternately B. c 13. 7.
: AB. AC. *W.W. to be Dem.*

PROP. XVIII.

5. C, 5. *If two numbers* A, B, *multi-*
3. B, 9. *plying any number* C, *produce*
, 15. BC, 45. *other numbers* AC, BC; *the*
numbers produced of them ſhal
the ſame proportion that the numbers multiplying.
(A. B :: AC. BC.)

For a AC = CA, and BC a = CB; ſo the ſame
nultiplying A and B produceth AC and BC. a 16. 7.
erefore A. B :: AC. BC. *W.W. to be Dem.* b 17. 7.

Schol.

Schol.

Hence is deduced the vulgar manner of reducing fractions ($\frac{3}{5}, \frac{7}{9}$) to the ſame denomination. For multiply 9 both by 3 and 5, and they produce $\frac{27}{45} = \frac{3}{5}$ becauſe by this, 3. 5 :: 27. 45. Likewiſe multiply 5 by 7 and 9; there ariſes $\frac{35}{45} = \frac{7}{9}$; becauſe 7.9 :: 35. 45.

PROP. XIX.

A, 4. B, 6. C, 8. D, 12.
AD, 48. BC, 48.

If there be four numbers in proportion (A.B :: C.D) *the number produced of the firſt and fourth* (AD) *is equall to the number which is produced of the ſecond and third* (BC.) *And if the number which is produced of the firſt and fourth* (AD) *be equall to that produced of the ſecond & third* (BC) *thoſe four numbers ſhall be in proportion* (A. B :: C. D.)

a 17. 7. b hyp. c 18. 7. 1. *Hyp.* For AC.AD [a] :: C.D [b] :: A.B [c] :: AC.BC. [d] therefore AD = BC. *W.W. to be Dem.*

d 9. 5. e hyp. f 7. 5. g 17. 7. h 18. 7. k 11. 5. 2. *Hyp.* Becauſe [e] AD = BC, therefore AC. AD [f] :: AC. BC. But AC. AD [g] :: C. D. and AC. BC [h] :: A.B. [k] therefore C.D :: A.B. *W.W. to be Dem.*

PROP. XX.

A.	B.	C.
4.	6.	9.

AC, 36. BB, 36.
D, 6.

If there be three numbers in proportion (A. B :: B. C.) *the number contained under the extremes* (AC) *is equall to the ſquare made of the middle* (BB.) *And if the number contained under the extremes be equall to that* (Bq,) *produced of the middle, thoſe three numbers ſhall be in proportion* ($\frac{A}{B} :: \frac{B}{C.}$)

a 1. ax. 7. b 19. 7. 1. *Hyp.* For take D = B. [a] therefore A.B :: D (B.) C. [b] wherefore AC = BD, [b] or BB. *Which was to be Dem.*

2.*Hyp*.Becauſe AC [c] = BD, [d] therefore A. B :: D (B.) C. *W.W.to be Dem.* c hyp. d 19. 7.

PROP. XXI.

A ... G .. B 5. E .... 10.
C .. H . D 3. F ......, 6.

Numbers AB, CD, *being the leaſt of all that have the ſame proportion with them* (E, F) *doe equally meaſure the numbers* E, F, *having the ſame proportion with them; the greater* AB *the greater* E, *and the leſſer* CD *the leſſer* F.

For AB.CD [a] :: E.F. [b] therefore alternately AB. E :: CD.F. [c] therefore AB is the ſame part or parts of E that CD is of F. Not parts; for if ſo, then let AG, GB be parts of the number E; and CH, HD, parts of the number F. [e] therefore AG. E :: CH. F. and by inverſion AG. CH [d] :: E. F [e] :: AB. CD. therefore AB, CD are not the leaſt in their proportion; which is contrary to the hypotheſis. Therefore, &c. a hyp. b 13. 7. c 20. def. 7. d 13. 7. e hyp.

PROP. XXII.

A, 4. D, 12.
B, 3. E, 8.
C, 2. F, 6.

If there be three numbers A, B, C; *and other numbers equall to them in multitude,* D, E, F; *which may be compared two & two in the ſame proportion: and if alſo the proportion of them be perturbed* (A.B :: E. F. *and* B.C :: D.E.) *then of equality they ſhall be in the ſame proportion* (A.C :: D. F.)

For becauſe A.B [a] :: E.F, therefore ſhall AF = BE; and becauſe B. C :: [a] D. E. [b] therefore BE = CD. [c] and conſequently AF = CD. [d] Wherefore A. C :: D.F. *W. W. to be Dem.* a hyp. b 19. 7. c 1. ax. 1. d 19. 7.

⁂

PROP.

PROP. XXIII.

A, 9. B, 4.
C ---- D ---
E --

Numbers prime the one to the ther, A, B, are the leaſt of all nu bers that have the ſame proport with them.

If it be poſſible, let C and D be leſſe then A a B, and in the ſame proportion; a therefore C m ſures A equally as D meaſures B, namely by t ſame number F; and ſo C ſhall be b as often in A 1 is in E; c likewiſe alternately, E as often in A a in C. By the like inference as many times as 1 is D, ſo many times ſhall E be in B. Therefore E m ſures both A and B; which conſequently are n prime the one to the other, *contrary to the Hypoth.*

a 21. 7.
b 15 def. 7.
c 15. 7.

PROP. XXIV.

A, 9. B, 4.
C ---
D --- E --

Numbers A, B, being the leaſ all that have the ſame proporti with them, are prime the one to t others.

If it be poſſible, let A and B have a common me ſure C; and let the ſame meaſure A by D, and B E; a therefore CD = A, b and CE = B. b Wherefo A.B :: D.E. But D and E are leſſer then A and B, being but parts of them. Therefore A and B are n the leaſt in their proportion, *againſt the Hypoth.*

a 9. ax. 7.
b 17. 7.

PROP. XXV.

A, 9. B, 4.
C, 3. D --

If two numbers A, B, be prime t one to the other, the number C me ſuring one of them A, ſhall be prime the other number B.

For if you affirme any other D to meaſure t numbers B and C, a then D meaſuring C does al meaſure A; and conſequently A and B are not prin the one to the other: *Which is againſt the Hypotheſ*

a 11. ax. 7.

PROP. XXVI.

A, 5. C, 8.
B, 3.
$\overline{AB}$, $\overline{15}$. E . . . .
F . . . .

If two numbers, A, B, be prime to any number C, the number alſo produced of them AB ſhall be prime to the ſame C.

If it be poſſible, let the number E be a common meaſure to AB, and C; and let $\frac{AB}{E}$ be = F; [a] thence AB = EF; [b] wherefore alſo E.A :: B.F. But becauſe A is prime to C, which E meaſures, [c] therefore E and A are prime to one another, [d] and ſo leaſt in their own proportion, [e] and conſequently they muſt meaſure B and F; namely F ſhall meaſure B, and A ſhall meaſure F. Therefore ſeeing E meaſures both B and C, they ſhall not be prime to one another: *contrary to the Hypoth.*

a 9. ax. 7.
b 19. 7.
c 25. 7.
d 23. 7.
e 21. 7.

PROP. XXVII.

A, 4. B, 5.
Aq, 16.
D, 4.

If two numbers A, B, be prime to one another, that alſo which is produc'd of one of them (Aq) ſhall be prime to the other B.

Take D = A; [a] therefore each of D, and A are prime to B. [b] wherefore A D or Aq is prime to B. *W.W. to be Dem.*

a 1. ax. 7.
b 26. 7.

PROP. XXVIII.

A, 5. C, 4.
B, 3. D, 2.
$\overline{AB}$, 15. $\overline{CD}$, 8.

If two numbers A, B be prime to two numbers C, D each to either of both, the numbers alſo produced of them AB, CD ſhall be prime to one another.

For being A and B are prime to C, [a] therefore ſhall AB alſo be prime to the ſame. And by the ſame reaſon

a 26. 7.

b 26. 7. ſon ſhall AB be prime to D. b Therefore AB is prime to CD. *W.W.to be Dem.*

PROP. XXIX.

A, 3. B, 2.
Aq, 9. Bq, 4.
Ac, 27. Bc, 8.

If two numbers A, B, *be prime to one another, and each multiplying himſelf produce another number* (Aq, *and* Bq;) *then the numbers produced of them* (Aq, Bq) *ſhall be prime to one another. And if the numbers given at firſt,* A, B, *multiplying the ſaid produced numbers* (Aq, Bq) *produce others* (Ac, Bc) *thoſe numbers alſo ſhall be prime to one another: and this ſhall ever happen about the extremes.*

For becauſe A is prime to B, a therefore Aq ſhall be prime to B. and Aq being prime to B, a therefore Aq ſhall alſo be prime to Bq. Again, becauſe A is as well prime to B and Bq, as Aq is to the ſaid B and Bq, b therefore ſhall A x Aq, that is, Ac, be prime to B x Bq, that is, to Bc: And ſo forth of the reſt.

a 27. 7.

b 28. 7.

PROP. XXX.

8 5
A ....B ..... C 13. D - - - -

If two numbers AB, BC, *be prime the one to the other; then both added together* (AC) *ſhall be prime to either of them* AB, BC. *And if both added together* AC *be prime to any one of them* AB, *the numbers alſo given in the beginning* AB, BC, *ſhall be prime to one another.*

1. *Hyp.* For if you would have AC, AB to be compoſed, let D be the common meaſure: a this ſhall meaſure the reſidue BC: and therefore AB, BC, are not prime to one another; *which is againſt the hypoth.*

a 12. ax. 7.

2. *Hyp.* AC, AB being taken for prime to one another, let D be the common meaſure of AB, BC. b But ſeeing that meaſures the whole AC, therefore AC, AB, are not prime to one another; *contrary to the hypoth.*

b 10. ax 7.

Coroll.

Hence, A number, which being compounded of two, is prime to one of them, is alſo prime to the other.

PROP. XXXI.

A, 5. B, 8. *Every prime number* A *is prime to every number* B, *which it meaſureth not.*

For if any common meaſure doth meaſure both, A, B, [a] then A will not be a prime number; *contrary to the Hyp.* a 11. def. 7.

PROP. XXXII.

A, 4. D, 3.
B, 6. E, 8.
AB, 24.

If two numbers A, B, *multiplying one another produce another* AB, *and ſome prime number* D *meaſure the number produced of them* AB; *then ſhall it alſo meaſure one of thoſe numbers,* A *or* B, *which were given at the beginning.*

Suppoſe the number D not to meaſure the number A, and let $\frac{AB}{D}$ be = E. [a] then AB = DE; [b] whence D.A :: B.E. [c] But D is prime to A; [d] therefore D and A are the leaſt in their proportion; [e] and conſequently D meaſures B as often as A meaſures E. *Which was to be Dem.* a 9. ax. 7. b 19. 7. c hyp. and 31. 7. d 23. 7. e 21. 7.

PROP. XXXIII.

A, 12. *Every compoſed number* A *is meaſured*
B, 2. *by ſome prime number* B.

Let one or more numbers [a] meaſure A, of which let the leaſt be B; that ſhall be a prime number: for if it be ſaid to be compoſed, then ſome [a] leſſer number ſhall meaſure it, [b] which ſhall alſo conſequently meaſure A. Wherefore B is not the leaſt of them which meaſure A. *contrary to the Hyp.* a 13. def. 7. b 11. ax. 7.

PROP.

PROP. XXXIV.

A, 9. *Every number* A, *is either a prime, or meaſured by ſome prime number.*

For A is neceſſarily either a prime or a compoſed number. If it be a prime, 'tis that we affirm. If compoſed, [a] then ſome prime number meaſureth it. *W.W. to be Dem.*

a 33. 7.

PROP. XXXV.

A, 6. B, 4. C, 8. H - - I - - K - - - -
D, 2.
E, 3. F, 2. G, 4. L - - -

How many numbers ſoever A, B, C, *being given, to find the leaſt numbers* E, F, G, *that have the ſame proportion with them.*

a 33. 7.
b 3. 7.

If A, B, C, be prime to one another, [a] they ſhall be the leaſt in their proportion. If they be compoſed, [b] let their greateſt common meaſure be D, which let meaſure them by E, F, G. Theſe are then leaſt in the proportion A, B, C.

c 9. ax. 7.
d 17. 7.
e 21. 7.
f 9. ax. 7.
g 1. ax. 1.
h 19. 7.
k ſuppoſ.
l 10. def. 7.

For D x E, F, G, [c] produceth ABC, [d] therefore they are all in the ſame proportion. But allow other numbers H, I, K to be the leaſt in the ſame proportion; [e] which ſhall therefore equally meaſure A, B, C, namely by the number L. [f] therefore L x H, I, K, ſhall produce A, B, C, [g] and conſequently ED = A = HL. [h] from whence E.H :: L. D. But E [k] ⊏ K; [l] therefore L ⊏ D, and ſo D is not the greateſt common meaſure of A, B, C. *Which is againſt the Hypoth.*

Coroll.

Hence, The greateſt common meaſure of how many numbers ſoever do's meaſure them by the numbers which are leaſt of all that have the ſame proportion with them. Whereby appears the vulgar method of reducing fractions to the leaſt termes.

PROP.

PROP. XXXVI.

"*Two numbers being given* A, B, *to find out the leaſt number which they meaſure.*

A, 5. B, 4.
AB, 20.
D - - - - - -
E - - - F - - -

1. *Caſe.* If A and B be prime the one to the other, AB is the number required. For it is manifeſt that A and B meaſure B. If it be poſſible, let A and B meaſure ſome other number D $\sqsupset$ AB, if you pleaſe by E, and F. [a] therefore AE = D = BF, [b] and ſo A.B :: F.E. But becauſe A and B [c] are prime the one to the other, [d] & ſo leaſt in their proportion, A ſhall [e] equally meaſure F as B does E. But B.E [f] :: B. AE (D) [g] Therefore AB ſhall alſo meaſure D, which is leſſe then it ſelf. *Which is Abſurd.*

a 9 ax. 7.
1. ax. 1.
b 19. 7.
c hyp.
d 23. 7.
e 21. 7.
f 17. 7.
g 20. def.

A, 6. B, 4. F - - - -
C, 3. D, 2. G - - - H - - -
AD, 12.

2. *Caſe.* But if A and B be compoſed one to another, [h] let there be found C and D the leaſt in the ſame proportion. [k] therefore AD = BC; and AD or BC ſhall be the number ſought for.

For it is [l] plain that B and D doe meaſure AD or BC. Conceive A and B to meaſure F $\sqsupset$ AD, namely A by G, and B by H. [m] therefore AG = F = BH. [n] whence A. B :: H. G [o] :: C. D. [p] and conſequently C equally meaſures H as D does G. But D. G [q] :: AD. AG (F.) therefore AD [r] meaſures F, the greater the leſſe. *Which is Abſurd.*

h 35. 7.
k 19. 7.
l 7 ax. 7.
m 9 ax. 7.
n 19. 7.
o conſtr.
p 21. 7.
q 17. 7.
r 20. def.

Coroll.

Hence, If two numbers multiply the leaſt that are in the ſame proportion, the greater the leſſe, and the leſſe the greater, the leaſt number which they meaſure ſhall be produced.

PROP. XXXVII.

A, 2. B, 3.
E 6
C---F---D

If two numbers A, B, meaſure any number CD, the leaſt number which they meaſure E ſhall alſo meaſure the ſame CD.

If you deny it, take E from CD as often as you can, and leave FD ⊐ E. therefore ſeeing A and B a meaſure E, b and E meaſures CF, c likewiſe A and B will meaſure CF. But a they meaſure the whole CD; d therefore alſo they meaſure the reſidue FD; and conſequently E is not the leaſt which A and B meaſure; *Contrary to the Hypoth.*

a hyp.
b conſtr.
c 11. ax. 7.
d 12. ax. 7.

PROP. XXXVIII.

A, 3. B, 4. C, 6.
D, 12.

Three numbers being given A, B, C, to find out the leaſt which they meaſure.

a 36. 7. a Find D to be the leaſt that two of them A and B do meaſure; which if the third C do alſo meaſure, it is manifeſt that D is the number ſought for. But if C do not meaſure D, let E be the leaſt that C and D do meaſure. E ſhall be the number required.

A, 2. B, 3. C, 4.
D, 6. E, 12.
F ---

For it appears by the 11. ax. 7. that A, B, C meaſure E; and it is eaſily ſhewn that they meaſure no other leſſe F. For if you affirm they do, b then D meaſures F, b and conſequently E meaſures the ſame F, the greater the leſſe. *Which is Abſurd.*

b 35. 7.

Coroll.

Hence it appears that, If three numbers meaſure any number, the leaſt alſo, which they meaſure, ſhall meaſure the ſame.

PROP. XXXIX.

A, 12.
B, 4. C, 3.

If any number B measure a number A, the number measured A, ſhall have a part C denominated of the number meaſuring B.

For becauſe $\frac{A}{B}$ [a] = C, [b] ſhall A = BC, [c] therefore $\frac{A}{C}$ = B. *W.W. to be Dem.*

[a] hyp.
[b] 9. ax. 7.
[c] 7. ax. 7.

PROP. XXXX.

A, 15.
B, 3. C, 5.

If a number A have any part whatſoever B, the number C, from which the part B is denominated, ſhall meaſure the ſame.

For being BC [a] = A, [b] thence $\frac{A}{C}$ = B. *W.W. to be Dem.*

[a] hyp. and 9. ax. 7.
[b] 7. ax. 7.

PROP. XXXXI.

G, 12.
H - - .

To find out a number G, which being the leaſt, containeth the parts given $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

[a] Let G be found the leaſt which the denominators 2, 3, 4, meaſure; [b] it is evident that G ha's the parts $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. If it be poſſible let H ⊐ G have the ſame parts; [c] therefore 2, 3, 4, meaſure H; and ſo G is not the leaſt which 2, 3, 4 meaſure: *againſt the ſuppoſ.*

[a] 36. 7.
[b] 39. 7.
[c] 40. 7.

The End of the ſeventh Book.

THE EIGHTH BOOK OF EUCLIDE'S ELEMENTS.

PROP. I.

A, 8. B, 12. C, 18. D, 27.
E - F - - G - - - H - - - -

IF there be divers numbers how many ſoever in continuall proportion, A, B, C, D, and their extremes A, D, prime to one another; then thoſe numbers A, B, C, D, are the leaſt of all numbers that have the ſame proportion with them.

For, if it be poſſible, let there be as many others E,
F, G, H, leſſe then A, B, C, D, and in the ſame pro-
a 14. 7. portion with them. a Therefore of equality A. D ::
E. H. and conſequently A & D are prime numbers,
b 23. 7. c 21. 7. b and ſo the leaſt in their proportion, c equally mea-
ſuring E and H, which are leſſe then themſelves.
Which is Abſurd.

PROP. II.

I.
A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.
Ac, 8. AqB, 12. ABq, 18. Bc, 27.

To find out the leaſt numbers continually proportionall, as many as ſhall be required, in the proportion given of A to B.

Let A and B be the leaſt in the proportion given; Then Aq, AB, Bq, ſhall be the three leaſt in the ſame continuall proportion that A is to B.

For AA. AB [a] :: A. B [a] :: AB. BB. Likewiſe becauſe A and B are [b] prime to one another, [c] ſhall Aq, Bq, be alſo prime to one another, [d] and ſo Aq, AB, Bq, are ∺ the leaſt in the proportion of A to B. a 17. 7. b 24. 7. c 29. 7. d 1. 8.

Moreover, I ſay Ac, AqB, ABq, Bc, are the four leaſt in the proportion of A to B. For AqA. AqB [e] :: A.B. [e] :: ABA (AqB.) ABB. [e] and A.B :: ABq. BBq (Bc.) Therefore ſince Ac, and Bc, are [f] prime to one another, likewiſe [g] ſhall Ac, AqB, ABq, Bc be the four leaſt ∺ in the proportion of A to B. In the ſame manner may you find out as many proportionall numbers as you pleaſe. *W. W. to be Done.* e 17. 7. f 29. 7. g 1. 8.

Coroll.

1. Hence, If three numbers, being the leaſt, are proportionall, their extremes ſhall be ſquares; if four, cubes.

2. How many extremes proportionall ſoever there be, being by this propoſ. found to be the leaſt in the given proportion, they are prime to one another.

3. Two numbers, being the leaſt in the given proportion, doe meaſure all the mean numbers whatſoever of the leaſt in the ſame proportion; becauſe they ariſe from the multiplication of them into certain other numbers.

4. Hence alſo it appears by the conſtruction, that the ſeries of numbers 1, A, Aq, AC; 1, B, Bq, Bc; Ac, AqB, ABq, BC, conſiſts of an equall multitude of numbers; and conſequently, the extreme numbers of how many ſoever the leaſt continually proportionals are the laſt of as many other continually proportionalls from a unite. as the extremes Ac, Bc, of the continually proportionals Ac, AqB, ABq, Bc, are the laſt of as many proportionals from a unite. 1, A, Aq, Ac; and 1, B, Bq, Bc,

5. 1, A, Aq, Ac; and B, BA, B Aq; and Bq, ABq, are ∺ in the proportion of 1 to A. Alſo B, Bq, Bc; and A, AB, ABq; and Aq, AqB are ∺ in the proportion of 1 to B.

PROP. III.

A, 8. B, 12. C, 18. D, 27. *If there be numbers continually proportionall, how many ſoever, A, B, C, D, being alſo the leaſt of all that have the ſame proportion with them; their extremes A, D, are prime to one another.*

a 2. 8. For if there be a found as many numbers the leaſt in the proportion of A to B, they ſhall be no other than A, B, C, D; therefore, by the ſecond *coroll.* of the precedent *prop.* the extremes A and D are prime to one another. *W.W. to be Dem.*

PROP. IV.

A, 6. B, 5. C, 4. D, 3.
H, 4. F, 24. E, 20. G, 15.
I. . K. . L. .

Proportions how many ſoever being given in the leaſt numbers (A to B, and C to D) to find out the leaſt numbers continually proportionall in the proportions given.

a 36. 7.
b 3. poſt 7.
a Find out E the leaſt number which B and C do meaſure; and let B meaſure E b as often as A does an other F, *viz.* by the ſame number H. b Alſo let C meaſure the ſaid E as often as D meaſures an other G. then F, E, G, ſhall be the leaſt in the proportions given. For AH c = F, and BC c = E; d therefore A.B :: AH. BH e :: F. E. In like manner C. D :: E.G; therefore F, E, G, are continually proportionall in the proportions given. And they are moreover the leaſt in the ſaid proportions; for conceive other numbers I, K, L, to be the leaſt; f then A and B muſt equally meaſure I and K, f and C and D likewiſe K and L; and ſo B and C meaſure the ſame K. g Wherefore alſo E meaſures the ſame number K, which is leſſe then it ſelf. *Which is Abſurd.*

c 9 ax. 7.
d 18. 7.
e 7. 5.
f 21. 7.
g 35. 7.

A, 6. B, 5. C, 4. D, 3. E, 5. F, 7.
H, 24. G, 20. I, 15. K, 21.

But three proportions being given; A to B, C to D, and E to F; find out as before three numbers H, G, I, the leaſt continually in the proportions of A to B, and C to D. Then if E meaſures I, [a] take another number K which may be equally meaſur'd by F; and thoſe four numbers H, G, I, K, ſhall be continually the leaſt in the given proportions; which we need go no other way to prove then we did in the firſt part. a 3. poſt. 5.

A, 6. B, 5. C, 4. D, 3. E, 2. F, 7.
H, 24. G, 29. I, 25.
M, 48. L, 40. K, 30. N, 105.

If E doe not meaſure I, let K be the leaſt which E and I doe meaſure; and as often as I meaſures K, let G as often meaſure L, and H alſo M. ſo likewiſe let F meaſure N as often as E meaſures K. The four numbers M, L, K, N ſhall be leaſt continually in the given proportions; which we may demonſtrate as before.

P R O P. V.

C, 4. E, 3.
D, 6. F, 16. ED, 18.
$\overline{CD}$, 24. $\overline{EF}$, 48.

Plane numbers CD, EF, *are in that proportion to one another which is composed of their ſides.*

For becauſe CD.ED [a] :: C.E; and ED.EF :: D.F. & $\frac{CD}{EF}$ [b] $= \frac{CD}{ED} + \frac{ED}{EF}$ [c] then ſhall be the proportion $\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F}$. *W.W. to be Dem.*

a 17. 7.
b 20. def. 4.
c 11. 5.

P R O P. VI.

A, 16. B, 24. C, 36. D, 54. E, 81.
F, 4. G, 6. H, 9.

If there be numbers continually proportionall, how many ſoever, A, B, C, D, E, *and the firſt* A *doe not meaſure the ſecond* B, *neither ſhall any of the other meaſure any one of the reſt.*

a 10. def. 7. Becauſe A does not meaſure B,[a] neither ſhall any one meaſure that which next followes; being A. B :: B.C :: C.D, &c.
b 35. 7. [b] Take three numbers, F,G,H, the leaſt in the proportion of A to B. therefore ſince A does not meaſure B, [c] neither ſhall F meaſure G. [c] therefore F is not a unite. But F and H are prime
c 8. ax 7. to one another; and ſo,[d] being of equality A.C :: F.
d 3. 3 H. and F does not meaſure H, [e] neither ſhall A mea-
e 14. 7. ſure C; and conſequently neither ſhall B meaſure D, nor C meaſure E, &c. becauſe A.C :: B.D :: C.E, &c. In like manner four or five numbers being taken the leaſt in the proportion of A to B, it will appear that A does not meaſure D and E; nor does B meaſure E and F, &c. Wherefore none of them ſhall meaſure any other. *W. W. to be Dem.*

PROP. VII.

A, 3. B, 6. C, 12. D, 24. E, 48.

If there be numbers continually proportionall how many ſoever A,B,C,D,E, *and the firſt* A *meaſure the laſt* E, *it ſhall alſo meaſure the ſecond* B.

a 6. 7. If you deny that A meaſures B, [a] then neither ſhall it meaſure E; *Which is contrary to the Hypoth.*

PROP. VIII.

A,24. C,36. D,54. B,81.
G,8. H,12. I,18. K,27.
E,32. L,48. M,72. F,108.

If between two numbers A, B, *there fall mean proportionall numbers in continuall proportion* C, D; *as many mean continually proportionall numbers as fall between them, ſo many alſo mean continually proportionall numbers ſhall fall between two other numbers* E,F, *which have the ſame proportion with them.* (L,M.)

a 35. 7. [a] Take G,H,I, K, the leaſt ∺ in the proportion
b 14. 7 of A to C; [b] of equality ſhall G. K :: A. B [c] :: E. F.
c hyp. But G, and K [d] are prime to one another.[e] Wherefore
d 3. 8. G meaſures E as often as K does F. Let H meaſure
e 21. 7. L, and I likewiſe M by the ſame number.[f] therefore
f conſtr. E,L,M,F are in ſuch proportion as G,H,I,K, that is as A,B,C,D. *W. W. to be Dem.*

PROP.

PROP. IX.

I.
E,2. F,3.
G,4. H,6. I,9.
A,8. C,12. D,18. B,17.

If two numbers A, B *be prime to one another, and mean numbers in continuall proportion* C, D *fall between them; as many mean numbers in continuall proportion as fall between them, so many means also* (E, G; and F, I) *shall fall in continuall proportion between either of them and a unitie.*

It is evident, that 1, E, G, A, and 1, F, I, B, are ∺; and as many as A, C, D, B. namely by the 4. Coroll. 2. 8. *W.W. to be Dem.*

PROP. X.

A,8. I,12. K,18. B,27.
E,4. DF,6. G,9.
D,2. F,3.
I.

If between two numbers A, B, *and a unite, numbers continually proportionall* (E, D, and F, G,) *do fall, how many mean numbers in continuall proportion fall between either of them and a unite, so many means also shall fall in continuall proportion between them,* I, K.

Nam E, DF, G, and A, DqF (I) DG (K) B are ∺ by 2. 8. therefore, &c.

PROP. XI.

A,2. B,3.
Aq,4. AB,6. Bq,9.

Between two square numbers Aq, Bq, *there is one mean proportionall number* A B : *and the square* Aq *to the square* Bq *is in double proportion of that of the side* A *to the side* B.

a It is manifest that Aq, AB, Bq, are ∺; b and consequently also $\frac{Aq}{Bq} = \frac{A}{B}$ doubly. *W.W. to be Dem.*

a 17. 7.
b 10. def. 5.

PROP. XII.

Ac,27. AqB,36. ABq,48. Bc,64.
A,3. B,4.
Aq,9. AB,12. Bq,16.

Between two cube numbers, Ac, Bc, *there are two mean proportionall numbers* AqB, ABq: *and the cube* Ac *is to the cube* Bc *in treble proportion of that in which the ſide* A *is to the ſide* B.

a 2. 8.
b 10. def. 5.

a For Ac, AqB, ABq, Bc, are ∺ in the proportion of A to B; b and therefore $\frac{Ac}{Bc} = \frac{A}{B}$ trebly. *W.W.* to be Dem.

PROP. XIII.

A,2. B,4. C,8.
Aq 4. AB,8. Bq,16. BC,32. Cq,64.
Ac,8. AqB,16. ABq,32. Bc,64. BqC,128. BCq,256. Cc,512.

If there be numbers in continuall proportion how many ſoever A, B, C; *and every of them multiplying it ſelf produce certain numbers; the numbers produced of them* Aq, Bq, Cq, *ſhall be proportionall: And if the numbers firſt given* A, B, C, *multiplying their products* Aq, Bq, Cq, *produce other numbers,* Ac, Bc, Cc, *they alſo ſhall be proportionall; and this ſhall ever happen to the extremes.*

a 8. 8.
b 14. 7.

For Aq, AB, Bq, BC, Cq a are ∺; b therefore of equality Aq. Bq :: Bq. Cq. *W.W. to be Dem.*

a Alſo Ac, AqB, ABq, Bc, BqC, BCq, Cc, are ∺; b therefore likewiſe of equality Ac. Bc :: Bc. Cc. *W.W. to be Dem.*

PROP XIV.

Aq,4. AB,12. Bq,36.
A,2. B,6.

If a ſquare number Aq *meaſure a ſquare number* Bq, *the ſide alſo of the one* (A) *ſhall meaſure the ſide of the other* (B:) *& if the ſide of one ſquare* A *meaſure the ſide of another* B, *the ſquare* Aq *ſhall likewiſe meaſure the ſquare* Bq.

1. *Hyp.* For Aq. AB a :: AB. Bq. therefore ſeeing by the hypotheſis Aq meaſures Bq, b it ſhall meaſure alſo AB. But Aq. AB :: A. B. c therefore alſo A meaſures B. *W.W. to be Dem.* a 2. & 11. 8. b 7. 8. c 20. def. 7.

2. *Hyp.* A meaſures B. c therefore Aq ſhall as well meaſure AB, as AB meaſures Bq; d and conſequently Aq meaſures Bq. *W.W. to be Dem.* d 11. ax. 7.

PROP. XV.

A, 2. B, 6. *If a cube number*
Ac, 8. AqB, 24. ABq, 72. Bc, 216. *Ac meaſure a cube number* Bc, *then the ſide of the one* (A) *ſhall meaſure the ſide of the other* (B:) *And if the ſide* A *of one cube* Ac *meaſure the ſide* B *of the other* BC, *alſo the cube* Ac *ſhall meaſure the cube* Bc.

1. *Hyp.* For Ac, AqB, ABq, Bc a are ∺, therefore Ac, b meaſuring the extreme Bc, ſhall alſo c meaſure the ſecond AqB. But Ac. AqB :: A. B. d therefore A ſhall alſo meaſure B. a 2. & 11. 8. b hyp. c 7. 8. d 20. def. 7.

2. *Hyp.* A meaſures B; d therefore Ac meaſures AqB, which alſo meaſures ABq, and that Bc; e therefore Ac ſhall meaſure Bc. *W.W. to be Dem.* e 11. ax. 7.

PROP. XVI.

A, 4. B, 9. *If a ſquare number* Aq *doe not*
Aq, 16. Bq, 81. *meaſure a ſquare number* Bq, *neither ſhall the ſide of the one* A *meaſure the ſide of the other* B: *And if* A *the ſide of the one ſquare* Aq *doe not meaſure* B *the ſide of the other* Bq, *neither ſhall the ſquare* Aq *meaſure the ſquare* Bq.

1. *Hyp.* For if you affirm that A meaſures B, a then Aq alſo ſhall meaſure Bq. *againſt the Hyp.* a 14. 8.

2. *Hyp.* If you maintain Aq to meaſure Bq; a then likewiſe A ſhall meaſure B. *contrary to the Hypoth.*

PROP.

PROP. XVII.

A, 2. B, 3.
Ac, 8. Bc, 27.

If a cube number Ac *doe not meaſure a cube number* Bc, *neither ſhall the ſide of one* A, *meaſure the ſide of the other* B: *And if* A *the ſide of one cube* Ac *do not meaſure* B *the ſide of the other* Bc, *neither ſhall the cube* Ac *meaſure the cube* BC.

a 15.8. 1. *Hyp*. Let A meaſure B; a then Ac ſhall meaſure Bc. *againſt the Hyp*.

2. *Hyp*. Let Ac meaſure Bc; then A ſhall meaſure B; *which is alſo againſt the Hyp*.

PROP. XVIII.

C, 6. D, 2.
CD, 12.
E, 9. F, 3. DE, 18.
EF, 27.

Between two like plane numbers C D *and* E F *there is one mean proportional number* DE: *And the plane* C D *is to the plane* EF *in double proportion of that which the ſide* C *hath to the homologous ſide (or of like proportion)* E.

* 21. def. 7. a 17. 7. b 11. 5. c 10. def. 5. Being * by the Hypoth. C.D :: E.F. therefore by inverſion C.E :: D.F. But C.E a :: CD. DE; a and D.F :: DE. EF. b therefore CD. DE :: DE. EF. c Wherefore the proportion of CD to EF is double to that of CD to DE, that is, to the proportion of C to E, or D to F.

Coroll.

Hence it is apparent, That between two like plane numbers there falls one mean proportionall in the proportion of the homologous ſides.

PROP.

PROP. XIX.

CDE, 30. DEF, 60. FGE, 120. FGH, 240.
CD, 6. DF, 12. FG, 24.
C, 2. D, 3. E, 5. F, 4. G, 6. H, 10.

Between two like ſolid numbers CDE, FGH, *there are two mean proportionall numbers* DFE, FGE. *And the ſolid* CDE *is to the ſolid* FGH, *in treble proportion of that which the homologous ſide* C *ha's to the homologous ſide* F.

Whereas by the * hyp. C.D :: F.G, & D.E :: G.H. * 21. def. 7.
therefore *a* by inverſion ſhall C.F :: D.G :: *a* E.H. a 13. 7.
But CD.DF *b* :: C.F, & DF.FG *b* :: D.G; *c* wherefore b 17. 7.
CD.DF :: DF.FG :: E.H. *d* and accordingly CDE. c 11. 5. d 17. 7.
DFE :: DFE.FGE :: E.H :: FGE.FGH. Therefore
between CDE, FGH, fall two mean proportionals
DFE, FGE. *e* And ſo it is plain that the proportion e 10. def. 5.
of CDE to FGH is treble to that of CDE to DFE,
or C to F. *W.W. to be Dem.*

Coroll.

Hereby it is manifeſt, That between two like ſolid numbers there fall two mean proportionals in the proportion of the homologous ſides.

PROP. XX.

A, 12, C, 18. B, 27.
D, 2. E, 3. F, 6. G, 9.

If between two numbers A, B, *there fall one mean proportionall number* C; *thoſe numbers* A, B, *are like plane numbers.*

a Take D and E the leaſt in the proportion of A a 35. 7.
to C, or C to B. then D meaſures A equally as E
does C, *viz.* by the ſame number F; *b* alſo D equally b 21. 7.
meaſures C, as E does B, *viz.* by the ſame number G.
c Therefore DF = A, and EG = B. *d* and conſe- c 9. ax. 7.
quently A and B are plane numbers. But becauſe d 16 def. 7.
EF *e* = C *e* = DG, *e* ſhall D.E :: F.G. and alter- e 19. 7.
nately

f 21.def.7. nately D.F :: E. G. *f* Therefore the plane numbers A and B are alſo like. *W.W. to be Dem.*

PROP. XXI.

A, 16. C, 24. D, 36. B, 54.
E, 4. F, 6. G, 9.
H, 2. P, 2. M, 4. K, 3. L, 3. N, 6.

If between two numbers A, B there fall two mean proportionall numbers C, D; theſe numbers A, B are like ſolid numbers.

a 2. 8. *a* Take E, F, G, the leaſt ∺ in the proportion of
b 20. 8. A to C. *b* then E and G are like plane numbers; let the ſides of this be H & P, & of that K and L. *c* therefore
c 21. def.7. d cor. 2. 8. H.K :: P.L :: *d* E.F. But E, F, G, doe *e* equally mea-
e 21. 7. ſure A, C, D. viz. by the ſame number M. and likewiſe the ſaid numbers E, F, G, doe equally meaſure the numbers C, D, B, viz. by the ſame number N. *f* Therefore A = EM = HPM, *f* and B = GN =
f 9. ax.7. KLN; *g* and ſo *g* A and B are ſolid numbers. But ſeeing
g 17. def.7. h 17. 7. that C *f* = FM, and D *f* = FN, therefore ſhall M.N
k 7. 5. l conſtr. *h* :: FM.FN *k* :: C.D *l* :: E.F :: H.K :: P.L. *m* where-
m 21. def.7. fore A and B are like ſolid numbers. *W.W. to be Dem.*

Lemma.

AE, BF, CG, DH,
A, B, C, D,
E, F, G, H.

If proportionall numbers A, B. C, D *meaſure proportionall numbers* AE, BF, CG, DH *by the numbers* E, F, G, H, *theſe numbers* (E, F, G. H) *ſhall be proportionall.*

a 19. 7. For being AEDH *a* = BFCG, *a* and AD = BC,
b 1. ax. 7. c 9. ax. 7. *b* thence will $\underset{AD}{\underline{ABDH}} = \underset{BC,}{\underline{BFCG}}$ *c* that is, EH = FG.

a Therefore E.F :: G.H. *W.W. to be Dem.*

Coroll.

Hence $\frac{Bq}{Aq} = \frac{B}{A} \times \frac{B}{A}$. *d* For 1.B :: B.Bq *d* and 1. A ::
d 15. def.7. A.Aq. *e* therefore $\frac{1}{A}$. B :: B.$\frac{Bq}{AAq.}$ *e* therefore $\frac{Bq}{Aq} = \frac{B}{A} \times \frac{B}{A.}$
e lem. prec.

In like manner $\frac{B}{Ac} \times \frac{B}{Ac} = \frac{Bc}{Acc.}$ and ſo of the reſt.

PROP.

PROP. XXII.

Aq, B, C.
4, 8, 16.

If three numbers Aq, B, C *be continually proportionall, and the firſt* Aq *a ſquare; the third* C *ſhall alſo be a ſquare.*

For becauſe Aq C a $=$ Bq, b thence is $C = \frac{Bq}{Aq}$ c $= Q.\frac{B}{A}$. But it is plain that $\frac{B}{A}$ is a number, d becauſe $\frac{Bq}{Aq}$ or C is a number. Therefore if three, &c.

a 20. 7.
b 7. ax. 7.
c cor. of the lem. prec.
d hyp. and 24. 8.

PROP. XXIII.

Ac, B, C, D.
8, 12, 18, 27.

If four numbers A, B, C, D *be continually proportionall, and the firſt of them* Ac *a cube, the fourth alſo* D *ſhall be a cube.*

For becauſe Ac D a $=$ BC, b therefore $D = \frac{BC}{Ac}$ c $= \frac{B}{Ac} \times C$; that is (becauſe AcC $=$ d Bq, and b thence $C = \frac{Bq}{Ac}$) $D = \frac{B}{Ac} \times \frac{Bq}{Ac}$ c $= \frac{Bc}{Acc}$ c $= C : \frac{B}{Ac}$. But it is evident e that $\frac{B}{Ac}$ is a number, becauſe $\frac{Bc}{Acc}$ or D is ſuppoſed a number. Therefore if four numbers, &c.

a 19. 7.
b 7. ax. 1.
c cor. of the prec. lem.
d 20. 7.
e 25. 8.

PROP. XXIV.

A, 16. 24. B, 36.
C, 4. 6. D, 9.

If two numbers A, B, *be in the ſame proportion one to another, that a ſquare number* C *is to a ſquare number* D, *and the firſt* A *be a ſquare number, the ſecond alſo* B *ſhall be a ſquare number.*

Between C and D being ſquare numbers, a and ſo between A and B having the ſame proportion, a falls one mean proportionall. Therefore b being A [illegible]

a 8. 8.
a 11. 8.

is

e 22. 8. is a ſquare number, *e* B alſo ſhall be a ſquare number. *W.W. to be Dem.*

Coroll.

1. Hence, If there be two like numbers AB, CD (A.B :: C.D) and the firſt AB be a ſquare, the ſecond alſo CD ſhall be a ſquare.

a 11, & 18. 8. *For AB.CD :: Aq.Cq.

2. From hence it appears, That the proportion of any ſquare number to any other not ſquare cannot poſſibly be declared in two ſquare numbers. Whence it cannot be Q.Q :: 1. 2. nor 1.5 :: Q.Q, &c.

P R O P. XXV.

C, 64. 96. 144. D, 216.
A, 8, 12. 18. B, 27.

If two numbers A, B, be in the ſame proportion one to another, that a cube number C is to a cube number D, *the firſt of them A being a cube number; the ſecond* B *ſhall likewiſe be a cube number.*

a 12. 8. b 8. 8. c hyp. d 23. 8. *a* Between the cube numbers C and D, *b* and ſo between A and B having the ſame proportion, fall two mean proportionals; therefore *c* becauſe A is a cube, *d* ſhall B be a cube alſo. *W.W. to be Dem.*

Coroll.

1. Hence, If there be two numbers ABC, DEF (A.B :: D.E, and B.C :: E. F;) and the firſt ABC be a cube, the ſecond DEF ſhall be a cube alſo.

a 12, & 19. 8. *For ABC.DEF :: Ac.Dc.

2. It is perſpicuous from hence, That the proportion of any cube number to any other number not a cube cannot be found in two cube numbers.

PROP. XXVI.

A,20. C,30. B,45.
D,4. E,6. F,9.

Like plane numbers A, B, are in the ſame proportion one to another, that a ſquare number is in to a ſquare number.

Between A and B a falls one mean proportionall number C; b take three numbers D, E, F the leaſt ∺ in the proportion of A to C. the extremes D, F, b ſhall be ſquare numbers. But of equality A. B c :: D.F. therefore A.B :: Q.Q. *W.W. to be Dem.* a 18.8. b 2.8. c 14.7.

PROP. XXVII.

A,16. C,24. D,36. B,54.
E,8. F,12. G,18. H,27.

Like ſolid numbers A, B, are in the ſame proportion one to another, that a cube number is in to a cube number.

a Between A and B fall two mean proportionall numbers, namely C and D: b take four numbers E, F, G, H the leaſt ∺ in the proportion of A to C; b the extremes E, H, are cube numbers. But A.B c :: E.H :: C.C. *W.W. to be Dem.* a 19.8. b 2.8. c 14.7.

Schol.

1. From hence is inferred, that no numbers in proportion ſuperparticular, ſuperbipartient, or double, or any other manifold proportion not denominated from a ſquare number, are like plane numbers. *See Clavius.*

2. Likewiſe, that neither any two prime numbers, nor any two numbers prime one to another, not being ſquares, can be like plane numbers.

The End of the eighth Book.

THE NINTH BOOK OF EUCLIDES ELEMENTS

PROPOSITION I.

A,6. B,54.
Aq,36. 108. AB,324.

If two like plane numbers A, B multiplying one another, produce a number AB, the number produced AB ſhall be a ſquare number.

a 17.7. b 18.8. c 8.8. For A.B a:: Aq. AB; wherefore ſince one mean proportionall b falls between A and B, c likewiſe one mean proportionall number ſhall fall between Aq and AB: therefore being the firſt Aq is a ſquare number, d the third AB ſhall be a ſquare number too. *W.W. to be Dem.* d 22.8.

Or thus. Let a b, c d, be like plane numbers; namely a. b:: c.d. x therefore a d = b c. and ſo likewiſe a b c d, or a d b c y = ad ad = Q: ad. x 19.7. y 1.&2.7.

PROP. II.

A,6. B,54.
Aq,36. AB,324.

If two numbers A, B, multiplying one another, produce a ſquare number AB, theſe numbers A,B are like plane numbers.

a 17.7. b 11.8. c 8.8. d 20.8. For A. B a:: Aq. AB; wherefore being between Aq, AB, b there falls one mean proportionall number, c likewiſe one mean ſhall fall between A and B. d therefore A and B are like planes. *W.W. to be Dem.*

PROP.

PROP. III.

A, 2. Ac, 8. Acc, 64. *If a cube number* Ac *multiplying it ſelf produce a number* Acc, *the number produced* Acc *ſhall be a cube number.*

For 1. A [a] :: A. Aq [b] :: Aq. Ac. therefore between 1 and Ac fall two mean proportionalls. But 1. Ac [b] :: Ac. Acc. [c] therefore between Ac and Acc, fall alſo two mean proportionalls: and ſo by conſequence ſeeing Ac is a cube, [d] Acc ſhall be a cube alſo. *Which was to be Dem.*

a 15. def. 7.
b 17. 7.
c 8. 8.
d 23. 8.

Or thus, aaa (Ac) multiplyed into it ſelf makes aaaaaa (*Acc*) this is a cube, whoſe ſide is aa.

PROP. IV.

Ac, 8. Bc, 27. Acc, 64. AcBc, 216. *If a cube number* Ac *multiplying a cube number* Bc *produce a number* AcBc, *the produced number* AcBc *ſhall be a cube.*

For Ac. Bc [a] :: Acc. AcBc. But between Ac and Bc, [b] two mean proportionall numbers fall; [c] therefore there fall as many between Acc and AcBc. So that whereas Acc is a cube number, [d] AcBc ſhall be ſuch alſo. *W. W. to be Dem.*

a 17. 7.
b 12. 8.
c 8. 8.
d 23. 8.

Or thus. AcBc = aaabbb (ababab) = G: ab.

PROP. V.

Ac, 8. B, 27. Acc, 64. AcB, 216. *If a cube number* Ac *multiplying a number* B *produce a cube number* AcB, *the number multiplyed* B *ſhall alſo be a cube.*

For Acc. AcB [a] :: Ac. B. But between Acc and AcB [b] fall two mean proportionalls; [c] therefore alſo as many ſhall fall between Ac and B. whence Ac being a cube number, [d] B ſhall be a cube number too. *W. W. to be Dem.*

a 17. 7.
b 12. 8.
c 8. 8.
d 23. 8.

PROP. VI.

A,8. Aq,64. Ac,512. *If a number A multiplying it ſelf produce a cube Aq, that number A it ſelf is a cube.*

a hyp. b 19. def. 7. c 5. 9. For becauſe Aq *a* is a cube, and AqA (Ac) *b* alſo a cube; therefore *c* ſhall A be a cube. *W.W. to be Dem.*

PROP. VII.

A,6. B,11. AB,66. D,2. E,3. *If a compoſed number A multiplying any number B, produce a number AB, the number produced AB ſhall be a ſolid number.*

a 13. def. 7. b 9. ax. 7. c 17. def. 7. Being A is a compoſed number, *a* ſome other number D meaſures it, conceive by E. *b* therefore A = DE: *c* whence DEB = AB is a ſolid number. *W.W. to be Dem.*

PROP. VIII.

1. a, 3. a^2, 9. a^3, 27. a^4, 81. a^5, 243. a^6, 729.

If from a unite there be numbers continually proportionall how many ſoever (1, a, a^2, a^3, a^4, &c.) *the third number from a unite* a^2 *is a ſquare number; and ſo are all forward, leaving one between* (a^4, a^6, a^8, &c.) *But the fourth* a^3 *is a cube number; and ſo are all forward, leaving two between* (a^6, a^9, &c.) *The ſeventh alſo* a^6 *is both a cube number and a ſquare; and ſo are all forward, leaving five between* (a^{12}, a^{18}, &c.)

For 1. a^2 = Q.a. and a^4 = aaaa = Q. aa. and a^6 = aaaaaa = Q. aaa, &c.

2. a^3 = aaa = C.a. and a^6 = aaaaaa = C.aa. and aaaaaaaaa = C.aaa, &c.

3. a^6 = aaaaaa = C.aa = Q.aaa. therefore, &c.

a hyp. b 20. 7. c 12. 8. Or according to *Euclide*; Becauſe 1, a *a* ∷ a. a^2. *b* ſhall a^2 = Q:a. therefore ſeeing a^2, a^3, a^4, are ∺, *c* the third a^4 ſhall be a ſquare number; and ſo likewiſe a^6, a^8, &c. Alſo becauſe 1. a *a* ∷ a^2. a^3. therefore

d 23. 8. ſhall a^3 *b* = a^2 x a = C: a. *d* therefore the fourth from a^3, namely a^6, ſhall be likewiſe a cube, &c. and conſequently a^6 is both a cube and a ſquare number, &c.

PROP. IX.

1. a, 4. a^2, 16. a^3, 64. a^4, 256, &c.
1. a, 8. a^2, 64. a^3, 512. a^4, 4096.

If from a unite there be numbers how many ſoever continually proportionall (1, a, a^2, a^3, &c.) *and the number following the unite* (a) *be a ſquare; then all the reſt*, a^2, a^3, a^4, &c. *ſhall be ſquares too. But if the number next the unite* (a) *be a cube, then all the following numbers* a^2, a^3, a^4, &c. *ſhall be cube numbers.*

1. *Hyp.* For a^2, a^4, a^6, &c. are ſquare numbers by the prec. *prop.* alſo being a is taken to be a ſquare, ^{a} therefore the third a^3 ſhall be a ſquare, and like- a 22. 8.
wiſe a^5, a^7, &c. and ſo all.

2. *Hyp.* a is taken to be a cube, ^{b} therefore a^4, a^7, b 23. 8.
a^{10} are cubes: but by the prec. a^3, a^6, a^9, &c. are c 20. 7.
cubes: laſtly becauſe 1. a :: a. aa. ^{c} therefore ſhall a^2 d 3. 9.
= Q: a. but a cube multiplyed into it ſelf ^{d} pro- e 23. 8.
duces a cube; therefore a^2 is a cube, ^{e} and conſequently the fourth from it a^5, and in like manner a^8, a^{11}, &c. are cubes, therefore all. *W. W. to be Dem.*

Peradventure more clearly thus. Let b be the ſide of the ſquare number a, and ſo the ſeries a, a^2, a^3, a^4, &c. will be otherwiſe expreſſed, thus, bb, b^4, b^6, b^8, &c. It is evident that all theſe numbers are ſquares, and may be thus expreſſed, Q: b, Q: bb: Q. bbb, Q: bbbb, &c.

In like manner, if b be the ſide of the cube a, the ſeries may be expreſſed thus, b^3, b^6, b^9, b^{12}, &c. or C: b, C: b^2, C: b^3, C: b^4, &c.

PROP. X.

1, a, a^2, a^3, a^4, a^5, a^6.
1, 2, 4, 8, 16, 32, 64.

If from a unite there be numbers how many ſoever continually proportionall (1, a, a^2, a^3, &c.) *and the number next the unite* (a) *be not a ſquare number; then is none of the reſt following a ſquare number, excepting* a^2 *the third from the unite, and ſo all forward, leaving one between* (a^4, a^6, a^8, &c.)

But if that (a) *which is next after the unite, be not a cube number, neither is any other of the following numbers a cube, ſaving* a^3 *the fourth from the unite, and ſo all forward, leaving two between,* a^6, a^9, a^{12}, &c.

1. *Hyp.* For it it be poſſible, let a^5 be a ſquare number; therefore becauſe $a . a^2$ a :: $a^4 . a^5$, and by inverſion $a^5 . a^4 :: a^2 . a$; and alſo a^5 and a^4 b ſquare numbers, and the first a^2 a ſquare, c therefore a ſhall be likewiſe a ſquare; *contrary to the Hyp.*

a hyp.
b ſuppoſ and 8. 9.
c 24. 8.

2. *Hyp.* If it may be, let a^4 be a cube; being d of equality $a^4 . a^6 :: a . a^3$, and inverſely $a^6 . a^4 :: a^3 . a$; and alſo being a^6 and a^4 are cubes, and the firſt a^3 a cube, e therefore a ſhall be a cube alſo; *againſt the Hyp.*

d 24. 7.
e 25. 8.

PROP. XI.

1, a, a^2, a^3, a^4, a^5, a^6.
1, 3, 9, 27, 81, 243, 729.

If there be numbers how many ſoever in continuall proportion from a unite (1, a, a^2, a^3, &c.) *the leſſe meaſureth the greater by ſome one of them that are amongſt the proportionall numbers.*

Becauſe 1 . a :: a . aa. a therefore $\frac{aa}{a} = a = \frac{aaa}{aa}$. Alſo becauſe 1 . aa b :: a . aaa. therefore $\frac{aaa}{a} = aa = \frac{a^4}{aa} = \frac{a^5}{a^3}$, &c. Laſtly becauſe 1 . a^3 b :: a . a^4. therefore $\frac{a^4}{a} = a^3 = \frac{a^6}{a^3}$, &c.

a 5. ax. 7. & 20. def. 7.
b 14. 7.

Coroll.

Hence, If a number that meaſures any one of proportionall numbers, be not one of the ſaid numbers, neither ſhall the number by which it meaſures the ſaid proportionall numbers, be one of them.

PROP. XII.

$1, a, a^2, a^3, a^4.$
$1, 6, 36, 216, 1296.$
$B, 3.$

If there be numbers how many ſoever in continuall proportion from a unite $(1, a, a^2, a^3, a^4.)$ *whatſoever prime number B measure the laſt* a^4, *the ſame (B) ſhall alſo meaſure the number (a) which follows next after the unite.*

If you ſay B does not meaſure a, a then B is prime to a^4; b and alſo B is prime to a^2; c and ſo conſequently to a^4, which it is ſuppoſed to meaſure: Which is *Abſurd.* a 31.7. b 27.7. c 27.7.

Coroll.

1. Therefore every prime number that meaſures the laſt, does alſo meaſure all thoſe other numbers that precede the laſt.

2. If any number not meaſuring that next to the unite, does yet meaſure the laſt, it is a compoſed number.

3. If the number next to the unite be a prime, no other prime number ſhall meaſure the laſt.

PROP. XIII.

$1, a, a^2, a^3, a^4,$
$1, 5, 25, 125. 625.$
H -- G -- F -- E --

If from a unite be numbers in continuall proportion how many ſoever $(a, a^2, a^3$, &c.) *and that after the unite (a) a prime; then ſhall no other meaſure the greateſt number, but thoſe which are amongſt the ſaid proportionall numbers.*

If it be poſſible, let ſome other E meaſure a^4, viz. by F. a then F ſhall be ſome other beſide a, a^2, a^3. But becauſe E meaſuring a^4, does not meaſure a, b therefore E ſhall be a compoſed number, c and ſome prime number meaſure it, d which does conſequently meaſure a^4. e and ſo is no other then a. therefore a meaſures E. So alſo may F be ſhewn to be a com-

a cor. 11.9. b 2. cor. 12.9. c 33.7. d 11. ax. 7. e 3. cor. 12.9.

poſed

poſed number, meaſuring a^4, and ſo that a meaſures F. Therefore ſeeing EF f $= a^4 = a \times a^3$, g ſhall a. E :: F. a^3. Conſequently, whereas a meaſures E, h likewiſe F ſhall equally meaſure a^3; viz. by the ſame number G. k Nor ſhall G be a, or a^2; therefore, as before, G is a compoſed number, and a meaſures it. Wherefore being that FG f $= a^3 = a^2 \times a$ g ſhall a. F :: G. a^2. and ſo becauſe A meaſures F, h G ſhall equally meaſure a^2, viz. by the ſame number H, k which is not a. Therefore being GH $= a^2 =$ aa, l thence H. a :: a. G. & becauſe a meaſures G (as before) m H alſo ſhall meaſure a, which is a prime number. *Which is impoſſible.*

f 9. ax 7.
g 19. 7.
h 20. def. 7.
k cor. 11. 9.
l 21. def. 7.
m 10. def. 7.

PROP. XIV.

A, 30.
B, 2. C, 3. D, 5.
E -- F ---

If certain prime numbers B, C, D, do meaſure the leaſt number A, no other prime number E ſhall meaſure the ſame, beſides thoſe that meaſured it at firſt.

If it is poſſible, let $\frac{A}{E}$ be $=$ F. a then A $=$ EF. b therefore every of the prime numbers B, C, D meaſures one of thoſe E, F. not E, which is taken to be a prime; therefore F, which is leſſe then it ſelf A; *contrary to the Hyp.*

a 9. ax 7.
b 32. 7.

PROP. XV.

A, 9. B, 12, C, 16.
D, 3. E, 4.

If three numbers continually proportionall , B, C, be the leaſt of all that have the ſame proportion with them; any two of them added together ſhall be prime to the third.

a Take D and E the leaſt in the proportion of A to B; b then A $=$ Dq, & b C $=$ Eq, b and B $=$ DE. But

a 35. 7.
b 2. 8.

But because D [c] is prime to E, [d] therefore shall $D + E$ be prime to both D and E. [*] therefore $D \times D + E = Dq + DE$ (ſ $A + B$) is prime to E, and so to G or Eq. *W. W. to be Dem.* c 24.7. d 30.7. * 16.7. e 3.2. f before.

[g] In like manner $DE + Eq$ ($B + C$) is prime to D, and consequently to $A = Dq$. *W. W. to be Dem.* g 27.7.

Lastly; because B [h] is prime to $D + E$, it shall also be prime to the ſquare of it [k] $Dq + 2\,DE + Eq$ ($A + 2\,B + C$;) [l] wherefore the ſaid B ſhall be prime to $A + B + C$, [l] and ſo likewiſe to $A + C$. *which was to be Dem.* h 26.7. k 4.2. l 30.7.

PROP. XVI.

A, 3. B, 5. C --- *If two numbers* A, B, *be prime to one another, the ſecond* B *ſhall not be to any other* C, *as the firſt* A *is to the ſecond* B.

If you affirm A. B :: B. C. then whereas A and B [a] are the leaſt in their proportion, [b] A ſhall meaſure B as many times as B does C; but A [c] meaſures it ſelf alſo; therefore A and B are not prime to one another. *againſt the Hyp.* a 23.7. b 21.7. c 6.ax.7.

PROP. XVII.

A, 8. B, 12. C, 18, D, 27. E --- *If there be numbers how many ſoever in continuall proportion* A, B, C, D, *and the extremes of them* A, D *be prime one to another, the laſt* D *ſhall not be to any other number* E, *as the firſt* A *is to the ſecond* B.

Suppoſe A. B :: D. E. then alternately A. D :: B. E. therefore ſeeing A and B are [a] the leaſt in their proportion, A [b] ſhall meaſure B, [c] and B likewiſe C, and C the following number D. [d] and ſo A ſhall *meaſure* the ſaid number D. Wherefore A and a 23.7. b 21.7. c 20 def.7. d 11.ax.7.

D are not prime to one another, *contrary to the Hypoth.*

PROP. XVIII.

A, 4. B, 6. C, 9.
Bq, 36.

Two numbers being given A, B, to conſider if there may be a third number found proportionall to them C.

a 9. ax 7.
b 20. 7.

If A meaſure Bq by any number C; a then AC = Bq. from whence b it is manifeſt that A. B :: B. C. *W. W. to be Dem.*

A, 6. B, 4. Bq, 16.

But if A doe not meaſure Bq; there will not be any third proportionall. For ſuppoſe A. B :: B. C. c then AC = Bq. = c and conſequently $\frac{Bq}{A} = C$. namely, A meaſures Bq. *Which is againſt the Hypoth.*

c 7. ax. 7.

PROP. XIX.

A, 8. B, 12. C, 18. D, 27.
BC, 216.

Three numbers being given A, B, C, to conſider if a fourth proportionall to them D may be found.

a 9. ax. 7.
b ax. 19. 7.

If A meaſures BC by any number D, a then AD = BC; b therefore it appears that A. B :: C. D. which was required.

But if A do not meaſure BC, then there can no fourth proportionall be found; which may be ſhewn as in the prec. *prop.*

PROP. XX.

A, 2. B, 3. C, 5.
D, 30. G - - - -

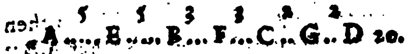
More prime numbers may be given then any multitude whatſoever of prime numbers A, B, C, propounded.

a 36. 7.
b 33. 7.

a Let D be the leaſt which A, B, C, meaſure; If D + 1 be a prime, the caſe is plain; if compoſed, b then ſome prime number, conceive G, meaſures D + 1;

which is none of the three A, B, C; For if it be, ſeeing it *c* meaſures the whole D + 1, *d* and the part taken away D, *e* it ſhall alſo meaſure the remaining unite. *Which is Abſ.* Therefore the propounded number of prime numbers is increaſed by D + 1, or at leaſt by G. *c ſuppoſ.* *d conſtr.* *e* 12. *ax.* 7.

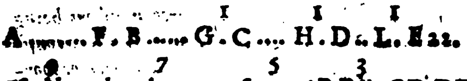
PROP. XXI.



If even numbers, how many ſoever, AB, BC, CD, be added together, the whole AD ſhall be even.

a Take $EB = \frac{1}{2} AB$, and $FC = \frac{1}{2} BC$, and $GD = \frac{1}{2} CD$. *b* It is plain that $EB + FC + GD = \frac{1}{2} AD$. *c* therefore AD is an even number. *Which was to be Dem.* a 6. *def.* 7. b 12. 7. c 6. *def.* 7.

PROP. XXII.



If odd numbers, how many ſoever, AB, BC, CD, DE, be added together, and their multitudes even, the whole alſo AE ſhall be even.

A unite being taken from each odde number, there will *a* remain AF, BG, CH, DL, even numbers, *b* and thence the number compounded of them will be even. adde to them the *c* even number made of the remaining unites, and the *d* whole AE will thereby be even. *W.W. to be Dem.* a 7. *def.* 7. b 21. 9. c *hyp.* d 21. 9.

PROP.

PROP. XXIII.

7 5 1
A .... B ..... C . E . D 15.
3

If odd numbers how many ſoever AB, BC, CD, *be added together, and the multitude of them be odde, the whole* AD *ſhall be odde.*

For CD one of the odde numbers being taken away, the number compounded of the others AC a is even. Whereto adde CD − 1, b the whole AE is alſo even; wherefore the unite being reſtored the whole AD c will be odd. *W. W. to be Dem.*

a 22. 9.
b 21. 9.
c 7. def. 7.

PROP. XXIV.

4 5 1
A .... B ..... D . C 10.
6

If an even number AB *be taken away from an even number* AC, *that which remains* BC *ſhall be even.*

For if BD (BC − 1) be odde, a BC (BD + 1) will be even. *W. W. to be Dem.* But if you ſay BD is even, becauſe AB b is even, c thence AD will be ſo; & conſequently AC (AD + 1) will be odde, *contrary to the Hypoth.* therefore BC is even. *W. W. to be Dem.*

a 7. def. 7.
b hyp.
c 21. 9.

PROP. XXV.

6 1 3
A ...... D . C ... B 10.
7

If from an even number AB, *an odde number* AC *be taken away, the remaining number* CB *ſhall be odde.*

For AC − 1 (AD) a is even. b therefore DB is even; c and conſequently CB (DB − 1) is odd. *W. W. to be Dem.*

a 7. def. 7.
b 24. 9.
c 7. def. 7.

PROP. XXVI.

4 6 1
A ..., C ...... D . B 11.

If from an odde number AB *be taken away an odde number* CB, *that which remaineth* AC *ſhall be even.*

For AB − 1 (AD) and CB − 1 (CD)[a] are even; [b]therefore AD − CD (AC) is even. *W.W.to be Dem.* a 7.def.7. b 24.9.

PROP. XXVII.

1   4   6
A . D .... C ...... B 11
5

If from an odde number AB be taken away an even number CB, the reſidue AC ſhall be odde.

For AB − 1 (DB)[a] is even, and CB is ſuppoſed to be even; [b]therefore the reſidue CD is even; [c]therefore CD + 1 (CA) is odde. *W.W.to be Dem.* a 7.def.7. b 24.9. c 7.def.7.

PROP. XXVIII.

A, 3.
B, 4.
AB, 12.

If an odde number A multiplying an even number B produce a number AB, the number produced AB ſhall be even.

For AB[a] is compounded of the odde number A taken as many times as a unite is conteined in B an even number. [b]Therefore AB is an even number. a hyp. & 15.def.7. b 21.9.

Schol.

In like manner, if A be an even number, AB ſhall be an even number alſo.

PROP. XXIX.

A, 3.
B, 5.
AB, 15.

If an odde number A multiplying an odde number B, produce a number AB, the number produced AB ſhall be odde.

For AB[a] is compounded of the odde number B taken as often as a unite is included in A likewiſe an odde number. [b]Therefore AB is an odd number. *W.W.to be Dem.* a 15.def.7. b 23.9.

Schol.

Schol.

$\frac{B,12}{A,3}$ (C,4. *1. An odd number A meaſuring an even number B, meaſures the ſame by an even number C.*

a 9 ax.7. b 29.9. For if C be affirmed to be odde, then becauſe a B = AC, b therefore B ſhall be odde, *againſt the Hyp.*

$\frac{B,15}{A,3}$ (C,5 *2. An odde number A meaſuring an odde number B, meaſures the ſame by an odde number C.*

a 28,9. For if C be ſaid to be even, a then AC, or B will be even, *contrary to the Hypoth.*

$\frac{B,15}{A,3}$ (C,5. *3. Every number (A and C) that meaſures an odde number B, is it ſelf an odde number.*

a 28,9. For if either A or C be affirmed to be even, B a ſhall be an even number, *againſt the Hypoth.*

PROP. XXX.

$\frac{B,24}{A,3}$ (C,8. $\frac{D,12}{A,3}$ (E,4.

If an odde number A meaſure an even number B, it ſhall alſo meaſure the half of it D.

a hyp. a Let $\frac{B}{A}$ be = C. b then C is an even number.

b 1.ſchol.
29.9.
c 9 ax.7. Therefore let E be = $\frac{1}{2}$ C, then B c = CA d = 2
d 1.2
e hyp. EA e = 2 D. f therefore EA = D ; g and conſe-
f 7.ax.1.
g 7.ax.7. quently $\frac{D}{A}$ = E. *W.W. to be Dem.*

PROP. XXXI.

A,5. B,8. C,16. D.... *If an odde number A be prime to any number B, it ſhall alſo be prime to the double thereof C.*

If it be poſſible, let ſome number D meaſure A, a 3.ſchol.29.9. and C, a then D meaſuring the odde number A ſhall b 30.9. be odde it ſelf, b & ſo ſhall meaſure B the half of the even number C. therefore A & B are not prime one to another. *Which is againſt the Hyp.*

Coroll.

It follows from hence that an odde number which is prime to any number of double progreſſion, is alſo prime to all the numbers of that progreſſion.

PROP. XXXII.

1. A, 2. B, 4. C, 8. D, 16. *All numbers* A, B, C, D, &c. *in double progreſſion from the binarie are evenly even only.*

It is evident that all theſe numbers 1, A, B, C, D, [a] are even, and [b] ∺, namely in a double proportion; [c] and ſo every leſſe meaſures the greater by ſome one of them. [d] Wherefore all are evenly even. But for that A is a prime number, [e] no number beſide theſe ſhall meaſure any of them. Therefore they are evenly even only. *W.W. to be Dem.*

[a] 6 def. 7.
[b] 20 def. 7.
[c] 12. 9.
[d] 8 def. 7.
[e] 13. 9.

PROP. XXXIII.

A, 30. B, 15. *If of a number* A, *the half* B *be*
D - - - E - - *odd, the ſame* A *is evenly odd only.*

Being an odde number B [a] meaſures A by two an even number, [b] therefore B is evenly odde. If you affirm it to be evenly even, [c] then ſome even number D meaſures it by an even number E. whence 2 B [d] = A [d] = DE. [e] wherefore 2, E ∷ D. B. and therefore as 2 [f] meaſures the even number E, [g] ſo D an even number meaſures B an odde. *Which is impoſſible.*

[a] hyp.
[b] 9 def. 7.
[c] 8 def. 7.
[d] 9 ax. 7.
[e] 19. 7.
[f] 6 def. 7.
[g] 10 def. 7.

PROP. XXXIV.

A, 24. *If an even number* A *be neither doubled from two, nor have it's half part odde, it is both evenly even and evenly odde.*

It is undoubtable, that A is evenly even, becauſe the half of it is not odde. But becauſe, if A be divided into two equall parts, and ſo continuing the bipartition

a 7. def. 7. tition, we ſhall at length light upon ſome a
number (not upon the number two; becauſe
not ſuppoſed to be doubled upward from
which ſhall meaſure A by an even number. (for
b 1. ſch. 29. therwiſe A it ſelf ſhould be odde, *againſt the*
Therefore A is alſo evenly odde. *W.W. to be Dem.*

PROP. XXXV.

A .... 8.
4 8
B .... F .... G 12.
C .... 18.
9 6 4 8
D .... H ...... L .... K .... N 27.

If there be numbers in continuall proportion
ny ſoever A, BG, C, DN, *and the number*
taken from the ſecond, and KN *from the laſt,*
the firſt A; *as the exceſſe of the ſecond* BF *is to the*
A, *ſo ſhall the exceſſe of the laſt* DK *be to all the*
bers that precede it, A, BG, C.

From DN take NL = BG, and NH = C.
a hyp. b 17. 5. cauſe DN. C (HN) a :: HN. BG (LN) a :
(BG.) A (KN.) b therefore by dividing each,
c 12. 5. DH. HN :: HL. LN :: LK. KN. c wherefore D
d 3. ax. 1. + BG + A :: LK (d BF.) KN (A.) *W.W. to be*

Coroll.

e 18. 5. Hence e by compounding, DN + BG + C
+ BG + C :: BG. A.

PROP. XXXVI.

1. A, 2. B, 4. C, 8. D, 16.
E, 31. G, 62. H, 124. L, 248. F, 496.
M, 31. N, 465.
P - - - - Q - - -

If from a unite be taken how many numbers
1, A, B, C, D, *in double proportion continually, unti*
whole added together E *be a prime number;* and

whole E multiplying the laſt produce a number F, that which is produced F ſhall be a perfect number.

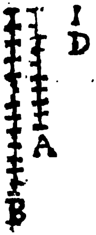
Take as many numbers E, G, H, L, likewiſe in
double proportion continually; then [a] of equality a 14.7.
A.D :: E.L. [b] therefore AL = DE [c] = F. [d] whence b 19.7.
$L = \frac{F}{2}$. Wherefore E, G, H, L, F, are ∺ in double c hyp. d 7.ax.7. e 35.9.
proportion. Let G − E be = M, and F − E = N;
[e] then M.E :: N.E + G + H + L. [f] But M = E. f 3.ax.1.
[g] therefore N = E + G + H + L. [h] therefore F g 14.5. h 2.ax.1.
= 1 + B + C + D + E + G + H + L = E +
N. Moreover becauſe D [k] meaſures DE(F) [l] therefore k 7.ax.7. l 11.ax.7.
every one, 1, A, B, C, [m] meaſuring D, as [m] alſo E, m 11.9.
G, H, L, does meaſure F. And further, no other number
meaſures the ſaid F. For if there do, let it be P,
which meaſures F by Q. [n] therefore PQ = F = DE; n 9.ax.7.
[o] therefore E. Q :: P. D. therefore ſeeing A a o 19.7.
prime number meaſures D, [p] and ſo no other P meaſures p 13.9.
the ſame, [q] conſequently E does not meaſure q 20.def.7.
Q. Wherefore E being ſuppoſed a prime number,
[r] it ſhall be prime to Q. [ſ] wherefore E and Q are the r 31.7.
leaſt in their proportion; [t] and ſo E meaſures P as ſ 23.7.
many times as Q does D; [u] therefore Q is one of t 21.7. u 13.7.
them A, B, C. Let it be B. ſeeing then of equality B.
D :: E. H. [x] and ſo BH = DE = F = PQ. [x] and ſo x 19.7.
alſo Q. B :: H. P. [y] therefore H = P. therefore P is y 14.5.
alſo one of them A, B, C, &c. *againſt the Hypoth.*
Wherefore no other beſide the foreſaid numbers
meaſures F, and [z] conſequently F is a perfect number. z 22.def.7.
Which was to be Demonſtrated.

The End of the ninth Book.

THE TENTH BOOK OF EUCLIDE'S ELEMENTS.

Definitions.

I. Commenſurable magnitudes are thoſe, which are meaſured by one and the ſame meaſure.

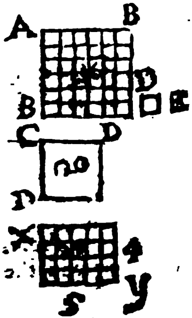


The note of commenſurability is ⊔, *as* A ⊔ B; *that is, the line* A *of* 8 *foot is commenſurable to the line* B *of* 13 *foot; becauſe* D *a line of one foot meaſures both* A *&* B. *Alſo* $\sqrt{18}$ ⊔ $\sqrt{50}$; *becauſe* $\sqrt{2}$ *meaſures both* $\sqrt{18}$ *and* $\sqrt{50}$. *For* $\sqrt{\frac{18}{2}} = \sqrt{9} = 3$. *and* $\frac{50}{2} = \sqrt{25} = 5$. *wherefore* $\sqrt{18}$. $\sqrt{50}$ :: 3. 5.

II. Incommenſurable magnitudes are ſuch, of which no common meaſure can be found.

Incommenſurability is denoted by this mark ⊔; *as* $\sqrt{6}$ ⊔ $\sqrt{25}$ (5;) *that is,* $\sqrt{6}$ *is incommenſurable to the number* 5, *or to a magnitude deſigned by that number; becauſe there is no common meaſure of them, as ſhall appear hereafter.*

III. Right lines are commenſurable in power, when the ſame ſpace does meaſure their ſquares.



The mark of this commenſurability is ⊤; *as* AB ⊤ CD. *i.e. the line* AB *of 6 foot is in power commenſurable to the line* CD, *which is expreſſed by* √ 20. *becauſe* E *the ſpace of one foot ſquare does as well meaſure* ABq (36) *as the rectangle* XY (20) *to which the ſquare of the line* CD (√ 20) *is equall. The ſame note* ⊤ *ſometimes ſignifies commenſurable in power only.*

IV. Lines incommenſurable in power are ſuch, to whoſe ſquares no ſpace can be found to be a common meaſure.

This incommenſurability is denoted thus; 5 ⊤ v√ 8. *i.e. the numbers or lines* 5, *and* v√ 8 *are incommenſurable in power, becauſe their ſquares* 25 *and* √ 8 *are incommenſurable.*

V. From which it is manifeſt, that to any right line given right lines infinite in multitude are both commenſurable and incommenſurable; ſome in length and power, others in power only. The right line given is called a Rationall line.

The note of which is ρ.

VI. And lines commenſurable to this line, whether in length and power, or in power only, are alſo called Rationall, ρ.

VII. But ſuch as are incommenſurable to it, are called Irrationall,

And denoted thus ρ.

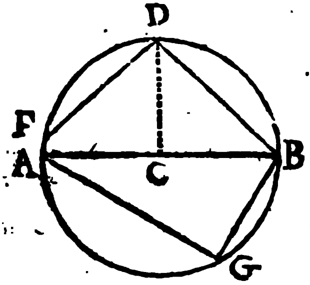
VIII. Alſo the ſquare which is made of the ſaid given right line is called Rationall, ρν.

IX. And likewiſe ſuch figures as are commenſurable to it, are Rationall ρα.

X. But ſuch as are incommenſurable, Irrationall ρα.

XI. And thoſe right lines alſo, which contain them in power, are Irrationall ρ.

Schol.



That the laſt ſeven definitions may be rendred more clear by an example, let there be a circle AD-BP, *whoſe ſemidiameter is* CB *inſcribe therein the ſides of the ordinate figures, as of a Hexagone* BP, *of a Triangle* AP, *of a ſquare* BD, *of a Pentagone* FD. *Therefore, if according to the* 5. def. *the ſemidiameter* CB *be the Rationall line given, expreſſed by the number* 2, *to which the other lines* BP, AP, BD, FD, *are to be compared, then* P ^{a} = BC = 2. *wherefore* BP *is* ρ' $\sqcap$ BC, *according to the* 6. def. *Alſo* AP ^{b} = √ 12 (*for* ABq (16) – BPq (4) = 12) *therefore* AB *is* ρ' $\underset{|}{\sqcap}$ BC *likewiſe accord. to the* 6. def. *and* APq (12) *is* ρν *by the* 9. def. *Moreover* BD ^{b} = √ DCq + BCq = √ 8; *whence* BD *is* ρ' $\underset{|}{\sqcap}$ BC; *and* BDq ρν *Laſtly,* FDq = 10 – √ 20 (*as ſhall appear by the praxis to be delivered at the* 10. 13.) *ſhall be* ρν, *according to the* 10. def. & *conſequently* FD = √ : 10 – √ 20 *is* ρ', *according to the* 11. def.

a cor. 15. 4.

b 47. 1.

A Poſtulate.

That any magnitude may be ſo often multiplied, till it exceed any magnitude whatſoever of the ſame kind.

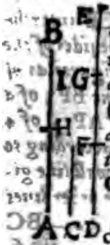
Axiomes.

1. A magnitude meaſuring how many magnitudes ſoever, does alſo meaſure that which is compoſed of them.

2. A magnitude meaſuring any magnitude whatſoever, does likewiſe meaſure every magnitude which that meaſures.

3. A magnitude meaſuring a whole magnitude and a part of it taken away, does alſo meaſure the reſidue.

PROP. I.

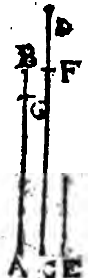


Two unequall magnitudes AB, C, *being given, if from the greater* AB *there be taken away more then half* (AH) *and from the reſidue* (HB) *be again taken away more then half* (HI) *and this be done continually, there ſhall at length be left a certain magnitude* IB, *leſſe then the leſſe of the magnitudes firſt given* C.

a Take C ſo often, till it's multiplex DE doe ſomewhat exceed AB, and there be DF = FG = GE = C. Take from AB more then half HA, and from the remainder HB more then half HI, and ſo continually, till the parts AH, HI, IB, be equall in multitude to the parts DF, FG, GE. Now it is plain, that FE, which is not leſſe then $\frac{1}{2}$ DE, is greater then HB, which is leſſe then $\frac{1}{2}$ AB $\sqsupset$ $\frac{1}{2}$ DE. And in like manner GE, which is not leſſe then $\frac{1}{2}$ FE, is greater then IB $\sqsupset$ $\frac{1}{2}$ HB. therefore C, or GE $\sqsubset$ IB. *W. W. to be Dem.* a post. 10.

The ſame may alſo be demonſtrated, if from AB the half AH be taken away, and again from the reſidue HB the half HI, and ſo forward.

PROP. II.

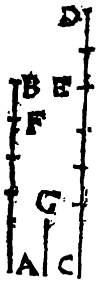


Two unequall magnitudes being given (AB, CD) if the leſſe AB be continually taken from the greater CD, by an interchangeable ſubſtraction, and the reſidue doe not meaſure the magnitude going before, then are the magnitudes given incommenſurable.

If it be poſſible, let ſome magnitude E be the common meaſure. Then becauſe AB taken from CD, as often as it can be, leaves a magnitude FD leſſe then it ſelf, and FD taken from AB leaves GB, and ſo forward; a therefore at length ſome magnitude GB ⊐ E ſhall be left. therefore E b meaſuring AB, c and ſo CF, b and the whole CD, d ſhall alſo meaſure the reſidue FD. c conſequently alſo AG; d wherefore it ſhall likewiſe meaſure the remainder GB, leſſe then it ſelf. *Which is Abſurd.*

a 1. 10.
b hyp.
c 2. ax. 10.
d 3. ax. 10.

PROP. III.



Two commenſurable magnitudes being given AB, CD, *to find out their greateſt common meaſure* FB.

Take AB from CD, and the reſidue ED from AB, and FB from ED, till FB meaſure ED (which will come to paſſe at length, a becauſe by the Hyp. AB ⊥ CD) FB ſhall be the magnitude required.

a 2. 10.

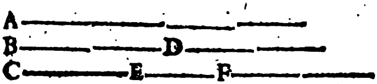
For FB b meaſures ED, c and ſo alſo AF; but it meaſures it ſelf too, d therefore likewiſe AB, c and conſequently CE, d and ſo the whole CD. Wherefore FB is the common meaſure of AB, CD. If you affirm, G to be a common meaſure greater then that, then G meaſuring AB and CD, e meaſures alſo CE and f the remainder ED, e and ſo AF; and f conſequently the remainder FB, the greater the leſſe. *Which is Abſurd.*

b conſtr.
c 2. ax 10.
d 1. ax. 10.
e 2. ax. 10.
f 3. ax. 10.

Coroll.

Hence, A magnitude that meaſures two magnitudes, does alſo meaſure their greateſt common meaſure.

PROP. IV.



Three commenſurable magnitudes being given A, B, C, *to find out their greateſt common meaſure.*

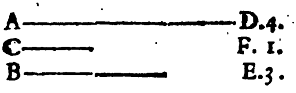
a Find out D the greateſt common meaſure of any two A, B; a alſo E the greateſt common meaſure of D and C. therefore E is the magnitude ſought for. a 3. 10.

a For it is clear, E meaſuring D and C, b does meaſure the three, A, B. C. Conceive another magnitude F greater then that to meaſure them; c then F meaſures D, c and conſequently E the greateſt common meaſure of D, and C, the greater the leſſe. *Which is Abſurd.* b conſtr. and 2. ax. 10. c cor. 3. 10.

Coroll.

Hence alſo it appeares, that if a magnitude meaſure three magnitudes, it ſhall likewiſe meaſure their greateſt common meaſure.

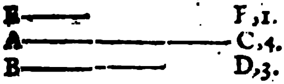
PROP. V.



Commenſurable magnitudes A, B, *have ſuch proportion one to another, as number hath to number.*

a C being found the greateſt common meaſure of A, B; as often as C is conteined in A and B, ſo often is 1 conteined in the numbers D and E; b therefore C. A :: 1. D; wherefore inverſly A. C :: D. 1. b but likewiſe C. B :: 1. E. c therefore of equality A. B :: D. E :: N. N. *W.W. to be Dem.* a 3. 10. b 27. def. 7.

PROP. VI.

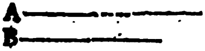


If two magnitudes A, B, have ſuch proportion one to another, as number C hath to number D, thoſe magnitudes A, B ſhall be commenſurable.

What part 1 is of the number C, [a] that let E be of A. Therefore becauſe E.A [b] :: 1.C. and A.B [c] :: C.D. [d] therefore of equality ſhall E.B :: 1. D. Wherefore ſeeing 1 [e] meaſures the number D, [f] likewiſe E meaſures B; but it [g] alſo meaſures A; [h] therefore A ⊤⊥ B. *W.W. to be Dem.*

a ſch. in 6.
b conſtr.
c hyp.
d 22. 5.
e 5. ax. 7.
f 20. def. 7.
g conſtr.
h 1. def. 10.

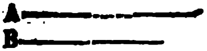
PROP. VII.



Incommenſurable magnitudes A, B, have not that proportion one to another, which number hath to number.

a 6. 10. If you affirm A.B :: N.N. [a] then A ⊤⊥ B. *againſt the Hyp.*

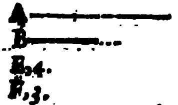
PROP. VIII.



If two magnitudes A, B, have not that proportion one to another, which number hath to number, thoſe magnitudes are incommenſurable.

a 5. 10. Conceive A ⊤⊥ B. [a] then A.B :: N.N *contrary to the Hyp.*

PROP. IX.



The ſquares deſcribed of right lines commenſurable in length, have that proportion one to another, that a ſquare number hath to a ſquare number. And ſquares, which have that pro-

portion one to another, that a ſquare number hath to a ſquare number, ſhall alſo have their ſides commenſurable in length. But ſuch ſquares as are made of right lines incommenſurable in length, have not that proportion one to another, which a ſquare number hath to a ſquare number. And ſquares which have not ſuch proportion one to another as a ſquare number hath to a ſquare number, have not their ſides commenſurable in length.

1. *Hyp.* A ⊔ B. I ſay Aq.Bq :: Q. Q.

For [a] let A. B :: number E. number F. therefore $\frac{Aq}{Bq}$ ([b] $\frac{A\ twice}{B}$) [c] = $\frac{E\ twice}{F}$ [d] = $\frac{Eq}{Fq}$. [e] therefore Aq.Bq :: Eq.Fq :: Q.Q. *W.W. to be Dem.*

2. *Hyp.* Aq.Bq :: Eq.Fq :: Q.Q. I ſay A ⊔ B. For $\frac{A}{B}$ twice ([f] $\frac{Aq}{Bq}$) [g] = $\frac{Eq}{Fq}$ [h] = $\frac{E}{F}$ twice. [i] therefore A. B :: E. F :: N.N. [k] wherefore A ⊔ B. *Which was to be Dem.*

3. *Hyp.* A ⊔ B. I deny that Aq.Bq :: Q.Q. For ſuppoſe Aq. Bq :: Q. Q. then A ⊔ B, as is ſhewn before, *againſt the Hyp.*

4. *Hyp.* Not Aq. Bq :: Q. Q. I ſay that A ⊔ B. For conceive A ⊔ B. then Aq Bq :: Q.Q. as above, *againſt the Hyp.*

a 5. 10.
b 20. 6.
c ſch. 23. 6.
d 11. 8.
e 11. 5.
f 20. 6.
g hyp.
h 11. 8.
i ſch. 23. 6.
k 6. 10.

Coroll.

Lines ⊔ are alſo ⊐. but not on the contrary. And lines ⊔̣ are not therefore ⊐̣. but ⊐̣ are alſo ⊔̣.

PROP. X.

If four magnitudes be proportionall (C.A :: B.D) *and the firſt* C *be commenſurable to the ſecond* A, *the third* B *ſhall be commenſurable to the fourth* D. *And if the firſt* C *be incommenſurable to the ſecond* A, *alſo the third* B *ſhall be incommenſurable to the fourth* D.

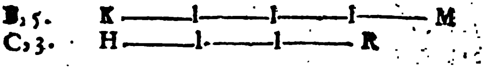
a 5.10. If C ⊓⊔ A, ᵃ then C.A :: N.N ᵇ :: B.D.
b 6.10. ᶜ therefore B ⊓⊔ D. But if ⊓⊔ A, ᶜ then ſhall not
c 7.10. C.A :: N.N :: B.D. ᵈ wherefore B ⊓⊔ D. *W.W.to be*
d 8.10. *Dem.*

Lemma 1.

To find out two plane numbers, not having the proportion which a ſquare number hath to a ſquare.

Any two plane numbers not like will ſatisfy this Lemma, as thoſe numbers which have ſuperparticular, ſuperbipartient, or double proportion; or any two prime numbers. *See ſchol.27.8.*

Lemma 2.



To find out a line HR, *to which a right line given* KM *hath the proportion of two numbers given* B, C.

a ſch.10.6. ᵃ Divide KM into as many equall parts as there are unites in the number B. and let as many of theſe, as there are unites in the number C, ᵇ make the right line HR. it is manifeſt that KM.HR :: B.C.

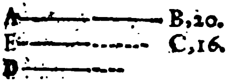
b 3.1.

Lemma 3.

To find out a line D, *to the ſquare of which the ſquare of a right line given* KM *hath the proportion of two numbers given* B, C.

a 2.lem.10.10. Allow B.C ᵃ :: KM.HR. and between KM and
b 13.6. HR, ᵇ find a mean proportionall D. Therefore
c 20.6. KMq.Dq. ᶜ :: KM.HR ᵈ :: B.C.
d conſtr.

PROP. XI.

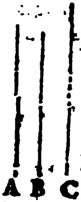


To find two right lines incommenſurable to a right line given A, one D in length only, the other E in power alſo.

1. Take the numbers B, C, [a] ſo that there be not B. C :: Q. Q. [b] and let B. C :: Aq. Dq. [c] it is plane that A ⊔ D. But Aq [d] ⊔ Dq. *W.W. to be Done.* a 1. lem. 10. b 3. lem. 10. c 9. 10. d 6. 10.

2. [d] Make A. E :: E. D. I ſay Aq ⊔ Eq. For A. D [e] :: Aq. Eq. therefore ſince A ⊔ D, as before: [f] therefore Aq ⊔ Eq. *W.W. to be Done.* d 13. 6. e 20. 6. f 10. 10.

PROP. XII.



Magnitudes (A, B) commenſurable to the ſame magnitude C, are alſo commenſurable one to the other.

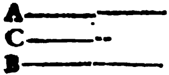
Becauſe A ⊔ C, and C ⊔ B, [a] let A. C :: N. N :: D. E, & C. B :: N. N :: F. G. [b] take three numbers H, I, K, the leaſt ∺ in the proportions of D to E, & F to G. Now becauſe A. C [c] :: D. E [c] :: H. I. and C. B [c] :: F. G :: I. K, [d] therefore of equality A. B :: H. K :: N. N. [e] therefore A ⊔ B. *W.W. to be Done.* a 5. 10. b 4. 8. c conſtr. d 22. 5. e 6. 10.

Schol.

Hence, Every right line commenſurable to a rationall line is alſo it ſelf rationall. And all right lines rationall are commenſurable to one another, at leaſt in power. Alſo, every ſpace commenſurable to a rationall ſpace is rationall too: and all rationall ſpaces are commenſurable one to another. But magnitudes whereof one is rationall, the other irrationall, are incommenſurable amongſt themſelves. 12. 10. and def. 6. def. 9. def. 7. & 10.

PROP.

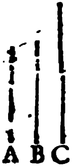
PROP. XIII.



If there be two magnitudes A, B, and one of them A commenſurable to a third C, but the other B incommenſurable, thoſe magnitudes A, B are incommenſurable.

a hyp. b 12. 10. Conceive B ⊤⊥ A. then being C [a] ⊤⊥ A; [b] therefore C ⊤⊥ B, *againſt the Hyp.*

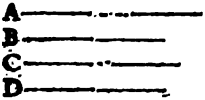
PROP. XIV.



If there be two magnitudes commenſurable A, B; and one of them A incommenſurable to any other magnitude C, the other alſo B ſhall be incommenſurable to the ſame C.

a hyp. b 12. 10. Imagine B ⊤⊥ C. then for that A [a] ⊤⊥ B, [b] therefore A ⊤⊥ C, *againſt the Hyp.*

PROP. XV.



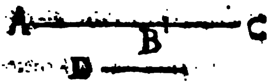
If four right lines be proportionall (A. B :: C. D) and the firſt A be in power more then the ſecond B by the ſquare of a right line commenſurable to it ſelf in length, then alſo the third C ſhall be more in power then the fourth D by the ſquare of a right line commenſurable to it ſelf in length. But if the firſt A be more in power then the ſecond B by the ſquare of a right line incommenſurable to it ſelf in length, then ſhall the third C be more in power then the fourth D by the ſquare of a right line incommenſurable to it ſelf in length.

a hyp.
b 22. 6.
c 17. 5.
d 22. 6.
e cor 4. 5.
f 22. 5.

For becauſe A. B [a] :: C. D. [b] therefore Aq. Bq :: Cq. Dq. [c] therefore by diviſion Aq — Bq. Bq :: Cq — Dq. Dq. [d] wherefore √: Aq — Bq. B :: √: Cq — Dq. D. [e] and ſo inverſely B. √: Aq — B :: D. √: Cq — Dq. [f] therefore of equality A. √: Aq — Bq :: C. √: Cq — Dq. conſequently if A ⊤⊥ or ⊤⊥ √ Aq

− Bq. g then likewiſe C ⌐⊔, or ⌐⊔ √: Cq − Dq. g 14. 10.
W. W. to be Dem.

PROP. XVI.



If two magnitudes commenſurable AB, BC *be compoſed, the whole magnitude* AC *ſhall be commenſurable to each of the parts* AB, BC. *And if the whole magnitude* AC *be commenſurable to either of the parts* AB, *or* BC, *thoſe two magnitudes given at firſt* AB, BC, *ſhall be commenſurable.*

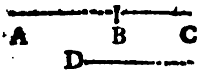
1. *Hyp.* a Let D be the common meaſure of AB, BC; b ſo D meaſures AC. c and therefore AC ⌐⊔ AB, and BC. *Which was to be Dem.* a 3. 10. b 1. ax. 10. c 1. def. 10.

2. *Hyp.* d Let D be the common meaſure of AC, AB. therefore D meaſures AC − AB (BC) and conſequently AB ⌐⊔ BC. *W. W. to be Dem.* d 3 ax. 10.

Coroll.

Hence it follows, if a whole magnitude compoſed of two be commenſurable to any one of them, the ſame ſhall be commenſurable to the other alſo.

PROP. XVII.



If two incommenſurable magnitudes AB, BC, *be compoſed, the whole magnitude alſo* AC *ſhall be incommenſurable to either of the two parts* AB, BC. *And if the whole magnitude* AC *be incommenſurable to one of them* AB, *the magnitudes firſt given* AB, BC, *ſhall be incommenſurable.*

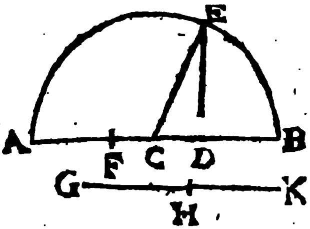
1. *Hyp.* If it can be, let D be the common meaſure of AC, AB. a therefore D meaſures AC − AB (BC) b and therefore alſo AB ⌐⊔ BC, *againſt the Hypoth.* a 3. ax. 10. b 1 def. 10.

e 16.10. 2. *Hyp.* Conceive AB ⊤⊥ BC, *e* therefore AC ⊤⊥ AB. *Againſt the Hyp.*

Coroll.

Hence alſo, If one magnitude, compoſed of two, be incommenſurable to any one of them, the ſame alſo ſhall be incommenſurable to the other.

PROP. XVIII.



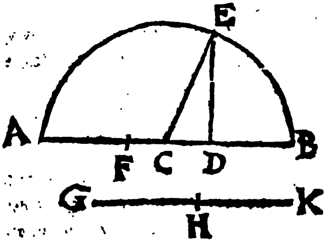
If there be two unequal right lines AB, GK, *and upon the greater* AB *a parallelogram* ADB *equall to the fourth part of the ſquare made of the leſſe line* GK, *and wanting in figure by a ſquare, be applyed, and divide the ſaid* AB *into parts commenſurable in length* AD, DB; *then ſhall the greater line* AB *be more in power then the leſſe* GK *by the ſquare of a right line* FD *commenſurable in length to the greater. And if the greater* AB *be in power more then the leſſe* GK *by the ſquare of the right line* FD *commenſurable unto it ſelf in length, and a parallelogram* ADB *equall to the fourth part of the ſquare made of the leſſe line* GK, *& wanting in figure by a ſquare, be applied to the greater* AB, *then ſhall it divide the ſame into parts* AD, DB *commenſurable in length.*

a 10.1.
b 28.6.
c 8.2.
d conſtr. and 4.2.
e 16.10.
g cor.16.10.
h cor.16.10.
k 12.10.
l 16.10.

a Divide GK equally in H, and *b* make the rectangle ADB = GHq. Cut off AF = DB, then is ABq *c* = 4 ADB *d* (4 GHq or GKq) + FDq. Now in the firſt place, if AD ⊤⊥ DB, then ſhall AB *e* ⊤⊥ BD *e* ⊤⊥ 2 DB *f* (AF + DB, or AB − FD) *h* therefore AB ⊤⊥ FD. *W. W. to be Dem.* But ſecondly, if AB ⊤⊥ FD, *h* then ſhall AB ⊤⊥ AB − FD (2 DB) *k* therefore AB ⊤⊥ DB. *l* wherefore AD ⊤⊥ DB. *W. W. to be Dem.*

PROP.

PROP. XIX.

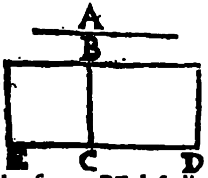


If there be two right lines unequall AB, GK, *and to the greater* AB *be applyed a parallelogram* ADB *equall to the fourth part of a ſquare made upon the leſſe* GK, *and wanting in figure by a ſquare, and alſo thus applyed divide the ſaid* AB *into parts* AD, DB *incommenſurable in length; the greater line* AB *ſhall be in power more then the leſſe* GK *by the ſquare of the right line* FD *incommenſurable to the greater in length: And if the greater line* AB *be more in power then the leſſe* GK *by the ſquare of a right line* FD *incommenſurable unto it ſelf in length, and if alſo upon the greater* AB *be applied a parallelogram* ADB *equall to the fourth part of the ſquare of the leſſe* GK *and wanting in figure by a ſquare, then ſhall it divide the ſaid greater line* AB *into parts incommenſurable in length* AD, DB.

Suppoſe all the ſame that was done and ſaid in the prec. prop. Therefore firſt, If AD ⊔ DB, *a* then ſhall AB ⊔ DB. *b* Wherefore AB ⊔ 2 DB (AB − FD) *c* therefore AB ⊔ FD. *W.W. to be Dem.* a 17. 10. b 13. 10. c cor 17. 10.

Secondly, If AB ⊔ FD, then AB ⊔ AB − FD (2 DB;) *d* wherefore AB ⊔ DB, *e* and conſequently AD ⊔ DB. *W.W. to be Dem.* d 13. 10. e 17. 10.

PROP. XX.



A rectangle BD *comprehended under right lines* BC, CD, *rationall and commenſurable in length, according to one of the foreſaid wayes, is rationall.*

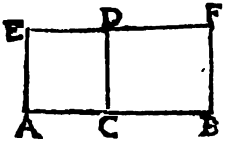
Let A be given ρ, and a the ſquare BE deſcribed upon BC. Becauſe DC. CE (BC) b :: BD. BE. and DC c ⊔ BC, d therefore ſhall the rectangle BD be ⊔ ſquare BE. wherefore ſeeing the ſquare BE e ⊔ Aq, ſhall alſo BD be ⊔ Aq. and ſo the rectangle BD is f ρ. W. W. to be Dem.

a 46. 1.
b 1. 6.
c hyp.
d 10. 10.
e hyp. and 9. def. 10.
f 9. 10.

Note. *There are three kinds of lines rationall commenſurable one to another. For either, of two lines rationall commenſurable in length one to the other, one is equall to the rationall line propounded, or neither of them is equall to it, notwithſtanding both of them are commenſurable to it in length; or laſtly both of them are commenſurable to the rationall line given only in power. And theſe are the wayes which the preſent theoreme ſpeaks of.*

In numbers, let there be BC, $\surd 8$ $(2 \surd 2)$ and CD $\surd 18$ $(3 \surd 2)$ then ſhall the rectangle BD $=$ $\surd 144 = 12$.

PROP. XXI.



If a rationall rectangle DB *be applyed to a rationall line* DC, *it makes the breadth thereof* CB *rationall, and commenſurable in length to that line* DC, *whereto* DB *is applyed.*

Let G be propounded ρ, and the ſquare DA deſcribed on BC. becauſe BD. DA :: a BC. CA; and BD, DA b are ρ c & ſo ⊔. d therefore BC ⊔ CA. but CD (CA) is ρ. e therefore BC is ρ. W. W. to be Dem.

a 1. 6.
b hyp.
c ſch. 12. 10.
d 10. 10.
e ſch. 12. 10.

In numbers, let there be the rectangle DB, 12. and DC, √ 8. then ſhall CB, √ 18. but √ 18 = 3 √ 2. and √ 8 = 2 x √ 2.

Lemma.

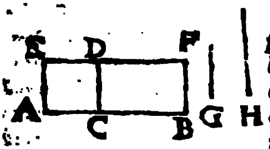
A————
B——————
C————

To find out two right lines rationall commenſurable only in power.

Let A be propounded ρ'. a Take B ⊓ A, a and C ⊓ B. b it is clear that B and C bee the lines required.

a 11. 10.
b ſch. 12. 10.

PROP. XXII.



A rectangle DB *comprehended under right lines rationall* DC, CB *commenſurable in power only, is irrationall: & the right line* H, *which conteineth that rectangle in power is irrationall, and called a Mediall line.*

Let G be the propounded ρ', and the ſquare DA deſcribed on DC, and let Hq = DB. Becauſe AC. CB a :: DA. DB. b & AC ⊓ CB, c ſhall be DA ⊓ DB (Hq.) d but Gq ⊓ DA. e thereſefore Hq ⊓ Gq. f wherefore H is ρ'. *Which was to be Dem.* and let it be called a Mediall line, becauſe AC. H :: H. CB.

a 1. 6.
b hyp.
c 10. 10.
d hyp. and 9. def. 10.
e 13. 10.
f 11. 10.

In numbers, let there be DC, 3. and CB, √ 6. then ſhall the rectangle be DB (Hq) √ 54. wherefore H is √√ 54.

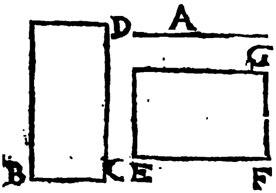
The note of a mediall line is μ, of a mediall rectangle μν, of more together μα.

Schol.

Every rectangle that can be conteined under two right lines rationall commenſurable only in power, is mediall, although it be conteined under two right lines irrationall: and every mediall rectangle may be conteined under two right lines rationall, commenſurable only in power. as for example, the

$\sqrt{24}$ is μν, becauſe it is conteined under $\sqrt{3}$, and $\sqrt{8}$, which are ρ, ⊤⊥. although it may be conteined under $v\sqrt{6}$, and $v\sqrt{96}$ irrationalls; for $\sqrt{24} = v\sqrt{576} = v\sqrt{6} \times v\sqrt{96}$.

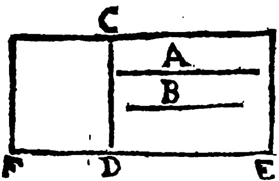
PROP. XXIII.



If the rectangle BD *made of a mediall line* A, *be applied on a rationall line* BC, *it makes the breadth* CD *rationall, and incommenſurable in length to the line* BC, *whereunto the rectangle BD is applied.*

a ſch.22.10. Becauſe A is μ, *a* therefore ſhall Aq be equall to
b 1.ax.1. ſome rectangle (EG) conteined under EF and FG
c 14.6. ρ ⊤⊥. *b* therefore D = EG. *c* whence BC. EF ::
d 22.6. FG. CD. *d* therefore BCq. EFq :: FGq. CDq. But
e hyp. BCq and EFq *e* are ρα, *f* and ſo ⊓⊔. *g* therefore FGq
f ſch.12.10. ⊓⊔ CDq. Wherefore being FG is ρ, *h* therefore CD
g 10.10. ſhall be ρ. Moreover, becauſe EF.FG *k* :: EFq.EG
h ſch.12.10. (BD;) for that EF ⊓⊔ FG, *l* ſhall EFq be ⊓⊔ BD.
k 1.6. But EFq *m* ⊓⊔ CDq. *n* therefore the rectangle
l 10.10. BD ⊓⊔ CDq. Whence being CDq.BD *o* :: CD.BC.
m ſch.12.10. *p* ſhall CD be ⊓⊔ BC. therefore, &c.
n 13.10.
o 1.6.
p 10.10.

PROP. XXIV.



A right line B *commenſurable to a mediall line* A *is alſo a mediall line.*

a 11.6. Upon CD ρ *a* make the rectangle CE = Aq; *a* and the rectangle CF = Bq. Becauſe Aq (CE) is μι, *b* and CD
b hyp. ρ, *c* therefore ſhall the latitude DE be ρ ⊤⊥ CD. But
c 23.10.

for that CE. CF *d* :: ED. DF. and CE *e* ⊔ CF; *f* therefore ED ⊔ DF. *g* therefore DF is ρ ⊔ CD. *h* whence the rectangle CF (Bq) is μν, and ſo B is μ. *W.W. to be Dem.*

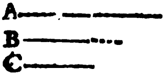
d 1. 6.
e *hyp.*
f 10. 10.
g 12. *and* 13. 10.
h 22. 10.

Obſ. *that the note* ⊐ *for the moſt part ſignifies commenſurable in power only, as in this and the precedent demonſtrations,* &c.

Coroll.

Hereby it is manifeſt that a ſpace commenſurable to a mediall ſpace, is alſo mediall.

Lemma.

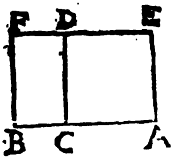


To find out two right lines mediall A, B, *commenſurable in length, and alſo two,* A, C, *commenſurable only in power,*

a Let A be any μ, *b* take B ⊔ A, and *c* C ⊐ A. *d* it appears to be done.

a *lem.* 22. 10. *and* 13 6.
b 2. *lem.* 10. 10.
c 3. *lem.* 10. 10.
d *conſtr. and* 24. 10.

PROP. XXV.

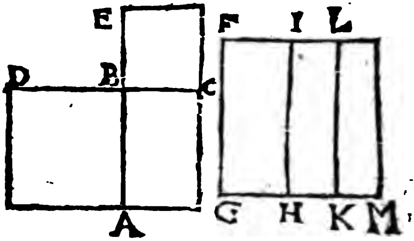


A rectangle DB *conteined under* DC, CB *mediall right lines commenſurable in length, is mediall.*

Upon DC deſcribe the ſquare DA. Being AC (DC.) CB *a* :: DA. DB, & DC ⊔ CB; *b* ſhall DA ⊔ DB. *c* therefore DB is μν. *Which was to be Dem.*

a 1. 6.
b 10. 10.
c 24. 10.

PROP. XXVI.

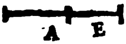


A rectangle AC *comprehended under mediall right lines* AB, BC *commenſurable only in power, is either rationall or mediall.*

a 46. 1. b cor. 16. 6. Upon the lines AB, BC, a deſcribe the ſquares AD, CE; and upon FG ρ' b make the rectangles FH, = AD, b and IK = AC, b and LM = CE.

c hyp. & 24. 10. d 23. 10. e 10. 10. f 20. 10. g ſch. 22. 6. h 1. 6. k 17. 6. l 12. 10 m 20. 10. n 22. 10. The ſquares AD, CE, that is, the rectangles FH, LM, c are μν and ⊓⊔. therefore GH, KM, having the ſame proportion d are ρ', e and ⊓⊔. f therefore GH x KM is ρν. But becauſe AD, AC, CE, that is, FH, IK, LM, g are ∺; h and ſo GH, HK, KM alſo ∺; k thence HKq = GH x KM. l therefore HK is ρ', or ⊓⊔, or ⊓⊤ IH (GF;) if ⊓⊔, m then the rectangle IK or AC is ρν. but if ⊓⊤, n then AC is μν. *Which was to be Dem.*

Lemma.

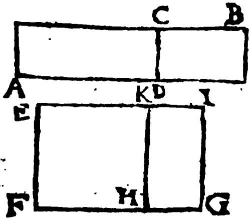


If A *and* E *be* ⊓⊤ *only, Then firſt,* ſhall Aq, Eq, Aq + Eq, Aq − Eq a ⊓⊔. *And ſecondly,* Aq, Eq, Aq + Eq, Aq − Eq ⊓⊔ AE and 2 E. For A. E b :: Aq. AE b :: AE. Eq. therefore ſeeing A c ⊓⊔ E, d ſhall Aq ⊓⊔ AE, e and 2 AE. alſo Eq d ⊓⊔ AE, e and 2 AE. wherefore becauſe Aq + Eq ⊓⊔ Aq and Eq; and Aq − Eq ⊓⊔ Aq and Eq. f therefore ſhall Aq + Eq, f and Aq − Eq be ⊓⊔ AB, and 2 AB.

a hyp. and 16. 10.
b 1. 6.
c hyp.
d 10. 10.
e 14. 10.
f 14. 10.

Hence alſo thirdly, Aq, Eq, Aq + Eq, Aq − Eq, 2 AE [g] ⌐⊔ Aq + Eq + 2 AE; and Aq + Eq − 2 AE. [g] and Aq + Eq + 2 AE ⌐⊔ Aq + Eq − 2 AE. [h] (Q.A − E.) g 14.16 an. 17.10. h cor.7.12.

PROP. XXVII.

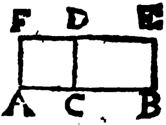


A mediall rectangle AB *exceedeth not a mediall rectangle* AC *by a rationall rectangle* DB.

Upon EF ρ', [a] make EG = AB, [a] and EH = AC. The rectangles AB, AC, *i.e.* EG, EH, [b] are $\mu\alpha$; [c] therefore FG and FH are ρ' ⌐Ч EF. Whence, if KG, [d] *i.e.* DB be $\rho'\nu$, [e] then ſhall HG be ⌐⊔ HK; [f] wherefore HG ⌐Ч FH. [g] and conſequently FGq ⌐⊔ FHq. But FH is ρ'. [h] therefore is FG ρ'. but FG was ρ' before. *Which is contradictory*.

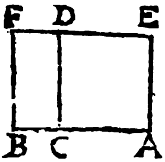
a cor.16.6. b hyp. c 23.10. d 3.ax 1. e 21.10. f 13.10. g lem.16.10. h ſch.12.10.

Schol.



1. *A rationall rectangle* AE *exceeds a rationall rectangle* AD *by a rationall rectangle* CE.

For AE [a] ⌐⊔ AD; [b] therefore AE ⌐⊔ CE. [c] wherefore CE is $\rho'\nu$. *Which*, &c. a hyp. b cor.16.10. c ſch.12.10.

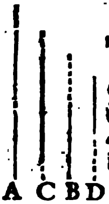


2. *A rationall rectangle* AD *joined with a rationall rectangle* CF *makes a rationall rectangle* AF.

For AD [a] ⌐⊔ CF. [b] wherefore AF ⌐⊔ AD and CF; [c] and ſo AF is $\rho'\nu$. *W. W. to be Dem.* a ſch.12.10. b 16 11. c ſch.12.10.

PROP. XXVIII.

To find out mediall lines (C *and* D,) *which contain a rationall rectangle* CD.



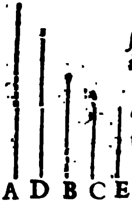
a lem.21 10.
b 13. 6.
c 12. 6.
d 22. 10.
e constr.
f 10. 10.
g 24. 10.
h 17. 6.
k sch. 12. 10.

a Take A and B $\rho\tau$ ⊤. b make A. C :: C.B. c and A.B :: C.D. I ſay the thing required is done. For AB (Cq) d is $\mu\tau$, d whence C is μ. but being that A. B e :: C.D. f therefore C ⊤ D. g and conſequently D is μ. Moreover by inverſion A.C :: B.D. *i.e.* C. B :: B.D. h therefore Bq = CD. But Bq is $\rho\tau$. k therefore C D is $\rho\tau$. *W.W. to be done.*

In numbers, let A be $\surd$ 2; and B $\surd$ 6. therefore C is $v\surd$ 12. make $\surd$ 2. $\surd$6 :: $v\surd$ 12. D. or $v\surd$ 4. $v\surd$ 36 :: $v\surd$ 12. D. then ſhall D be $v\surd$ 108. but $v\surd$ 12 x $v\surd$ 108 = $v\surd$ 1296 = $\surd$ 36 = 6. therefore CD is 6. likewiſe C.D :: 1. $\surd$ 3. wherefore C ⊤ D.

PROP. XXIX.

To find out mediall right lines commenſurable in power only, D *and* E, *containing a mediall rectangle* DE.



a lem 21.10.
b 13 6.
c 12. 6.
d 17. 6.
e 22. 10.
f conſtr.
g 10. 10.
h 24. 10.
k conſtr. and cor 4 5.
l 16 6.
m 22. 6.

a Take A, B, C, ρ ⊤. make A. D b :: D.B. c and B. C :: D.E. I ſay the thing deſired is performed.

For AB d = Dq. and AB e is $\mu\tau$, therefore D is μ; and B f ⊤ C, g whence D ⊤ E. therefore h E is μ. Moreover B.C f :: D.E. and by inverſion B.D :: C.E. i.e. D.A :: C.E. l therefore DE = AC. But AC m is $\mu\tau$. therefore DE is $\mu\tau$. *W.W. to be Done.*

In numbers, let A be 20. and B, $\surd$ 200, and C, $\surd$ 80. Therefore D is $\surd\surd$ 80000; and E $v\surd$ 12800. Therefore DE = $\surd\surd$ 1024000000 = 32000. and D.E :: $\surd$ 10. 2. wherefore D ⊤ E.

Schol.

Schol.

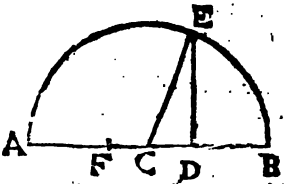
A,6. C,12.
B,4. D,8.
$\overline{AB}$, 24. $\overline{CD}$, 96.

A,6. C,5.
B,4. D,8.
$\overline{AB}$, 24. $\overline{CD}$, 40.

To find out two plane numbers, like or unlike.

Take any four numbers proportionall A.B :: C.D. it is manifeſt that AB and CD are like plane numbers. And you may find out as many unlike plane numbers, as you pleaſe, by help of *Schol.* 27. 8.

L*emma.*



To find out two ſquare numbers (DEq *and* CDq) *ſo that the number compoſed of them* (CEq) *be ſquare alſo.*

Take AD, DB like plane numbers (of which let both be equall, or both odde) *viz.* AD, 24. and DB, 6. The totall of theſe (AB) is 30; the difference (FD) 18. half of which (CD) is 9. [a] Now the like plane numbers AD, DB, have one mean number proportionall, namely DE. therefore it is evident that every of thoſe numbers CE, CD, DE, are rationall, and by conſequence CEq ([b] CDq + DEq) is the ſquare number required.

[a] 18. 8.
[b] 47. 1.

Whereby it will be eaſy to find out two ſquare numbers, the exceſſe of which is a ſquare or not a ſquare number. namely by the ſame conſtruction [c] ſhall CEq − CDq be = DEq.

[c] 3. *ax.* 1.

But if AD, DB be plane numbers unlike, the mediall proportionall line (DE) ſhall not be a rationall number, and ſo neither ſhall the exceſſe (DEq) of the ſquare numbers, CEq, CDq, be a ſquare number.

Lemma 2.

2. *To find out two ſuch ſquare numbers* B, C, *as the number compounded of them* D *is not ſquare. Alſo to divide a ſquare number* A *into two numbers* B, C, *not ſquares.*

A, 3. B, 9. C, 36. D, 45.

1. Take any ſquare number B, and let C be = 4 B, and D = B + C. I ſay the matter is done.

For B is Q. by the conſtr. likewiſe becauſe B. C ::
a 24. 8. 1. 4 :: Q. Q. *a* therefore C alſo ſhall be a ſquare number. But becauſe B + C. (D) C :: 5. 4 :: not Q.
b cor. 24. 8. Q. *b* therefore ſhall not D be a ſquare number. *W. W. to be done.*

A, 36. B, 24. C, 12. D, 3. E, 2. F, 1.

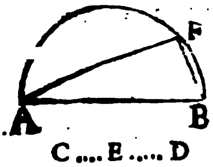
2. Let A be ſome ſquare number. Take D, E, F, plane numbers unlike, and let D be = E + F. make D. E :: A. B. and D. F :: A. C. I ſay the thing required is done.

For becauſe D. E + F :: A. B + C. and D = E
a 14. 5. + F, *a* therefore ſhall A = B + C. Now ſuppoſe
b 21 def. 7. B to be ſquare, *b* then A and B, *c* and conſequently
c 26. 8. D and E are like plane numbers. *Which is contrary to the Hyp.*

The ſame abſurdity will follow if C be ſuppoſed a ſquare number. Therefore, &c.

PROP.

PROP. XXX.



To find out two ſuch rationall right lines AB, AF, *commenſurable only in power, as the greater* AB *ſhal be in power more then the leſſe* AF *by the ſquare of a right line* BF *commenſurable in length to the greater.*

Let AB be the line given ρ'. [a] Take the ſquare numbers CD, CE, ſo that CD − CE (ED) be not Q.[b] and let there be CD. ED :: ABq. AFq. In a circle deſcribed upon the diameter AB [c] draw AF, and alſo BF. Then I ſay AB, AF, are the lines required.

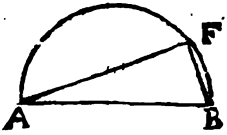
a 1.lem.29.10.
b 3.lem. 10. 10.
c 2. 4.

For ABq. AFq [d] :: CD. ED. [e] therefore ABq ⊓ AFq. but AB is ρ'. [f] therefore AF is alſo ρ'. But becauſe CD is Q: and ED not Q: [g] therefore ſhall AB be ⊓ AF. Moreover by reaſon of the [h] right angle AFB, is ABq [k] = AFq + BFq; therefore ſeeing ABq. AFq :: CD. ED. by converſion of proportion ſhall ABq. BFq :: CD. CE :: Q. Q. [l] therefore AB ⊓ BF. *W. W. to be Done.*

d conſtr.
e 6. 10.
f ſch. 12. 10.
g 9. 10.
h 31. 3.
k 47. 1.
l 9. 10.

In numbers, let there be AB, 6; CD, 9; CE, 4; wherefore ED, 5. Make 9. 5 :: 36. (Q:6) AFq. then AFq ſhall be 20. and conſequently AF √ 20. therefore BFq = 36 − 20 = 16. wherefore BF is 4.

PROP. XXXI.



To find out two rationall lines AB, AF *commenſurable only in power, ſo that the greater* AB *ſhall be in power more then the leſſe* AF *by the ſquare of a right line* BF *incommenſurable in length to the greater.*

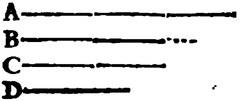
Let AB be the line given ρ'. [a] Take the ſquare numbers CE, ED, ſo that CD = CE + ED be not Q. and in the reſt follow the conſtruction of the *preced. prop. I ſay then* the thing required is done.

a 2. lem. 29. 10.

For, as above, AB, AF, are ρ' ╤. alſo ABq. BFq :: b 9 10. CD. ED. therefore being CD is not Q. AB, BF[b] ſhall be ⊓. *Which was to be Done.*

In numbers, let there be AB, 5. CD, 45. CE = 36. ED = 9. Make 45. 9 :: 25 (ABq.) 5 (AFq.) therefore AF = √ 5. conſequently BFq = 45 − 25 = 20. wherefore BF = √ 20.

PROP. XXXII.

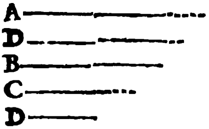


To find out two mediall lines C, D, *commenſurable only in power, comprehending a rational rectangle* CD, *ſo that the greater* C *be more in power then the leſſer* D *by the ſquare of a right line commenſurable in length to the greater.*

a 30. 10. [a] Take A and B ρ' ╤; ſo as √ Aq − Bq ⊓ A.
b 13. 6. [b] and make A. C :: C. B. [c] and A. B :: C. D. I ſay the
c 12. 6. thing is done.
d conſtr. For becauſe A and [d] B are ρ' ╤. [e] therefore ſhall
e 22. 10. C ([f] √ AB) be μ. [g] & thence alſo C ╤ D.[h] there-
f 17. 6. fore D is likewiſe μ. Furthermore, whereas A, B [d] ::
g 10. 10. C. D; and inverſely A. C :: B. D :: C. B; and Bq is ρ'.
h 24. 10. therefore ſhall CD ([k] Bq) be ρ'ν. Laſtly, becauſe √
k 17. 6. Aq − Bq [d] ⊓ A, [l] ſhall √ Cq − Dq be ⊓ C. there-
l 15. 10. fore, &c. But if √ Aq − Bq ⊓ Aq, then ſhall √ Cq − Dq be ⊓ C.

In numbers; let there be A 8, B √ 48 (√: 64 − 16) therefore C = √ AB = ν√ 3072. and D = ν√ 1728. wherefore CD = ν√ 5308416 = √ 2304.

PROP. XXXIII.



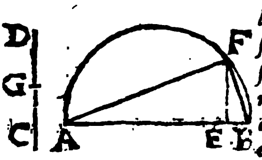
To find out two mediall lines D, E, *commenſurable in power only, comprehending a mediall rectangle* DE, *ſo that the greater* D *ſhall be more in power then the leſſe* E, *by the ſquare of a right line commenſurable to the greater in length.*

a Take A and C ρ' ⊐̸⊏, ſo that √ Aq − Cq ⊐⊏ A. a 30. 10.
take alſo B ⊐̸⊏ A and C, and make A. D c :: D. B b lem. 21. 10.
:: C. E. then D, and E are the lines ſought for. c 13. 6.
d 12. 6.
For becauſe A and C e are ρ', e and B ⊐̸⊏ A and C, e conſtr.
therefore ſhall B be ρ', and D (√. AB) g ſhall be μ. f ſch. 12. 10.
ut becauſe A. D :: C. E, therefore inverſely A. C :: g 22. 10.
). E. wherefore ſeeing A ⊐̸⊏ C, therefore D ſhall be h 10. 10.
⊐̸⊏ E. therefore E is μ. Furthermore, l being D. B :: k 24. 10.
E. and BC is μν. alſo DE, equall to it, is μν. Laſtly l 22. 10.
ecauſe A. C :: D. E. e ſeeing √ Aq − Cq ⊐⊏ A. m 16 6.
herefore √ Dq − Eq ⊐⊏ D. therefore, &c. But if n 15. 5.
√ Aq − Cq ⊐⊏ A. then √ Dq − Eq ⊐⊏ Eq.

In numbers, let there be A 8, C √ 48. B √ 28.
hen D ν√ 3072. and E ν√ 588. wherefore D. E ::
√ 3. and DE = √ 1344.

PROP. XXXIV.

To find out two right lines AF, BF, *incommenſurable in power, whoſe ſquares added together make a rationall figure, and the rectangle contained under them mediall.*



a Let there be found AB, CD, ρ' ⊐̸⊏; ſo that √ a 31. 10.
ABq − CDq ⊐⊏ AB. divide CD equally in G. c make b 10. 1.
the rectangle AEB = GCq. Upon AB the dia- c 28. 6.
meter draw the ſemicircle AFB, erect the perpendi- d 12. 6
cular EF, and draw AF, BF. Theſe are the lines re- e cor. 8. 6. & 17. 6.
quired. f 7. 5.
For AE. BE d :: BA x AE. AB x BE. But BA x AE g 19. 10.
= AFq; and AB x BE = FBq. f therefore AE. EB h 10. 10.
:: AFq. FBq. therefore being AE g ⊐⊏ EB, h AFq
ſhall be ⊐⊏ FBq. Moreover ABq (k AFq + FBq) l is k 31. 3. and 47. 1.
ρ'. Laſtly EFq l = AEB l = CGq. m therefore EF = l conſtr.
CG. therefore CD x AB = 2 EF x AB. But CD x m 1. ax. 1.
AB n is μν. o therefore AB x EF, p or AF x FB, is μν. n 22. 10.
W. W. to be Dem. o 24. 10.
p ſch. 22. 6.

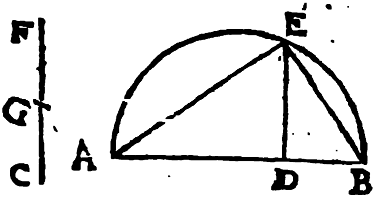
The

The Explication of the ſame by numbers.

Let AB be 6. CD √ 12. then CG $= \sqrt{\frac{12}{4}} = \sqrt{3}$. But AE $= 3 + \sqrt{6}$. and EB $= 3 - \sqrt{6}$. whence AF ſhall be √: 18 + 216. and FB √: 18 − √ 216 − Alſo AFq + FBq is 36, and AF x FB $= \sqrt{108}$.

But AE is found in this manner. Becauſe BA (6.) AF :: AF . AE. therefore 6 AE = AFq = AEq + 3 (EFq.) therefore 6 AE − AEq = 3. Put 3 + e = AE. then 18 + 6e − 9 − 6e − ee, that is 9 − ee = 3. or ee = 6. wherefore e $= \sqrt{6}$. & ſo AE $= 3 + \sqrt{6}$.

PROP. XXXV.



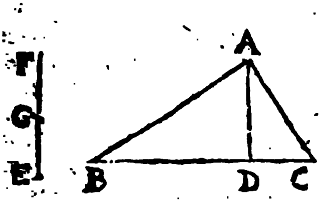
To find out two right lines AE, EB, *incommenſurable in power, whoſe ſquares added together make a mediall figure, and the rectangle contained under them rationall.*

a 32.10. ^a^ Take AB and CF μ ⊓, ſo that AB x CF be ρν, and √ ABq − CFq ⊔ AB. and let the reſt be done as in the prec. *prop.* AE, EB are the lines required.

For, as it is ſhewn there, AEq ⊔ EBq. alſo ABq (AEq + EBq) is μν. and laſtly AB x CF ^b^ is ρν. ^c^ therefore alſo AB x DE, that is, AE x EB, is ρν. therefore, &c.

b conſtr.
c ſch. 12. 10.
d ſch. 22. 6.

PROP. XXXVI.

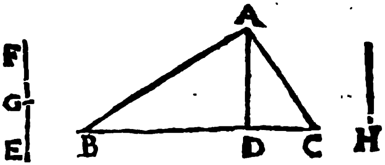


To find out two right lines BA, AC, *incommensurable in power, whoſe ſquares added together make a mediall figure, & the rectangle alſo contained under them mediall, and incommenſurable to the figure compoſed of the ſquares.*

a Take BC and EF μ ⊐⊦, ſo that BC x EF be μν. and √ BCq − EFq ⊐⊏ BC. and ſo forward, as in the prec. BA, AC, ſhall be the lines ſought for. a 33.10.

For (as above) BAq ⊐⊏ ACq. alſo BAq + ACq is μν. and BA x AC is μν. Laſtly BC b ⊐⊏ EF, and c ſo BC ⊐⊏ EG; likewiſe BC. EG d :: BCq. BC x EG (BC x AD, or BA x AC) e therefore BCq (ABq + ACq) ⊐⊏ BA x AC. therefore, &c. b conſtr. c 13. 10. d 1.6. e 14 10.

Schol.



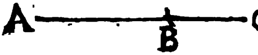
To find out two mediall lines incommenſurable both in length and power.

a Take BC μ. and let BA x AC be μν, and ⊐⊏ BCq (BAq + ACq) b make BA. H :: H. AC. then I ſay BC & H are μ ⊐⊦. For BC is μ, a and BA x AC (c Hq) is μν. wherefore H is alſo μ. d Likewiſe BA x AC ⊐⊏ BCq; therefore Hq ⊐⊏ BCq. therefore, &c. a 38.10. b 13. 6. c 17.6. d 14.10.

Here

Here begin the ſenaries of lines irrationall by compoſition.

PROP. XXXVII.



If two rationall lines AB, BC, *commenſurable only in power, be added together, the whole line* AC *is irrationall, and is called a binomiall line, or of two names.*

a hyp.
b lem. 26.10.
c 11. def. 10.

For becauſe AB a ⊔ BC, thence b ſhall ACq be ⊔ ABq. But AB a is ρ'. c therefore AC is 'ρ. *Which was to be Dem.*

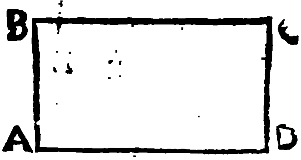
PROP. XXXVIII.

If two mediall lines AB, BC, *only in power commenſurable, be compounded, and contain a rationall rectangle, the whole line* AC *is irrationall, and called a firſt bimediall line.*

a hyp
b lem. 26.10.
c 11. def. 10.

For being that AB a ⊔ BC, b ſhall ACq be ⊔ AB x BC, ρ'ν. c therefore AC is 'ρ. *W. W. to be Dem.*

Lemma.



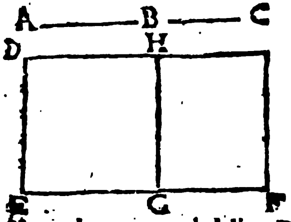
A rectangle AC, *conteined under a rationall line* AB *and an irrationall line* BC, *is irrationall.*

a hyp.
b 21.10.

For if the rectangle AC be affirmed ρ'ν, a then being AB is ρ', b the breadth BC ſhall be alſo ρ'. *againſt the Hyp.*

PROP.

PROP. XXXIX.



If two mediall lines AB, BC, *commenſurable only in power, conteining a mediall rectangle, be compounded, the whole line* AC *ſhall be irrationall, and is called a Second bimediall line.*

Upon the propounded line DE ρ' a make the rectangle DF = ACq; b and DG = ABq + BCq. a cor 16.6. b 47.1. and 22.6.

Becauſe ABq c ⊓⊔ BCq, d therefore ABq + BCq, *i.e.* DG, ⊓⊔ ABq: but ABq e is μν, e therefore DG is μν. But the rectangle ABC is taken μν; e and conſequently 2 ABC (f HF) is μν. g therefore EG and GF are ρ'. Being alſo that DG h ⊓⊔ HF; and DG. HF :: k EG.GF; l therefore EG ⊓⊔ GF. m therefore the whole EF is ρ'. n wherefore the rectangle DF is ρ'ν. o therefore √DF, *i.e.* AC, is ρ'. *W.W. to be Dem.*

c hyp. d 16.10. e 24.10. f 4.2. g 23.10. h lem.26.10. k 1.6. l 10.13. m 37.10. n lem.38.10. o 11.def.10.

PROP. XXXX.

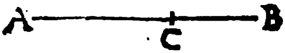
If two right lines AB, BC *commenſurable only in power, be added together, making that which is compoſed of their ſquares rationall, and the rectangle conteined under them mediall, the whole right line* AC *is irrationall and is called a Maior line.*

For whereas ABq + BCq a is ρ'ν. and b ⊓⊔ 2 ABC c μν; and ſo ACq (d ABq + BCq + 2 ABC) e ⊓⊔ ABq + BCq ρ'ν. f therefore ſhall AC be ρ'. *Which was to be Dem.*

a hyp. b sch.12.10. c hyp. & 24.10. d 4.2. e 17.10. f 11.def.10.

PROP.

PROP. XLI.



If two right lines AC, CB, *incommenſurable in power, be added together, having that which is made of their ſquares added together mediall, and the rectangle contained under them rationall, the whole right line* AB *ſhall be irrationall, and is called* A *line containing in power a rationall and a mediall rectangle.*

a hyp. and ſch. 12. 10. For 2 rectangles ACB ^{a} ῥν, ^{b} ⊔ ACq + CBq ^{c} μν. ^{d} therefore 2 ACB ^{d} ⊔ ABq. wherefore ^{e} AB

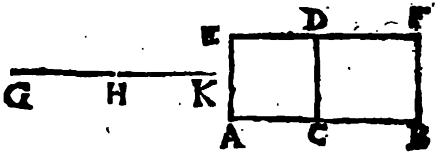
b ſch. 12. 10. is ῥ. *W.W. to be Dem.*

c hyp.

d 17. 10.

e 11. def. 10.

PROP. XLII.



If two right lines GH, HK, *incommenſurable in power be added together, having both that which is compoſed of their ſquares mediall, and the rectangle contained under them mediall, and incommenſurable to that which is compoſed of their ſquares, the whole right line* GK *is irrationall, & is called* A *line containing in power two mediall figures.*

Upon the propounded line FB ῥ make the rectangles AF = GKq, & CF = GHq + HKq. Being GHq

a hyp. + HKq (CF) ^{a} is μν, the breadth CB ^{b} ſhall be ῥ.

b 23. 10. Alſo becauſe 2 rectangles GHK (^{c} AD) ^{a} is μν, there-

c 4. 2. fore AC ^{b} ſhall be ῥ. Moreover becauſe the rectangle

d 1. 6. AD ^{a} ⊔ CF, ^{d} and AD. CF :: AC. CB. ^{e} thence

e 10. 10. ſhall ^{e} AC be ⊔ CB. ^{f} wherefore A is ^{g} ῥ. therefore

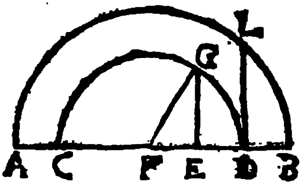
f 17. 10. the rectangle AF. i.e. GKq is ῥς; ^{h} and conſequently GK is ῥ. *W.W. to be Dem.*

g lem. 38. 10.

h 21 def. 10.

PROP.

PROP. XLIII.

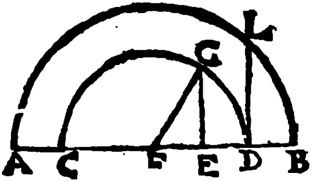


line of two names , or binomiall, AB , *can at one*
only D *be divided into its names* AD, DB.

it be poſſible, let the binomiall line AB be divi-
it the point E, into other names AE, EB. It is
feſt that the line AB is in both caſes divided
ually, ſince AD ⊐⊏ DB, and AE ⊐⊏ EB.
cauſe the rectangles ADB, AEB *a* are μα; *a* and
of ADq, DBq, AEq, EBq is ῥα. *b* and ſo ADq
Bq *b* and AEq + EBq are alſo ῥα. *b* therefore
| + DBq −: AEq + EBq. *i. e.* 2 AEB − 2
3 is ῥν. *d* therefore AEB − ADB is ῥν. therefore
ceeds μν by ῥν. *e Which is Abſurd.*

a 37. 10.
b ſch. 27. 10.
c ſch 5. 2.
d ſch. 12. 10.
e 27. 10.

PROP. XLIV.



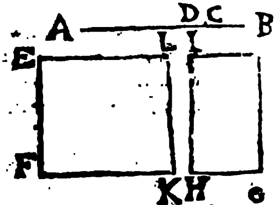
firſt bimediall line AB, *is in one point only* D *di-*
l into its names AD, DB.

onceive AB to be divided into other names
EB. whereupon every one ADq, DBq, EBq, will
. and the rectangles ADB, AEB, and the
les of them, ῥα. *b* therefore 2 AEB − 2 ADB.
ADq + DBq −: AEq + EBq is ῥν. W. is Abſ.

a 38. 10.
b ſch. 27. 10.
c ſch. 5. 2.
d 27. 10.

P PROP.

PROP. XLV.



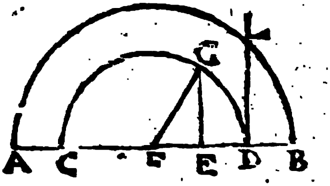
A ſecond bimediall line AB*, is divided into its names* AC,CB, *onely at one point* C.

Suppoſe there were other names AD, DB. Upon the propounded line EF [a] make the rectangles EG = ABq, and EH = ACq + CBq; as alſo EK = ADq + DBq.

[a] 39. 10.
[b] 16 *and* 24. 10.
[c] 23 10.
[d] 24. 10.
[e] 4. 2.
[f] lem. 26. 10.
[g] 1. 6.
[h] 10. 10.
[k] 37. 10.

Becauſe ACq, BCq [a] are μα ⊔; [b] ACq + CBq (EH) ſhall be μα. [c] therefore the breadth FH is ρ'. [a] moreover the rectangle ACB, [d] and ſo 2 ACB ([e] IG) is μα, [c] therefore HG is alſo ρ'. And ſince EH is [f] ⊔ IG, [g] & EH.IG :: FH.HG, [h] therefore FH, HG ſhall be ⊔. [k] therefore FG is a binomial, whoſe names are FH,HG. By the ſame reaſon FG is binomiall, and the names of it FK,KG: *contrary to the* 43. *of this Book.*

PROP. XLVI.

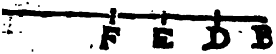


A Major line AB *is at one point only* D *divided into its names* AD,DB.

[a] 40 10.
[b] sch 27. 10.
[c] sch. 5. 2.
[d] 27. 10.

Imagine other names AE,EB. whereupon the rectangles ADB,AEB, are [a] μα. [a] and as well ADq + DBq, as AEq + EBq are ρ'α. [b] therefore ADq + DBq − : AEq + EBq, [c] i.e. 2 AEB − 2 ADB is ρ'. [d] *Which is impoſſible.*

PROP. XLVII.

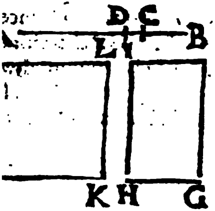


A line AB *containing in power a rationall and a medi-figure is divided at one point only* D *into its names* DB.

onceive other names AE , EB. then both AEq + Bq, and ADq + DBq are μα. a and the rectan-EB, ADB are ρα. b therefore 2 AEB − 2 ADB, ADq + DBq − : AEq + EBq is ρν. *Which is* rd.

a 41. 10.
b ſch.27. 10.
c ſch.5. 2.
d 27. 10.

PROP. XLVIII.



A line AB *containing in power two mediall rectangles, is at one point only* C *divided into its names* AC, CB.

If you would divide AB into other names AD , DB. draw upon ne propounded EF ρ the rectangles EG = and EH = ACq + CBq, and EK = ADq + then becauſe ACq + CBq, namely EH, a is e breadth FH ſhall be ρ. Alſo becauſe 2 ACB, is, IG, is a μν, HG b ſhall be likewiſe ρ. There-whereas EH c ⊓ IG. and EH. IG d :: FH. hence FH e ſhall be ⊓ HG. f therefore FG is omiall, and the names of it FH, HG. In like er FK, KG ſhall be the names of it , *againſt the his Book.*

a 42. 10.
b 23. 10.
c 4. 2.
d 1. 6.
e 10. 10.
f 37. 10.

Second Definitions.

ationall line being propounded, and the binoall divided into its names , the greateſt of ſe names is more in power then the leſſe by a ure of a right line commenſurable to the ter in length; then

I. If the greater name be commenſurable in length to the rationall line propounded, the whole line is called a firſt binomiall line.

II. But if the leſſer name be commenſurable in length to the rationall line propounded, the whole line is called a ſecond binomiall.

III. If neither of the names be commenſurable in length to the rationall line propounded, it is called a third binomiall.

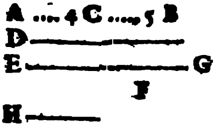
Furthermore, if the greater name be more in power then the leſſe by the ſquare of a right line incommenſurable to the greater in length then

IV. If the greater name be commenſurable to the propounded rationall line in length, it is called a fourth binomiall.

V. If the leſſer name be ſo, a fift.

VI. If neither, a ſixt.

PROP. XLIX.



To find out a firſt binomiall line, EG.

a ſch. 29. 10.
b 2. lem. 10. 10.
c 3. lem. 10. 10.

[a] Take AB, AC, ſquare numbers, whoſe exceſſe CB is not Q. let D be propounded ῥ. [b] Take EF ⊤⊥ D, and [c] make AB. CB :: EFq. FGq. then EG ſhall be a 1 bin.

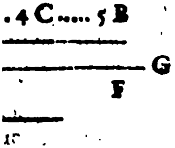
d conſtr.
e 6. def. 10.
f 6. 10.
g ſch. 12. 10.
h 9. 10.

For EF [d] ⊤⊥ D, [e] therefore EF is ῥ. [f] alſo EFq ⊤⊥ FGq. [g] therefore FG is alſo ῥ. likewiſe [d] becauſe EFq. FGq :: AB. CB :: Q. not Q. [h] therefore EF ⊤⊥ FG. Laſtly becauſe by converſion of proportion, EFq. EFq − FGq :: AB. AC :: Q. Q. thence EF [k] ſhall be ⊤⊥ √ EFq − FGq. [l] therefore EG is a 1 binomiall. *W. W. to be Done.*

k 9. 10.
l 1. def. 48. 10.

In numbers thus; let there be D 8. EF 6. AB 9. CB 5. wherefore becauſe 9. 5 :: 36. 20, therefore FG is √ 20. and conſequently EG is 6 + √ 20.

PROP. L.



To find out a second binomiall line, EG.

Take AB and AC ſquare numbers, the exceſſe of which is CB not Q. Let D be the line propounded ρ'. take FG ⊔⊔ D, and make CB. AB ::
q.EFq. then EG will be the line deſired.

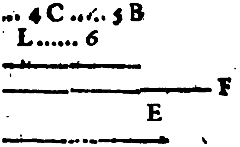
Prove is as the prec.

or FG ⊔⊔ D. wherefore FG is ρ'. Alſo EFq ⊔⊔
q. therefore EF is ρ'. Likewiſe becauſe FGq. EFq
B.AB :: not Q.Q. thence FG is ⊔⊔ EF. Laſtly
g CB. AB :: FGq. EFq, and inverſely AB. CB
Fq. FGq. therefore as in in the foregoing *prop.*
⊔⊔ √ EFq − FGq. *a* whereby EG is a 2. bino-
ll. *W. W. to be Done.*

a 2.*def*.48. 10.

n numbers; let there be D 8, FG 10, AB 9, CB
en EF is √ 180. wherefore EG is 10 + √

PROP. LI.



To find out a third binomiall line, DF.

a Take AB, AC, ſquare numbers, the exceſſe of which CB is not Q. and let L be a
nber not Q next greater then CB, *viz.* by a unite
two. Let G be the line propounded ρ'. *b* Make L.
:: Gq.DEq. *b* and AB.CB :: DEq.EFq. then DF
ll be a 3.bin.

a sch.29. 10.

b 3.*lem*. 10. 10.

For becauſe DEq *c* ⊔⊔ Gq, *d* DE is ρ'. alſo Gq.
q :: L. AB :: not Q.Q. *e* therefore G ⊔⊔ DE.
kewiſe being that DEq *c* ⊔⊔ EFq, *d* alſo EF is ρ'.
oreover becauſe DEq. EFq :: AB. CB :: Q not Q.

c conſtr. & 10.

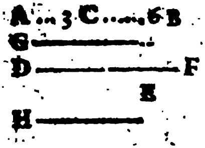
d ſch.12. 10.

e 6. 10.

f 9. 10. Q.[f] is DE ⊤⊥ EF. and being that by constr. and of
g sch. 27. 8. equality Gq.EFq :: L.CB :: not Q.Q. ([g] for L and
h 9. 10. CB are not like plane numbers.) [h] therefore shall G
be also ⊤⊥ EF. Lastly, as in the prec. prop. $\sqrt{DEq}$
k 3. def. 48. 10. $- EFq$ ⊤⊥ DE. [k] therefore DF is a 3. bin. which was to be Done.

In numbers; let there be AB, 9. CB, 5. L, 6. G, 8. then shall be DE $\sqrt{96}$, & EF $\sqrt{\frac{480}{9}}$. wherefore DF $= \sqrt{96} + \sqrt{\frac{480}{9}}$.

PROP. LII.



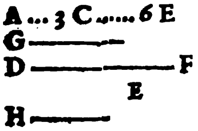
To find out a fourth binomiall line DF.

a sch. 29. 10 [a] Take any square number AB, and divide it into AC, CB not squares. Let G be the line propounded; [b] take DE ⊤⊥ G. [c] and make AB.
b 2 lem. 10. 10. CB :: DEq.EFq. then DF shall be a 4. bin.

c 3. lem. 10. 10. For, as in the 49 of this Book, DF may be shewn to be a binom. and also because by constr. and conversion of proportion DEq.DEq $-$ EFq :: AB.AC
d 9. 10. :: Q not Q. [d] shall DE be ⊤⊥ $\sqrt{DEq - EFq}$.
e 4. def. 48. 10. [e] therefore DF is a 4.bin.

In numbers, let G be 8, DE, 6. then EF shall be $\sqrt{24}$. therefore DF is $6 + \sqrt{24}$.

PROP. LIII.



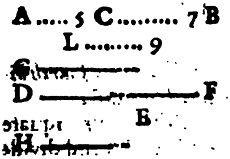
To find out a fift binomiall line DF.

Take any square number AB, whose segments AC, CB are not Q. Let G be the line propounded; take EF ⊤⊥ G. and make CB.AB :: EFq.DEq. then shall DF be a 5. bin.

For DF ſhall be a bin. as in the 50. of this book. and becauſe by conſtruction, and inverſion, DEq. EFq :: AB. CB and ſo by converſion of proportion, DEq. DEq − EFq :: AB. AC :: Q. not Q. a therefore ſhall DE be ⊥⊥ $\surd$ DEq − EFq. b therefore DF is 5 bin. *W. W. to be Done.* a 9. 10. b 5. def. 48. 10.

In numbers, let there be G, 7. EF, 6. then DE ſhall be $\surd 54$ wherefore DF is $6 + \surd 54$.

PROP. LIV.



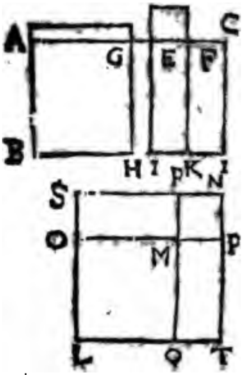
To find out a ſixt binomiall line.

Take AC, CB, prime numbers, ſo that AC + CB (AB) be not Q. take alſo any number ſquare L. Let G be the line propounded ρ. a and make L. AB :: Gq. DEq. and AB. CB :: DEq. EFq. then DF ſhall be a 6. binomiall. a 3. lem. 10. 10.

For DF may be demonſtrated bin. as in the 51. of this Book. and alſo by reaſon that DE and EF ⊥⊥ G. laſtly likewiſe becauſe by conſtr. and converſion of proportion DEq. DEq − EFq :: AB. AC :: not Q. Q. (For AB is prime to AC, b and ſo unlike to it) c therefore DE ⊥⊥ $\surd$ DEq − EFq. d therefore DF is a 6. bin. *Which was required.* b sch. 27. 8. c 9. 10 d 6. def. 48. 10.

In numbers, let there be G 6. DE $\surd 48$. then EF ſhall be $\surd 28$. wherefore DF is $\surd 48 + \surd 28$.

Lemma.



Let AD *be a rectangle, and the ſide thereof* BC *divided unequally in* E; *alſo let the leſſer portion* EC *be equally divided in* F. *upon the line* AE [a] *make the rectangle* AGE = EFq. *and from the points* G, E, F [b] *draw* GH, EI, FK, *parallel to* AB. [c] *Let the ſquare* LM *be made, equall to the rectangle* AH, *and upon* OMP *produced the ſquare* MN = GI *& let the right lines* LOS, LQT, NRS, NPT *be produced.*

a 28. 6.
b 31. 1.
c 14. 2.

I ſay 1. MS, MT, *are rectangles.* For by reaſon of the right angles of the ſquares OMQ, RMP, [a] ſhall QMR be a right line. [b] therefore RMO, QEP, are right angles. wherefore the parallelograms MS, MT are rectangles.

a ſch.15.1.
b 13. 1.

2. *Hence it is plain that* LS [c] = LT, *and conſequently that* LN *is a ſquare.*

c 2.ax.31.

3. *The rectangles* SM, MT, EK, FD *are equall.* For becauſe the rectangle AGE [d] = EFq. [e] thence ſhall AE.EF :: EF.GE, [f] and ſo AH.EK :: EK.GI. that is by conſtr. LM.EK :: EK. MN. [g] but LM. SM :: SM. MN, therefore EK [h] = SM [k] = FD [l] = MT.

4. *Hence* LN [m] = AD.

5. *Being that* EC *is equally divided in* F, *it is plain that* EF, FC, EC *are* ⊤⊥.

6. If AE ⊤̶⊥ EC, and AE ⊤⊥ √ AEq − ECq, [o] then ſhall AG, GE, AE, be ⊤⊥. alſo, becauſe AG. GE :: AH. GI. [p] therefore ſhall AH, GI, *i. e.* LM, MN, be ⊤⊥. Likewiſe thereupon

d hyp.
e 17. 6.
f 1. 6.
g ſch.22.6.
h 9. 5.
k 36. 1.
l 43. 1.
m 2.ax.1.
n 16. 10.
o 18. and 16. 10.
p 10. 10.

7. OM ⊔ MP. For by the Hyp. AE ⊔ EC. *q* therefore EC ⊔ GE. *q* wherefore EF ⊔ GE. but EF. GE :: EK. GI. *r* therefore EK ⊔ GI, that is, SM ⊔ MN. but SM. MN :: OM. MP. *r* therefore OM ⊔ MP. — *q* 14. 10. *r* 10. 10.

8. If AE be supposed ⊔ $\surd$ AEq − ECq *s* it is apparent that AG, GE, AE, are ⊔. whence LM ⊔ MN. for AG. GE :: AH. GI :: LM. MN. — *s* 18. and 15. 10.

These being well considered, we shall easily dispatch the six following Propositions.

PROP. LV.

If a space AD *be conteined under a rationall line* AB, *and a first binomiall line* AC (AE + EC) *the right line* OP *which conteineth that space in power is irrationall, and called a binomiall line.*

All that being supposed which is described and demonstrated in the next foregoing *Lemma*, it is manifest that the right line OP containeth in power the space AD. *a* Likewise AG, GE, AE, are ⊔. therefore seeing AE *b* is ῥ ⊔ AB. *c* shall also AG and GE be ῥ ⊔ AB. *d* therefore the rectangles AH, GI, that is, the squares LM, MN are ῥa. therefore OM, MP are ῥ *e* ⊔. *f* and consequently OP is a binomiall. *W. W. to be Dem.* — *a* hyp. and lem. 54. 10. *b* hyp. *c* sch. 12. 10. *d* 20. 10. *e* lem. 54. 10. *f* 37. 10.

In numbers, let there be AB 5. AC 4 + $\surd$ 12. wherefore the rectangle AD = 20 + $\surd$ 300 = to the square LN. therefore OP is $\surd$ 15 + $\surd$ 5. namely a 6 binomial.

PROP. LVI.

If a space AD *be comprehended under a rationall line* AB, *and a second binomiall* AC (AE + EC) *the right line* OP, *which containeth that space* AD *in power, is irrationall, and called a first mediall line.*

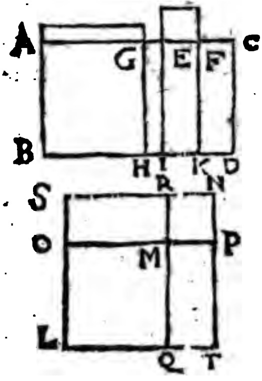
The foreſaid *Lemma* of the 54. of this Book being again ſuppoſed, then ſhall OP be = √ AD. a alſo AE, G, GE are ⊐⊏. therefore ſince AE b is ρ́ ⊐⊏ AB, likewiſe AG, GE c ſhall be ρ́ ⊐⊏ AB. therefore the rectangles AH, GI, *i.e.* OMq, MPq. d are μα. e Moreover OM ⊐̸⊏ MP. Laſtly, EF ⊐⊏ EC, and EC f ⊐⊏ AB. g wherefore EK, *i.e.* SM, or OMP, is ρν. h Conſequently OP is a firſt bimediall. *Which was to be Dem.*

a hyp. and lem. 54. 10.
b hyp.
c ſch. 12. 10.
e lem. 54. 10.
f hyp. 12. 10.
g 20. 10.
h 38. 10.

In numbers, let there AB, 5; and AC, √ 48 + 6. then the rectangle AD = √: 1200 + 30 = OPq: therefore OP is u√ 675 + u√ 75. *viz.* a firſt bimedial.

See Scheme 57.

PROP. LVII.



If a ſpace AD *be contained under a rationall line* AB *and a third binomiall line* AC (AE + EC) *the right line* OP *which containeth in power the ſpace* AD, *is irrationall, and called a ſecond bimediall line.*

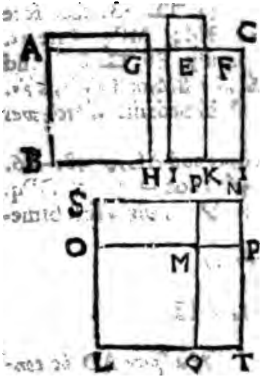
As above, OPq = AD. alſo the rectangles AH, GI, that is OMP, MPq are μα. a Likewiſe EK or OMP is μν. b therefore OP is a ſecond bimediall.

a hyp. and 22. 10.
b 39. 10.

In numbers; let there be AB 5. AC √ 32 + √ 24. wherefore AD is √ 800 + √ 600 = OPq. and ſo OP is v√ 450 + v√ 50. that is a 2. bimed.

PROP.

PROP. LVIII.



If a ſpace AD be comprehended under a rationall line B and a fourth binomiall AC (AE + EC) *the right line OP containing the ſpace AD in power, is that irrationall line which is called a Major line.*

For again, OMq [a] ⊓ MPq; & the rectangle AI, i.e. OMq + MPq [b] is ρ'ν. [c] alſo EK or OMP is μν. [d] therefore OP (√AD) is a *Major* line. *W.W. to be Dem.*

a lem. 54. 10.
b hyp. and 20. 10.
c hyp. and 22. 10.
d 40. 10.

In numbers; let there be AB 5. and AC 4 + √ 8. then the rectangle AD is 20 + √ 200. wherefore OP is √: 20 + √ 200.

PROP. LIX.

If a ſpace AD *be contained under a rationall line* AB, *and a fift binomiall* AC, *the right line* OP *which containeth the ſpace* AD *in power, is that irrationall line, which is a line containing a rationall and a mediall rectangle in power.*

Again OMP ⊓ MPq. and the rectangle AI or OMq + MPq is μν. [a] Likewiſe the rectangle EK or OMP is ρ'ν. [b] therefore OP (√ AD) contains in power ρ'ν and μν. *W.W. to be Dem.*

a as in the prec.
b 41. 10.

In numbers, let there be AB 5. and AC 2 + √ 8. then the rectangle AD = 10 + √ 200 = OPq. Wherefore OP is √: 10 + 200.

PROP. LX.

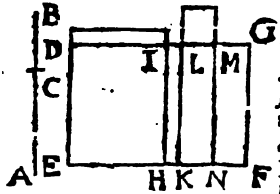
If a ſpace AD *be contained under a rationall line* AB *and a ſixt binomiall* AC (AE + EC) *the line* OP *containing the ſpace* AD *in power is irrationall, which containeth in power two mediall rectangles.*

As often before, OMq ⊤⊥ MPq, and OMq + MPq is μν. and alſo the rectangle (EK) OMP is μν. a therefore OP = √ AD contains in power 2 μν. *W.W. to be Dem.*

a 42. 10.

In numbers, let there be AB 5. AC √ 12 + √8: therefore the rectangle AD or OPq is √ 300 + √ 200. and ſo OP is √: √ 300 + √ 200.

Lemma.



Let a right line AB *be unequally divided in* C, *and let* AC *be the greater portion, and upon ſome line* DE *apply the rectangles* DF = ABq, *and* DH = ACq, *and* IK = CBq. *and let* LG *be divided equally in* M, *and alſo* MN *drawn parallel to* GF.

I ſay 1. The rectangle ACB is = LN or MF. a For 2 ACB = LF.

a 4.2. and 3. ax.1.

2. DL ⊏ LG. for DK (ACq + CBq) b ⊏ LF (2 ACB) therefore being DK, LF are of equall altitude, c DL ſhall be ⊏ LG.

b 7.2. c 1.6.

3. If AC ⊤⅃ CB, d then ſhall the rectangle DK be ⊤⊥ ACq and CBq.

d 16.10.

4. *Alſo* DL ⊤⊥ LG. for ACq + CBq e ⊤⊥ 2 ACB, *i. e.* DK ⊤⊥ LF. but DK. LF e :: DL. LG. f therefore DL ⊤⊥ LG.

e lem.26.10. f 10.10.

5. *Moreover* DL ⊤⊥ √ DLq − LGq. For ACq. ACB g :: ACB. CBq. that is, DH. LN :: LN. IK. e wherefore DI. LM :: LM. IL. h therefore DI x IL = LMq. therefore ſeeing ACq k ⊤⊥ CBq, that is DH ⊤⊥ IK. and l ſo DI ⊤⊥ IL. m ſhall DL be ⊤⊥ √ DLq − LGq. *W.W.* to be Dem.

g 1.6. h 17.6. k hyp. l 10.10. m 18.10.

6. But if ACq *be put* ⊔ CBq, [a] *then ſhall* DL *be* ⊔ √ DLq − LGq. a 19. 10.

This Lemma is preparatory to the 6 following Propoſitions.

PROP. LXI.

The ſquare of a binomiall line (AC + CB) *applyed unto a rationall line* DE, *makes the latitude* DG *a firſt binomiall line.*

Thoſe things being ſuppoſed, which are deſcribed a hyp.
and demonſtrated in the next preceding *Lemma*; be- b lem. 60. 10
cauſe AC, CB [a] are ρ́ ⊔, [b] the rectangle DK ſhall c ſch. 12. 10.
be ⊔ ACq. [c] and ſo DK is ρ́ν. [d] therefore DL ⊔ d 21. 10.
DE ρ́. but the rectangle ACB, and ſo 2 ACB (LF) e 22. & 24. 10.
[e] is μν. [f] therefore the latitude LG is ρ́ ⊔ DE. f 23. 10.
[g] therefore alſo DL ⊔ LG alſo DL ⊔ √ DLq g 13. 10.
− LGq. from whence [k] it follows that DG is a firſt h lem. 60. 10.
binomiall. *W.W. to be Dem.* k 1 def. 48. 10.

PROP. LXII.

The ſquare of a firſt bimediall line (AC + CB) *being applied to a rationall line* DE, *makes the latitude* DG *a ſecond binomiall line.*

The aforeſaid *Lemma* being again ſuppoſed; The
rectangle DK ⊔ ACq. [a] therefore DK is μν. [b] there- a 24. 10.
fore the latitude DL is ρ́ ⊔ DE. But becauſe the b 23. 10.
rectangle ACB, and ſo LF (2 ACB) [c] is ρ́ν, [d] ſhall c hyp. and ſch. 12. 10.
LG be ρ́ ⊔ DE. [e] therefore DL, LG are ⊔. [f] alſo d 23. 10.
DL ⊔ √ DLq − LGq. [g] from whence it is clear e 13. 10. f lem. 60. 10.
that DG is a ſecond binomiall. *W.W. to be Dem.* g 2 def. 48. 10.

PROP. LXIII.

The ſquare of a ſecond bimediall line (AC+CB) *applied to a rationall line* DE *makes the breadth* DG *a third binomiall line.*

a hyp. and As in the *prec.* DL is ρ' ⌐⊔ DE. Furthermore be-
24.10. b 23.10. cauſe the rectangle ACB, and ſo LF (2 ACB) a is
c lem.60.10 μν. b therefore ſhall LG be ρ' ⌐⊔ DE. c Moreover
d 3 def.48. DL ⌐⊔ LG. and alſo DL ⌐⊔ √DLq − LGq.
10. d therefore DG is a third binomiall. *W.W. to be Dem.*

PROP. LXIV.

The ſquare of a Major line (AC+CB) *applied to a rationall line* DE, *makes the breadth* DG *a fourth binomiall line.*

a hyp. and Again ACq+CBq *i.e.* DK a is ρν. b therefore DL
ſch 12.10. is ρ' ⌐⊔ DB. alſo ACB, and ſo LF (2 ACB) c is μν.
b 21.10. d therefore LG is ρ' ⌐⊔ DE. e and conſequently DL
c hyp and 24.10. ⌐⊔ LG. Laſtly becauſe AC ⊤⊔ BC. f ſhall DL be
d 13.10. e 13.10. ⌐⊔ DLq − LGq. g whence DG is a fourth bino-
f lem.60.10. miall. *W.W. to be Dem.*
g 4.def.48. 10.

PROP. LXV.

The ſquare of a line containing in power a rationall and a mediall rectangle (AC+CB) *applied to a rationall line* DE *makes the latitude* DG *a fift binomiall.*

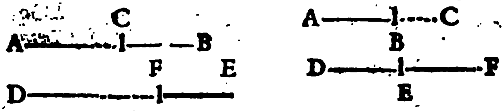
a 23.10. Again, DK is μν. a therefore DL is ρ' ⌐⊔ DE. alſo
b 21.10. LF is ρν. b therefore LG is ρ' ⌐⊔ DE. c therefore DL
c 13.10. ⌐⊔ LG. d likewiſe DL ⌐⊔ √DLq − LGq. e and
d lem.60. 10. ſo by conſequence DG is a fift binomiall. *Which was*
e 5.def.48. 10. *to be Dem.*

PROP. LXVI.

The ſquare of a line containing in power two mediall rectangles (AC+CB) *applied to a rationall line* DE, *makes the latitude* DG *a ſixt binomiall line.*

As before, DL and LG and LG are, ⊓⊔ DE. *a hyp.*
But for that ACq + CBq (DK) *a* ⊓⊔ ACB, *b* 14 10.
b and so DK ⊓⊔ LF (2 ACB) and alſo DK. LF *c* :: *c* 1. 6.
DL. LG. *d* therefore ſhall DL be ⊓⊔ LG. *e* Laſtly *d* 10. 10.
DL ⊓⊔ √DLq − LGq. *f* by which it appears that *e lem.* 60. 10.
DG is a ſixt binomiall. *f* 6 *def.* 48. 10.

Lemma.



Let AB, DE *be* ⊓⊔ *and make* AB. DE :: AC. DF.
I ſay 1. AC ⊓⊔ DF. as appears by 10. 10. alſo *a* 19. 5.
CB ⊓⊔ FE, *a* becauſe AB. DE :: CB. FE.

2. AC. CB :: DF. FE. For AC. DF :: AB. DE :: CB. FE. therefore inverſely AC. CB :: DF. FE.

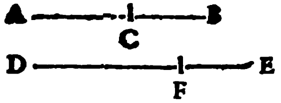
3. *The Rectangle* ACB ⊓⊔ DFE. For ACq. ACB
b :: AC. CB *c* :: DF. EF :: DFq. DFE. wherefore by *b* 1. 6.
inverſion ACq. DFq :: ACB. DFE. therefore being *c before.*
ACq ⊓⊔ DFq. *d* ſhall ACB be ⊓⊔ DFE. *d* 10. 10.

4. ACq + CBq ⊓⊔ DFq + FEq. For becauſe
ACq. CBq *e* :: DFq. FEq. therefore by addition *e* 22. 6.
ACq + CBq. CBq :: DFq + FEq. FEq. therefore
being CBq ⊓⊔ FEq, *f* ſhall alſo ACq + CBq be *f* 10. 10.
⊓⊔ DFq + FEq.

5. *Hence*, If AC ⊓⊐ or ⊓⊐ CB, *g* then likewiſe *g* 10. 10.
ſhall DE be ⊓⊐ or ⊓⊐ EF.

PROP.

PROP. LXVII.



A line DE, *commensurable in length to a binomiall line* (AC + CB) *is it ſelf a binomiall line, and of the ſame order.*

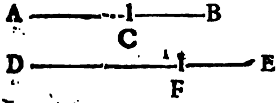
Make AB. DE :: AC.DF. *a* then are AC, DF ⫫
a lem.66.10. *a* and CB, FE ⫫. whence being that AC and CB
b hyp. *b* are ρ' ⫪, *c* thence DF, FE ρ' ⫪. therefore DE is
c lem.66.10. and ſch.12.10. a binomiall. But for that AC. CB *d* :: DF. FE. if AC
⫫ or ⫫ √ ACq − BCq, *d* then in like manner
d 19.10. DF ⫫ or ⫫ √ DFq − FEq. alſo if AC ⫫ or
e 12.10. and 14.10. ⫫ ρ' propounded, *e* then ſhall DF be ⫫ or ⫫ ρ'
propounded. But if CB ⫫ or ⫫ ρ'. likewiſe FE
f 14.10. ⫫ or ⫫ ρ'. If both AC, CB, ⫫ ρ'. *f* then alſo
g by def. 48. 10. both DF, FE, ⫫ ρ'. *g* That is, whatſoever binomiall AB is, DE ſhall be of the ſame order. *W.W. to be Dem.*

PROP. LXVIII.

A line DE *commenſurable in length to a bimediall line* (AC + CB) *is alſo a bimediall line, and of the ſame order.*

a 12.6. Make AB. DE :: AC. DF. *b* therefore AC ⫫
b lem.66.10. DF. and CB ⫫ FE. therefore ſeeing AC and CB
c hyp. *c* are μ, *d* alſo DF and FE ſhall be μ. and for that AC
d 24.10. *e* ⫪ CB, *e* therefore FD ⫪ FE. *f* therefore DE is
e 10.10. 2 μ. Wherefore if the rectangle ACB be ρν. becauſe
f 38.10. DFE *b* ⫫ ACB, *g* likewiſe DFE is ρν. and if that
g ſch.12.10. be μν, *h* this ſhall be μν too. *k* That is, whether AB be
h 24.10. 1 bimed. or 2 bimed. DF ſhall be of the ſame order.
k 38. or 39. 10. *W.W. to be Dem.*

PROP. LXIX.



A line DE *commenſurable to a Major line* (AC+CB) *is it ſelf a Major line.* a hyp.

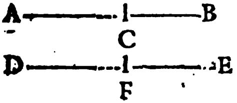
Make AB.DE :: AC.DF. Becauſe AC *a* ⊤ CB, b lem.66.10
b thence DF ⊤ FE. alſo ACq + CBq *a* is ῥν. and
ſo being DFq + FEq *b* ⊓⊔ ACq + CBq, *c* alſo c ſch.12. 10.
DFq + FEq is ῥν. laſtly, the rectangle ACB *a* is μν. d 24.10.
d therefore the rectangle DFE is μν. (becauſe DFE e 40.10.
is *b* ⊓⊔ ACB) *e* wherefore DE is *a Major* line. *W.W. to be Dem.*

PROP. LXX.

A line DE *commenſurable to a line containing in power a rationall and a mediall rectangle* (AC+CB) *is a line containing in power a rationall and a mediall rectangle.*

Again make AB. DE :: AC. DF. Becauſe AC
a ⊤ CB, *b* alſo DF ⊤ FE. likewiſe becauſe ACq a hyp.
+ CBq *a* is μν, *c* therefore DFq + FEq ſhall be μν. b lem.66. 10.
laſtly, becauſe the rectangle ACB *a* is ῥν, *d* alſo DFE c 24.10.
is ῥν. Therefore DE contains in power ῥν and μν. d ſch.12. 10.
W.W. to be Dem. e 41. 10.

PROP. LXXI.

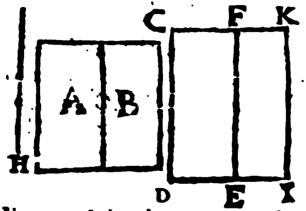


A line DE *commenſurable to a line containing two mediall rectangles in power* (AC + CB) *is alſo a line containing in power two mediall rectangles.*

Divide DE, as in the prec. Becauſe ACq *a* ⊓⊔ a hyp.
CBq, *b* thence ſhall DFq be ⊓⊔ FEq. alſo for that b lem.66.10.
ACq + CBq *a* is μν, *c* ſhall DFq + FEq be alſo μν. c 24. 10.
And in like manner becauſe ACB *a* is μν, *d* alſo DFE d 24. 10.
is μν. *Laſtly,* becauſe ACq + CBq ⊓⊔ ACB,

e 14. 10. ^e^ ſhall DFq + FEq be ⊤⊔ DFE. *f* From whence
f 42. 10. it follows that DE contains in power 2 μα. *Which was to be Dem.*

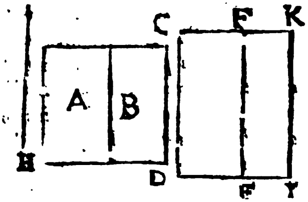
PROP. LXXII.



If a rationall rectangle A and a mediall B, be composed together, these four irrationall lines will be made; either a binomiall, or a first bimediall, or a major, or a line containing in power a rationall and a mediall rectangle.

Namely, if Hq = A + B, then H ſhall be one of
a cor. 16. 6. the four lines which the Theoreme mentions. For
upon CD the propounded ρ', *a* make the rectangle
b 2. ax. 1. CE = A, and FI = B. *b* and ſo CI = Hq. Whereas
c 21. 10. then is A ρ'ν, likewiſe CE is ρ'ν. *c* therefore the latitude
CF is ρ' ⊤⊔ CD. and becauſe B is μν, alſo FI ſhall
d 23. 10. be μν. *d* therefore FK is ρ' ⊤⊔ CD. *e* therefore CF,
e 13. 10. FK are ρ' ⊤⊔. and ſo the whole CK *f* is binom.
f 37. 10. wherefore if A ⊏ B, *i.e.* CE ⊏ FI, *g* then CF ⊏
g 1. 6. FK. therefore if CF ⊤⊔ √ CFq − FKq, *h* likewiſe
h 1. def. 48. 10 CK ſhall be a 1. bin. and conſequently H = √ CI
k 55. 10. *k* is a bin. If CF be ſuppoſed ⊤⊔ √ CFq − FKq,
l 4 def. 48. 10. *l* then ſhall CK be a 4. bin. wherefore H (√ CI) *m* is
m 58. 10. a major line. But if A ⊐ B, *g* then ſhall CF be ⊐
n 2. def. 48. 10. FK. conſequently if FK ⊤⊔ √ FKq − CFq, *n* then
o 56 10. ſhall CK be a 2. bin. *o* wherefore H is a firſt 2 μ. laſtly
p 5. def. 48. 10. if FK ⊤⊔ √ FKq − CFq, *p* then CK ſhall be a fift
q 59. 10. binom. *q* whence H ſhall contein in power ρ'ν and μν.
W. W. to be Dem.

PROP. LXXIII.



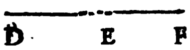
If two mediall rectangles A,B, *incommensurable to one another be composed together, the two remaining irrationall lines are made, either a second bimediall, or a line containing in power two mediall rectangles.*

As H containing in power A + B is one of the ſaid irrationall lines. For upon CD propounded ρ' draw the rectangle CE = A, and FI = B. whence Hq = CI. Therefore becauſe CE and FI [a] are μα. [b] the latitudes CF,FK, ſhall be ρ' ˥⊔ CD. alſo becauſe CE [a] ˥⊔ FI, and CE.FI [c] :: CF.FK, [d] therefore CF ˥⊔ FK. [e] therefore CK is a 3.bin. namely, if CF ˥⊔ √ CFq − FKq. whence H = √ CI [f] ſhall be a ſecond 2 μ. But if CF ˥⊔ √ CFq − FKq, [g] then CK ſhall be a 6 binom. [h] and conſequently H conteins in power 2 μα. *W. W. to be Dem.*

a hyp.
b 23.10.
c 1.6.
d 10.10.
e 3.def.48.10.
f 57.10.
g 6.def 48.10.
h 60.10.

Here begin the Senaries of lines irrationall by Subtraction.

PROP. LXXIV.



If from a rationall line DF *a rationall line* DE, *commenſurable in power only to the whole* DF, *be taken away, the reſidue* EF *is irrationall, and is called an Apotome or reſiduall line.*

For EFq [a] ˥⊔ DEq; [b] but DEq is ρ'; [c] therefore EF is ρ'. *W.W. to be Dem.*

In numbers; let there be DF, 2. DE, √ 3. then EF *ſhall be* 2 − √ 3.

a lem.26.10.
b hyp.
c 10. & 11. def.10.

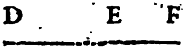
PROP. LXXV.

If from a mediall line DF, *a mediall line* DE *commenſurable only in power to the whole* DF, *and comprehending with the whole* DF *a rationall rectangle, be taken away, the remainder* EF *is irrationall, and is called a firſt reſiduall line of a mediall.*

a ſch. 16 10. For EFq *a* ⊐⊏ to the rectangle FDE. therefore
b hyp. ſeeing FDE *b* is *ρ'ν*, *c* EF ſhall be *ρ'*. *W.W. to be Dem.*
c 20. and 11. def. 10.

In numbers; let DF be *v*√ 54, and DE *v*√ 24. therefore EF is *v*√ 54 − *v*√ 24.

PROP. LXXVI.



If from a mediall line DF, *a mediall line* DE *be taken away being commenſurable only in power to the whole* DF, *and comprehending together with the whole line* DF *a mediall rectangle, the remainder* EF *is irrationall, and is called a ſecond reſiduall of a mediall line.*

a hyp. Becauſe DFq and DEq *a* are *μα* ⊐⊏, *b* therefore
b 16. 10. ſhall DFq + DEq be ⊐⊏ DEq. *c* wherefore DFq +
c 24. 10. DEq is *μν*. alſo the rectangle FDE; *c* and ſo 2 FDE,
d cor. 7. 2. *a* is *μν*. therefore EFq (*d* DFq + DEq − 2 FDE)
e 27. 10. *e* is *ρ'ν*. wherefore EF is *ρ'*. *W.W. to be Dem.*

In numbers, let DF be *v*√ 18. and DE *v*√ 8. then EF *v*√ 18 − *v*√ 8.

PROP. LXXVII.

If from a right line AC *be taken away a right line* AB *being incommenſurable in power to the whole* BC, *and making with the whole* AC *that which is compoſed of their ſquares rationall, and the rectangle contained under them mediall, the remainder* BC *is irrationall, and is called a Minor line.*

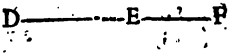
a hyp. For ACq + ABq *a* is *ρ'ν*. but the rectangle ACB
b ſch. 12. 10. *a* is *μν*. *b* therefore 2 CAB ⊐⊏ ACq + ABq (2 *c* CAB
c 7. 2.

$+ BCq.)$ [d] therefore $ACq + ABq \sqsupset\!\sqsubset BCq.$ [e] therefore BC is ρ'. *W.W.to be Dem.*

d 17. 10.
e 11. def. 10

In numbers, let AC be $\surd:18+\surd 108$; AB $\surd:18 - \surd 108$. then BC is $\surd: 18 - \surd: 108 - \surd:18 - \surd 108$.

PROP. LXXVIII.



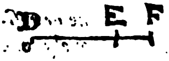
If from a right line DF *be taken away a right line* DE, *being incommensurable in power to the whole line* DF, *and with the whole* DF *making that which is composed of their squares mediall, and the rectangle contained under the same lines rationall, the line remaining* EF *is irrationall, and is called a line making a whole space mediall with a rationall space.*

For 2 FDE [a] is ρ'r. [b] and $DFq + DEq$ is μν. [c] therefore 2 FDE $\sqsupset\!\sqsubset$ $DFq + DEq$ [d] $(2\ FDE + EFq)$ [e] therefore EF is ρ'. *W.W.to be Dem.*

a hyp. & sch. 12. 10.
b hyp.
c sch. 12. 10.
d 7. 2.
e sch. 12. 10. and 11. def. 10.

In numbers, let DF be $\surd: \surd 216 + \surd 72$; DE $\surd: \surd 216 - \surd 72$. therefore EF is $\surd: \surd 216 + \surd 72 - \surd: \surd 216 - \surd 72$.

PROP. LXXIX.



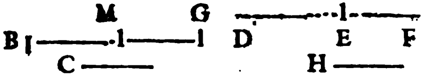
If from a right line DF *be taken away a right line* DE, *incommensurable in power to the whole* DF, *and which together with the whole makes that which is composed of their squares mediall, and the rectangle contained under them also mediall and incommensurable to that which is composed of their squares, the remainder is irrationall, and is called a line making a whole space mediall with a mediall space.*

For 2 FDE, and $FDq + DEq$ [a] are μα; [b] therefore EFq ([c] $DFq + DEq - 2\ FDE$) is ρ'. [d] and so consequently EF is ρ'. *W.W. to be Dem.*

a hyp. [illegible] 10.
b 27. 10.
c [illegible]
d 11. def. [illegible]

In numbers, let DF be $\surd: \surd 180 + \surd 60$; DE $\surd: \surd 180 - \surd 60$. then EF shall be $\surd: \surd 180 + \surd 60 - \surd: \surd 180 - \surd 60$.

Lemma.



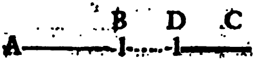
If there be the ſame exceſſe between the firſt magnitude BG *and the ſecond* C (MG) *as is between the third magnitude* DF *and the fourth* H(EF;) *then alternately, the ſame exceſſe ſhall be between the firſt magnitude* BG *and the third* DF, *as is between the ſecond* C *and the fourth* H.

a hyp. For becauſe that *a* to the equalls BM, DE, are added the equalls MG, EF, that is, C, H; the exceſſe of the wholes BG, DF, *b* ſhall be equall to the exceſſe of the parts added C, H. *W.W. to be Dem.* b 19 ax. 1.

Coroll.

Hence, Four magnitudes Arithmetically proportionall, are alternately alſo Arithmetically proportionall.

PROP. LXXX.

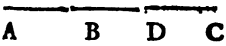


To an Apotome or reſiduall line AB *onely one rationall right line* BC, *being commenſurable in power onely to the whole* AB, *is congruent, or can be joyned.*

If it be poſſible, let ſome other line BD be added to it; *a* then the rectangles ACB, ADB, *b* and ſo conſequently the doubles of them are μμ. wherefore ſeeing ACq + BCq — 2ACB *c* = ABq *c* = ADq + DBq — 2ADB. therefore alternately ACq + BCq —: ADq + BDq *d* = 2ACB —: 2ADB. But ACq + BCq —: ADq + BDq *e* is ῥ. *f* therefore 2ACB —: 2ADB is ῥ. *g* Which is Abſurd.

a 22. 10.
b 24. 10.
c 7. 2.
d lem. 79. 10.
e hyp. and 39. 10.
f ſch. 12. 10.
g 27. 10.

PROP. LXXXI.

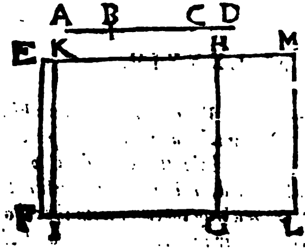


To a firſt mediall reſiduall line AB *onely one mediall right line* BC, *being commenſurable only in power to the whole, and comprehending with the whole line a rationall rectangle, can be joyned.*

Conceive BD to be ſuch a line as may be joyned to it; then becauſe ACq and BCq, as well as ADq and BDq [a] are μν ⊥. [b] alſo ACq + BCq, & ADq + BDq ſhall be μν. [c] but the rectangles ACB, ADB, [d] and ſo 2 ACB and 2 ADB are ρ̀ν. [e] therefore 2 ACB —: 2 ADB, [f] that is ACq + BCq —: ADq + BDq is ρ̀ν. [g] *Which is Abſurd.*

a *hyp.*
b 10 *and* 24. 10.
c *hyp.*
d *sch.* 12. 10.
e *sch.* 27. 10.
f 7. 2. *and lem.* 79. 10.
g 27. 10.

PROP. LXXXII.



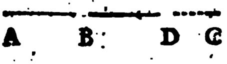
Vnto a ſecond mediall reſiduall line AB *onely one mediall right line* BC, *commenſurable onely in power to the whole, and with it conteining a mediall rectangle, can be joyned.*

If it be poſſible, let ſome other line BD be added to it; and upon EF ρ̀ make the rectangle EG = ACq + BCq, as alſo the rectangle EL = ADq + BDq. likewiſe EI = ABq. Now 2 ACB + ABq = ACq + BCq = EG. therefore ſeeing EI = ABq, [a] alſo KG ſhall be = 2 ACB. moreover ACq and BCq [b] are μν ⊥. [c] therefore EG (ACq + BCq) is μν. [d] therefore the breadth EH is ρ̀ ⊥ EF. [e] Further, the rectangle ACB [f] and ſo 2 ACB (KG) is μν. [d] therefore KH is alſo ρ̀ ⊥ EF. laſtly, becauſe ACq + BCq (EG) [g] ⊥ 2 ACB (KG) and EG. KG [h] :: EH. KH. [k] therefore

a 4. 2. *and* 3.
b *hyp.*
c 24. 10.
d 23. 10.
e *hyp.*
f 24. 10.
g *lem* 26. 10.
h 1. 6.
k 10. 10.

l 74. 10. ſore EH ⊤⊥ HK. [l] therefore EK is a reſiduall line, whereto KH is congruent. by the ſame reaſon alſo ſhall KM be congruent to the ſaid EK. *which is repugnant to the* 80. prop. *of this Book.*

PROP. LXXXIII.

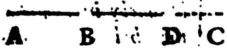


To a Minor line AB *onely one right line* BC *can be joined being incommenſurable in power to the whole, and making together with the whole line that which is compoſed of their ſquares rationall, and the rectangle which is conteined under them mediall.*

Conceive any other BD to be congruent to it; Therefore whereas ACq + BCq, and ADq + BDq [a] are ρ'α, their exceſſe (2 [b] ACB −: 2 ADB) [c] is ρ'τ. *Which is Abſurd;* becauſe ACB and ADB are μσ by the Hyp.

a hyp.
b lem. 97. 10.
c ſch. 27. 10.
d 27. 10.

PROP. LXXXIV.



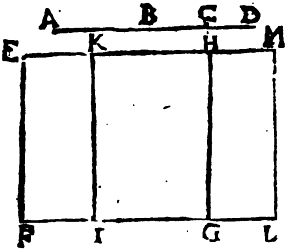
Unto a line (AB) *making with a rationall ſpace a whole ſpace mediall onely one right line* BC *can be joined, being incommenſurable in power to the whole,* ~~*& making together with the whole*~~ *that which is compoſed of their ſquares mediall, and the rectangle which is contained under them rationall.*

Suppoſe ſome other BD to be congruent alſo to it; [a] then the rectangles ACB, ADB, [b] and ſo 2 ACB and 2 ADB are ρ'α. therefore 2 ACB −: 2 ADB, [c] that is, ACq + BCq −: ADq + BDq [d] is ρ'τ. *Which is Abſurd.* ſince ACq + BCq, and ADq + BDq are μσ by the Hyp.

a hyp.
b ſch. 12. 10.
c lem. 79. 10.
d ſch. 27. 10.

PROP.

PROP. LXXXV.



To a line AB, which with a mediall ſpace makes a whole ſpace mediall, can be joined onely one right line BC, incommenſurable in power to the whole, and making with the whole both that which is compoſed of their ſquares mediall, and the rectangle which is contained under them mediall and incommenſurable to that which is compoſed of their ſquares.

Thoſe things being ſuppoſed which are done and ſhewn in the 82. *prop.* of this Book; it is clear that EH and KH are ⊐⊏ EF. Beſides, being that ACq + CBq, that is, the rectangle EG,[a] is ⊐⊏ ACB.[b] & ſo EG ⊐⊏ 2 ACB (KG;) & EG.KG[c] :: EH.KH; ſhall EH be ⊐⊏ KH. therefore EK is a reſiduall line, & the line congruent to it is KH. In like manner may KM be ſhewn to be congruent to the ſaid reſiduall EK, againſt the 80. *prop.* of this Book.

a hyp.
b 14. 10.
c 1. 6.

Third Definitions.

A Rationall line & a reſiduall being propounded, if the whole be more in power then the line joined to the reſiduall, by the ſquare of a right line commenſurable unto it in length; then

I. If the whole be commenſurable in length to the rationall line propounded, it is called a firſt reſiduall line.

II. But if the line adjoined be commenſurable in length to the rationall line propounded, it is called a ſecond reſiduall line.

III. If neither the whole nor the line adjoined be commenſurable in length to the rationall line *propounded, it is called* a third reſiduall line.

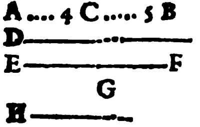
Moreover, If the whole be more in power then the line adjoined by the ſquare of a right line incommenſurable to it in length, then

IV. If the whole be commenſurable in length to the rationall line propounded, it is called a fourth reſiduall line.

V. But if the line adjoined be commenſurable in length to the rationall line propounded, it is a fift reſiduall.

VI. If neither the whole nor the line adjoined be commenſurable in length to the rationall line propounded, it is termed a ſixt reſiduall line.

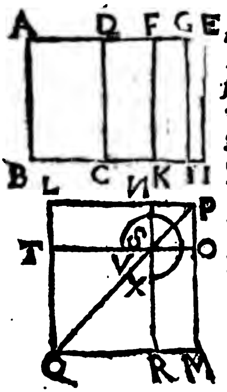
PROP. LXXXVI, 87,88,89,90,91.



To find out a firſt, ſecond, third, fourth, fift, and ſixt reſiduall line.

Reſiduall lines are found out by ſubducting the leſſe names or parts of binomialls from the greater. Ex.gr. Let $6 + \sqrt{20}$ be a firſt binom. then ſhall $6 - \sqrt{20}$ be a firſt reſiduall. So that it is not neceſſary to repeat more concerning the finding of them out.

Lemma.



Let AC *be a rectangle conteined under the right lines* AB, AD. *Let* AD *be drawn forth to* E, *and* DE *equally divided in* F. *and let the rectangle* AGE *be* = FEq. *& the rectangles* AI, DK, FH, *finiſhed. Then let the ſquare* LM = AH *be made, and the ſquare* NO = GI; *and the lines* NSR, OST, *produced.*

I ſay 1. the rectangle AI = LM + NO = TQq + SOq. which appears by the conſtr.

2. *The rectangle* DK = LO. For becauſe the rectangle AGE *a* = FEq. *b* thence are AG, FE, GE, ∺ *c* and ſo AH, FI, GI ∺, *c* that is LM, FI, NO, ∺; but LM, LO, NO *d* are ∺; therefore FI = *e* LO *f* = DK = *g* NM. — a conſtr. b 17. 6. c 1. 6. d ſch. 22. 6. e 9. 5. f 36. 1.

3. *Hence,* AC = AI − DK − FI = LM + NO − LO − NM = TR. — g 43. 1.

4. *h It is manifeſt that* DF, FE, DE, *are* ⊏⊐. — h 16. 10.

5. *If* AE ⊏⊐ DE, *and* AE ⊏⊐ √ AEq − DEq, *k then ſhall* AG, GE, AE *be* ⊏⊐. — k 18. and 10. 10.

6. *Alſo, becauſe* AE *l* ⊏⊐ DE, *m thence ſhall* AE, FE *be* ⊏⊐. *n and ſo* AI, FI; *that is,* LM + NO *and* LO *are* ⊏⊐. — l hyp. m 13. 10. n 1. 6. and 10. 10.

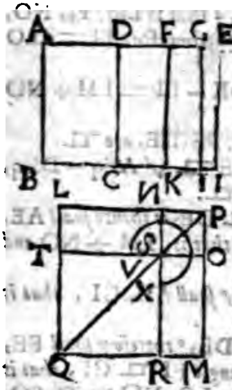
7. *Becauſe* AG * ⊏⊐ GE, *n ſhall* AH, GI, *that is* LM, NO *be* ⊏⊐. — * before.

8. *But becauſe* AE *l* ⊏⊐ DE, *o therefore ſhall* FE, GE *be* ⊏⊐, *n and ſo the rectangle* FI ⊏⊐ GI, *that is* LO ⊏⊐ NO. *wherefore ſeeing* LO. NO *p* :: TS. SO. *q therefore ſhall* TS, SO *be* ⊏⊐. — o 14. 10. p 1. 6. q 10. 10.

9. *If* AE *be put* ⊏⊐ √ AEq − DEq, *r then ſhall* AG, GE, AE *be* ⊏⊐. — r 19. 10. and 17. 10.

10. *ſ Wherefore the rectangles* AH, GI, *that is* TOq, SOq *ſhall be* ⊏⊐. — ſ 1. 6. and 10. 10.

PROP. XCII.



If a ſpace AC *be contained under a rationall line* AB, *and a firſt reſiduall line* AD (AE − DE) *the right line* TS, *which containeth the ſpace* AC *in power, is a reſiduall line.*

Uſe the foregoing *Lemma* for a preparatory to the demonſtration of this prop. Therefore TS = $\sqrt{}$ AC. Alſo AG, GE, AE, are ⊓; therefore ſince AE ⊓ *a* AB ρ', *b* alſo AG and GE ſhall be ⊓ AB. *c* therefore the rectangles AH & GI, that is, TOq and SOq are $\rho'\alpha$. *d* Likewiſe TO, SO, are ρ' ⊤. *e* and conſequently TS is a reſiduall line. *W. W. to be Dem.*

a hyp.
b 13.10.
c 20.10.
d lem.91.10.
e 74.10.

PROP. XCIII.

See the prec. Scheme.

If a ſpace AC *be contained under a rationall line* AB *and a ſecond reſiduall* AD (AE − DE) *the right line* TS, *containing the ſpace* AC *in power, is a firſt mediall reſiduall line.*

Again, by the foregoing *Lemma*, AG, GE, AE are ⊓. therefore *a* ſince AE is ρ' ⊓ AB, *b* alſo AG, GE, ſhall be ρ' ⊓ AB. *c* therefore the rectangles AH, GI, that is, TOq, SOq are $\mu\alpha$. *d* likewiſe TO ⊤ SO. Laſtly, becauſe DE *e* ⊓ AB ρ', *f* the right angle DI, and the half thereof DK or LO, that is TOS ſhall be $\rho'\nu$. *g* from whence it follows that TS ($\sqrt{}$ AC) is a firſt mediall reſiduall. *W. W. to be Dem.*

a hyp.
b 13.10.
c 22.10.
d lem 74.10.
e hyp.
f 20.10.
g 75.10.

PROP. XCIV.

See Scheme. 92.

a ſpace AC *be contained under a rationall line* AB *third reſiduall* AD (AE − DE) *the right line ntaining in power the ſpace* AC *is a ſecond mediall iall line.*

in the former, TO and SO are μ. Therefore a hyp.
ſe DE *a* is ρ´ ⊓⊥ AB. *b* the rectangle DI, *c* and b 22.10.
K, or TOS, ſhall be μν, *d* therefore $TS = \sqrt{AC}$ c 24.10
cond mediall reſiduall. *W.W. to be Dem.* d 76.10.

PROP. XCV.

See Scheme 92.

ſpace AC *be conteined under a rationall line* AB *fourth reſiduall* AD (AE − DE) *the right line nteining the ſpace* AC *in power, is a Minor line.*

before, TO *a* ⊓⊥ SO. Therefore becauſe AE a lem. 91. 10.
⊓⊥ AB, *c* ſhall AI (TOq + SOq) be ρ´ν. but, as b hyp.
e, the rectangle TOS is μν. *d* therefore $TS = \sqrt{}$ c 20. 10.
a *Minor* line. *W.W. to be Dem.* d 77. 10.

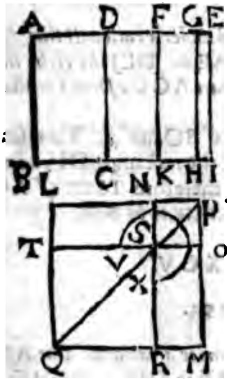
PROP. XCVI.

See Scheme 92.

ſpace AC *be contained under a rationall line* AB *fift reſiduall* AD (AE − DE) *the right line* TS *ning in power the ſpace* AC, *is a line which maith a rationall ſpace the whole ſpace mediall.*

again TO ⊓⊥ SO. therefore ſince AE *a* is a hyp.
AB. *b* alſo AI, that is TOq + SOq ſhall be b 22. 10.
t, as in the 93. the rectangle TOS is ρ´ν. *c* whence c 78. 10.
$= \sqrt{AC}$ is a line which with ρ´ν makes a
μν. *W.W. to be Dem.*

PROP. XCVII.

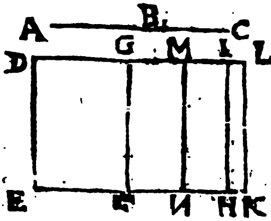


If a ſpace AC *be contained under a rationall line* AB, *and a fixt reſiduall* AD (AB — DE) *the right line* TS *containing in power the ſpace* AC, *is a line making with a mediall rectangle, a whole ſpace mediall.*

As often above, TO ⊏⊐ SO. alſo, as in 96, TOq + SOq is μν. but the rectangle TOS is ρν, as in 94. a Laſtly TOq + SOq ⊏⊐ TOS. b therefore TS = √ AC is a line which with μν makes a whole μν. *W.W. to be Dem.*

a lem. 91. 10. b 79. 10.

Lemma.



* cor. 16. 6.

Upon a right line DE * *apply the rectangles* DF = ABq, *and* DH = ACq, *and* IK = BCq. *and let* GL *be biſected in* M, *and the line* MN *drawn parallel to* GF.

Then 1. *the rectangle* DK *is* = ACq + BCq. *as the conſtruction manifeſts.*

2. *The rectangle* ACB = GN or MK. For DK a = ACq + BCq b = 2 ACB + ABq. but ABq c = DF. therefore GK c = 2 ACB. and d conſequently GN or MK = ACB.

a conſtr. b 7. 2. c 3. ax. 1. d 7. ax. 1. e 1. 6.

3. *The rectangle* DIL = MLq. For becauſe ACq. ACB e :: ACB. BCq, that is DH. MK :: MK.

IK. [e] thence is DI. ML :: ML. IL. [f] therefore DIL = MLq. f 17. 6.

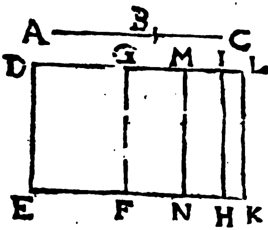
4. *If* AC *be taken* ⊤⅃ BC, *then* DK *ſhall be* ⊤⊔ ACq. For ACq + BCq (DK) [g] ⊤⊔ ACq. g 16. 10.

5. *Likewiſe* DL ⊤⊔ √ DLq − GLq. For becauſe DH (ACq) ⊤⊔ IK (BCq) [h] thence ſhall DI be ⊤⊔ IL. [k] therefore √ DLq − GLq ⊤⊔ DL. h 10. 10. k 18. 10.

6. *Alſo* DL ⊤⊔ GL. For ACq + BCq ⊤⊔ [l] 2 ACB, that is, DK ⊤⊔ GK. [m] therefore DL ⊤⊔ GL. l lem. 26. 10. m 10. 10.

7. *But if* AC *be taken* ⊤⅃ BC, [n] *then* DL ſhall be ⊤⅃ √ DLq − GLq. n 19. 10.

PROP. XCVIII.



The ſquare of a reſiduall line AB (AC − BC) *applied to rationall line* DE, *makes the breadth* DG *a firſt reſiduall line.*

Doe as is enjoined in the *Lemma* next preceding. Then becauſe AC, BC, [a] are ρ' ⊤⅃. [b] alſo DK (ACq + BCq) ſhall be ⊤⊔ ACq. [c] therefore DK is ρ'ν. [d] wherefore DL is ρ' ⊤⊔ DE. [e] Likewiſe the rectangle GK (2 ACB) is μν. [f] therefore GL is ρ' ⊤⊔ DE. [g] and conſequently DL ⊤⊔ GL. [h] But DLq ⊤⊔ GLq. [k] therefore DG is a reſiduall, [l] and that of the firſt order (becauſe [m] AC ⊤⅃ BC, and therefore DL ⊤⊔ √ DLq − GLq.) *W. W. to be Dem.* a hyp. b lem. 97. 10. c sch. 12. 10. d 21. 10. e 22, and 24. 10. f 23. 10. g 13. 10. h sch. 12. 10. k 74. 10. l 1. def. 85. 10. m lem. 97. 10.

PROP.

PROP. XCIX.

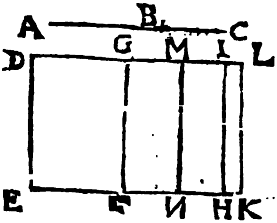
See the following Scheme.

The ſquare of a firſt mediall reſiduall line AB (AC − BC) *applied to a rationall line* DE, *makes the breadth* DG *a ſecond reſiduall line.*

a hyp.
b lem.97.10.
c 24.10.
d 23.10.
e hyp. & ſch. 12.10.
f 21.10.
g 13.10.
h ſch.12.10.
k 74 10.
l lem 97 10.
m 2.def. 85.10.

Suppoſing the foregoing *Lemma*; becauſe AC and BC *a* are μ ⊓⊔, *b* thence ſhall DK (ACq + BCq) be ⊓⊔ ACq. *c* wherefore DK is μν. *d* therefore DL is ρ' ⊓⊔ DE. *e* alſo GK (2 ACB) is ρν. *f* therefore GL is ρ' ⊓⊔ DE; *g* wherefore DL ⊓⊔ GL. *h* But DLq ⊓⊔ GLq. *k* therefore DG is a reſiduall line; and becauſe DL is ⊓⊔ √ DLq − GLq, *m* therefore ſhall DG be a ſecond reſiduall. *W. W. to be Dem.*

PROP. C.



The ſquare of a ſecond mediall reſiduall line AB (AC − BC) *applied to a rationall line* DE; *makes the breadth* DG *a third reſiduall line.*

a 23.10
b lem 26.10.
c 1. 6. and 10.10.
d ſch.12. 10.
e 74.10.
f 3.def.85. 10.
g lem.97.10.

Again DK is μν. *a* wherefore DL is ρ' ⊓⊔ DE. alſo DGK is μν *a* whence GL is ρ' ⊓⊔ DE. *b* likewiſe DK ⊓⊔ GK. *c* wherefore DL ⊓⊔ GL. *d* but DLq ⊓⊔ GLq. *e* therefore DG is a reſiduall line, and that of *f* the third order, *g* becauſe DL ⊓⊔ √ DLq − GLq. *W. W. to be Dem.*

PROP. CI.

See the foregoing Scheme.

The ſquare of a Minor *line* AB (AC − BC) *applied* *to a rationall line* DE , *makes the breadth* DG *a fourth reſiduall.*

As before, ACq + BCq, that is DK, is ῥν. *a* there- a 21.10.
fore DL is ῥ ⊐⊏ DE. but the rectangle ACB, and * hyp.
ſo GK (2 ACB) * is μν. *b* wherefore GL is ῥ ⊐⊏ b 25. 10.
DE. *c* therefore DL ⊐⊏ GL. *d* but DLq ⊐⊏ GLq. c 23.10.
and becauſe * ACq ⊐⊏ BCq, *e* thence ſhall DL be d ſch. 12. 10.
⊐⊏ √ DLq − GLq. *f* therefore DG ha's the con- e lem. 97. 10.
ditions required to a fourth reſiduall. *W.W. to be Dem.* f 4 def. 85. 10.

PROP. CII.

See Scheme 100.

The ſquare of a line AB (AC − BC) *which makes with a rationall ſpace the whole ſpace mediall, applied to a rationall line* DE , *makes the breadth* DG *a fift reſiduall line.*

For, as above, DK is μν. *a* wherefore DL is ῥ ⊐⊏ a 23.10.
DE. alſo GK is ῥν. *b* whence GL is ῥ ⊐⊏ DE. *c* there- b 21.10.
fore DL ⊐⊏ GL. *d* but DLq ⊐⊏ GLq. Moreover c 13.10.
DL *e* ⊐⊏ √ DLq − GLq. wherefore DG *f* is a fift d ſch. 13. 10.
reſiduall. *W.W. to be Dem.* e lem. 97. 10.
f 5. def. 85. 10.

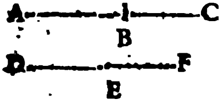
PROP. CIII.

See the laſt Scheme.

The ſquare of a line AB (AC − BC) *making with a mediall ſpace the whole ſpace mediall, applied to a rationall line* DE, *makes the breadth* DG *a ſixt reſiduall line.*

As above DK and GK are μα; *a* wherefore DL a 23.10.
and GL are ῥ ⊐⊏ DE, alſo DK *b* ⊐⊏ GK. *c* whence b hyp. and lem. 97. 10.
DL ⊐⊏ GL. *d* therefore DG is a reſiduall. *b* And c 10. 10.
whereas ACq ⊐⊏ BCq. and ſo DL ⊐⊏ √ DLq d 74. 10.
− GLq, *e* therefore DG ſhall be a ſixt reſiduall. e 6. def. 85. 10.
W.W. to be Dem.

PROP. CIV.



A right line DE *commenſurable in length to a reſiduall* AB (AC − BC) *is it ſelf alſo a reſiduall, and of the ſame order.*

Lemma.

Let AB.DE :: AC.DF *and* AB ⊤⊔ DE.

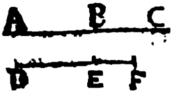
I ſay AC + BC ⊤⊔ DF + EF. For AC.BC [a] :: DF.EF. therefore by addition AC + BC.BC :: DF + EF.EF. therefore by inverſion AC + BC.DF + EF :: BC.EF. [a] but BC ⊤⊔ EF. [b] therefore AC + BC ⊤⊔ DF + EF. *W.W. to be Dem.*

a lem. 66. 10.
b 10. 10.

[a] Make AB.DE :: AC.DF. [b] therefore AC + BC ⊤⊔ DF + EF. therefore ſeeing AC + BC [c] is a binomiall, [d] DF + EF ſhall be a binomiall too, and of the ſame order. [e] wherefore DF − EF is a reſiduall of the ſame order with AC − BC. *W.W. to be Dem.*

a 12. 6.
b lem. 103. 10.
c hyp.
d def. 10.
e by def. 85. 10.

PROP. CV.

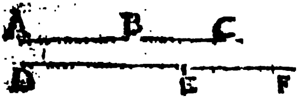


A right line DE *commenſurable to a mediall reſiduall line* AB (AC − BC) *is it ſelf a mediall reſiduall, and of the ſame order.*

Again [a] make AB.DE :: AC.DF. [b] whence AC + BC ⊤⊔ DF + EF. [c] therefore DF + EF is a bimediall of the ſame order with AC + BC, [d] and conſequently DF − EF ſhall be a mediall reſiduall of the ſame order with AC − BC. *Which was to be Demonſtrated.*

a 12. 6.
b lem. 103. 10.
c 68. 10.
d 75 and 76. 10.

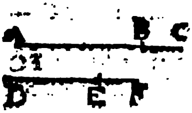
PROP. CVI.



A right line DE *commenſurable to a Minor line* AB (AC − BC) *is it ſelf alſo a Minor line.*

Make AB.DE :: AC. DF. ^a then is AC + BC ⊏⊐ DF + EF. but AC + BC ^b is a *Major* line; ^c therefore DF + EF is alſo a *Major* line; ^d and conſequently DF − EF is a *Minor* line. *W. W. to be Dem.* a lem. 104. 10. b hyp. c 69. 10. d 77. 10.

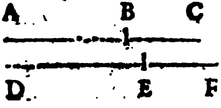
PROP. CVII.



A right line DE *commenſurable to a line* AB (AC − BC) *which makes with a rationall ſpace the whole ſpace mediall, is it ſelf alſo a line making with a rationall ſpace the whole ſpace mediall.*

For, accordingly as in the former, we may ſhow DF + EF to contain in power ῤν and μν. ^a whence DF − EF is a line making, &c. a 78. 10.

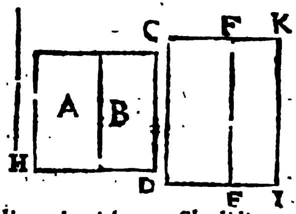
PROP. CVIII.



A right line DE *commenſurable to a line* AB (AC − BC) *which with a mediall ſpace makes the whole ſpace mediall, is it ſelf a line making with a mediall ſpace the whole ſpace mediall.*

For according to the preced. DF + EF ſhall contain in power 2 μα. ^a therefore DF − EF ſhall be, as in the *prop*. a 79. 10.

PROP. CIX.



A mediall -
gle B being
from a rationa
angle A+B, th
line H which c
eth in power th
remaining A,
of these two irr
lines. viz. either a reſiduall line, or a Minor line.

Upon CD ρ' make the rectangles CI = A
and FI = B. whence CE [a] = A = Hq. wherefo
cauſe CI [b] is ρ'ν. [c] therefore CK is ρ' ⊔ CD. but
FI [b] is μν, [d] ſhall FK be ρ' ⊔ CD, [e] when
⊔ FK. [f] therefore CF is a reſiduall line. Whe
if CK be ⊔ √ CKq − FKq, [g] then CF ſhal
firſt reſiduall. [h] therefore √ CE (H) is a reſ
line. But if CK ⊔ √ CKq − FKq, [k] then CI
be a fift reſiduall; and conſequently H (√
[l] ſhall be a Minor line. *W.W. to be Dem.*

[a] 3. ax. 1.
[b] hyp. and conſtr.
[c] 21. 10.
[d] 23. 10.
[e] 13. 10.
[f] 74. 10.
[g] 1. def. 85. 10.
[h] 92. 10.
[k] 4 def. 85. 10.
[l] 95. 10.

PROP. CX.

See the prec. Scheme.

A rationall rectangle B being taken away from
diall rectangle A + B, other two irrationall li
made, namely either a firſt mediall reſiduall line, o
making with a rationall ſpace the whole ſpac
diall.

Upon CD the propounded ρ' make the recta
CI = A + B, and FI = B. [a] whence CE =
Hq. Therefore becauſe CI [b] is μν; [c] ſhall CK be
CD. but becauſe FI [b] is ρ'ν. [d] thence FK ρ' ⊔
[e] whence CK ⊔ FK. [f] therefore CF is a reſi
[g] and that a ſecond. If CK ⊔ √ CKq −
[h] then H (√ CE) is a firſt mediall reſiduall.
CK ⊔ + CKq − FKq, [k] then ſhall CF be a
ſiduall; and [l] conſequently H (√ CE) ſhall be
making μν with ρ'ν. *W.W. to be Dem.*

[a] 3 ax. 1.
[b] hyp. and conſtr.
[c] 23 10.
[d] 21. 10.
[e] 13. 10.
[f] 74 10.
[g] 2. def. 85. 10.
[h] 93 10.
[k] 5 def. 85. 10.
[l] 96. 10.

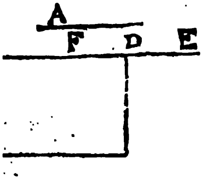
PROP. CXI.

See the ſame Scheme.

A mediall ſpace B being taken away from a me-
ll ſpace A + B ; *which is incommenſurable to the*
le A + B, *the other two irrationall lines are made;*
. *either a ſecond mediall reſiduall line, or a line ma-*
g with a mediall ſpace the whole ſpace mediall.

Upon CD ρ' make the rectangles CI = A + B,
l FI = B. [a] wherefore CE = A = Hq. Becauſe a 3. ax. 1.
refore CI is μν, [b] thence CK is ρ' ⊤⊥ CD. and in b 23. 10.
e manner FK ρ' ⊤⊥ CD. Likewiſe becauſe CI c hyp.
⊥ FI, [d] therefore CK ⊤⊥ FK. [e] wherefore CF is d 10. 10.
ſiduall, [f] namely a third. If CK ⊤⊥ √ CKq − e 74. 10. f 3. def. 85. 10
q, [g] whence H (√ CE) ſhall be a ſecond mediall g 94. 10. h 6. def. 85.
duall. but if CK ⊤⊥ √ CKq − FKq [h] then ſhall 10.
be a ſixt reſiduall. [k] wherefore A ſhall be a line k 97. 10.
king μν with μν. *W.W. to be Dem.*

PROP. CXII.



A reſiduall line A *is not the ſame with a binomiall line.*

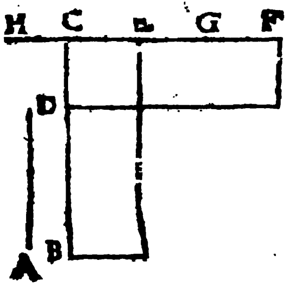
Upon BC propounded ρ' make the rectangle CD = Aq. Therefore ſeeing A is a reſiduall, [a] BD ſhall a 98. 10.
firſt reſiduall, to which let DE be the line con-
ent or that may be adjoined. [b] wherefore BE, DE, b 74. 10.
ρ' ⊤⊥. [c] and BE ⊤⊥ BC. If you conceive A to c 1. def. 85. 10.
binomiall, then BD is a firſt bin. whoſe names
e BF, FD; and let BF be ⊏ FD. [d] therefore BF, d 37. 10.
are ρ' ⊤⊥; and BF [e] ⊤⊥ BC. therefore ſince BC e 1. def. 48. 10.
. BE, [f] ſhall BE be ⊤⊥ BF. [g] and thence BE ⊤⊥ f 12. 10.
. [h] therefore FE is ρ'. Likewiſe becauſe BE ⊤⊥ g cor. 16. 10.
, [k] ſhall FE be ⊤⊥ DE. [l] wherefore FD is a reſi- h ſch 12. 10. k 14. 10.
ll, & ſo FD is ρ'. but it was ſhewn ρ'. which are re- l 74. 10.
nant. Therefore A is falſely conceived to be a bi-
niall. *W.W. to be Dem.*

The names of the 13. irrationall lines diffe one from another.

1. A Mediall line.
2. A binomial line, of which there are cies.
3. A first bimediall line.
4. A second bimediall.
5. A Major line.
6. A line containing in power a rational ficies and a mediall ſuperficies.
7. A line containing in power two med perficies.
8. A reſiduall line; of which there alſo ſi
9. A firſt mediall reſiduall line.
10. A ſecond mediall reſiduall line.
11. A Minor line.
12. A line making with a rationall ſup the whole ſuperficies mediall.
13. A line making with a mediall ſuperfi whole ſuperficies mediall.

Being the differences of breadths do argue dif of right lines, whoſe ſquares are applied to ſome r line, and it is demonſtrated in the preced. Propoſiti the breadths which ariſe from applying of the of theſe 13. lines do differ one from another, it e ly followes that theſe 13. lines do alſo differ one fr other.

PROP. CXIII.

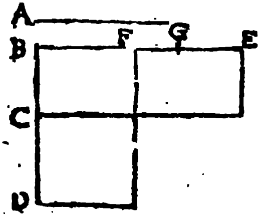


The ſquare tionall line A to a binomi (BD + DC) *the breadth* B *ſiduall line*, *names* BH, C *commenſurabl names* BD, D *the binomiall* *in the ſame proportion* (EH. BD :: CH.DC:) *and moreover, the reſiduall line* EC *which is made; is of the ſame order with* BC *the binomiall.*

Upon DC the leſſe name [a] make the rectangle DF = Aq = BB. whence BC.CD [b] :: FC. CE. therefore by diviſion, BD DC :: FE. EC. And whereas BD [c] ⊏ DC, [d] thence FE ſhall be ⊏ EC. Take EG = EC, & make FG.GE :: BC.CH. Then EH, & CH ſhall be the names of the reſiduall EC, whereunto all is agreeable that is propounded in the theoreme. For being that by addition FE. GE (EC) :: EH. CH. therefore FH. EH [e] :: EH. CH [f] :: FE. EC [f] :: BD. DC. wherefore ſince BD [g] ⊥ DC. [h] thence ſhall EH be ⊥ CH, [k] and FHq ⊥ EHq. Therefore becauſe FHq. EHq [k] :: FH. CH. [l] ſhall FH be ⊥ CH, [l] and ſo FC ⊥ CH. Moreover CD [g] is ῥ, and DF (Aq) [g] is ῥν. [m] therefore FC is ῥ, [n] ⊥ CD. whence alſo CH is ῥ ⊥ CD. [n] therefore EH, CH are ῥ and ⊥, as before. [o] therefore EC is a reſiduall line, to which CH may be joined. Furthermore EH.CH [f] :: BD.DC. and ſo by inverſion EH.BD :: CH.DC. whence becauſe CH [f] ⊥ DC, [p] ſhall EH be ⊥ BD. But ſuppoſe BD ⊥ √ BDq − DCq. [q] then ſhall EH be ⊥ √ EHq − CHq. Alſo if BD ⊥ ῥ propounded, then ſhall EH be ⊥ to the ſame ῥ. [s] that is, if BC be a firſt binomiall, [r] EC ſhall be a firſt reſiduall. In like manner, if DC be to the ⊥ propounded ῥ, [r] then is CH ⊥ to the ſame ῥ. [u] that is, if BC be a ſecond binomiall, [t] EC ſhall be a ſecond reſiduall: and if this be a third binom. then that ſhall be a third reſiduall, &c. But if BD be ⊥ √ BDq − DCq, [y] then ſhall EH be ⊥ √ EHq − CHq. therefore if BC be a 4, 5, or 6 binomiall, EG ſhall be likewiſe a 4, 5, or 6 reſiduall. *W.W. to be Dem.*

[a] cor. 16 6.
[b] 14 6.
[c] hyp.
[d] 14 5.
[e] 12. 5.
[f] before.
[g] hyp.
[h] 10. 10.
[k] cor. 20. 6.
[l] 16. 10.
[m] 21 10.
[n] ſch. 12. 10.
[o] 74. 10.
[p] 10. 10.
[q] 15. 10.
[r] 12. 10.
[s] 1. def. 48. 10.
[t] 1. def. 85. 10.
[u] 2. def. 48. 10.
[x] 2. def. 85. 10.
[y] 15. 10.

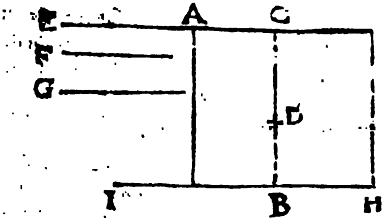
PROP. CXIV.



The ſquare of a rationall line A applied to a reſiduall line BC (BD — CD) *makes the breadth* BE *a binomiall; whoſe names* BE , GE *are commenſurable to the names* BD , BC *of the reſiduall line* BC, *and in the ſame proportion. and moreover, the binomiall line which is made* (BE) *is of the ſame order with the reſiduall line* (BC.)

a cor.16.6. [a] Make the rectangle DF = Aq. and BF.FE [b] ::
b 12. 6. EG.GF. whence for that DF = Aq = CE, [c] there-
c 14. 6. fore BD. BC :: BE. BF. therefore by converſion of
d 19. 5. proportion BD. CD :: BE. FE :: EG. GF :: [d] BG.
e hyp. EG. but BD [e] ⊔ CD. [f] therefore BG ⊔ GE.
f 10. 10. therefore becauſe BGq. GEq [g] :: BG. GF. [h] ſhall BG
g cor.20.6. be ⊔ GF. [k] and ſo BG ⊔ BF. moreover BD [e] is
h 10.10. ρ', and the rectangle DF(Aq) [e] is ρ'. [l] therefore BF is
k cor. 16.10. ρ' ⊔ BD. [m] therefore alſo BG is ρ' ⊔ BD. [n] therefore
l 21. 10. BG, GE are ρ' ⊔. [o] wherefore BE is a binomiall.
m 12.10. Laſtly, becauſe BD.CD :: BG. GE. and inverſely
n ſch.12. 10. BD.BG :: CD. GE. and BD ⊔ BG. [p] thence ſhall
o 37. 10. CD be ⊔ GE. therefore if CB be a firſt reſiduall,
p 10. 10. BE ſhall be a firſt binomiall, &c. as in the prec. therefore, &c.

PROP. CXV.



Ifa ſpace AB *be contained under a reſiduall line* AC *E — AE) and a binomiall* CB, *whoſe names* CD, *are commenſurable to the names* CE, AE, *of there- rall line; and in the ſame proportion* (CE. AE ::). DB) *then the right line* F *which containeth in power ſpace* AB, *is irrationall.*

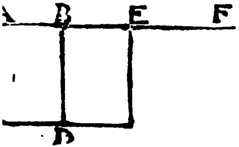
Let G be ῥ. and make the rectangle CH = Gq; en ſhall BH (HI — IB) be a reſiduall line, and *a* ⊓ CD *b* ⊓ CE. *a* and BI ⊓ DB. *a* and HI. :: CD. DB *b* :: CE. EA. therefore by inverſion . CE :: BI. EA. *c* therefore BH. AC :: HI. ; :: BI. EA. wherefore ſince *d* HI ⊓ CE, ence BH ⊓ AC. *f* therefore the rectangle HC ⊓ . But HC (Gq) *b* is ῥv. *g* therefore BA (Fq) is ῥv: l conſequently F is ῥ. *W.W. to be Dem.*

a 113. 10.
b hyp.
c 19. 5.
d 12. 10.
e 10. 10.
f 1. 6 *and*
g *ſch* 12. 10.

Coroll.

Hereby it appears that a rationall ſuperficies may contained under two irrationall right lines.

PROP. CXVI.



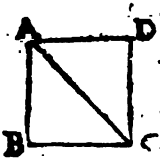
Of a mediall line AB *are produced infinite irrationall lines* BE, BF, &c. *whereof none is of the ſame kind with any of the precedent.*

et AC be propounded ῥ. & AD a rectangle con-
tained

tained under AC, AB. a therefore AD is $\dot{\rho}\nu$. Take BE $=\surd$ AD. b then BE is $\dot{\rho}$, and the ſame with none of the former. For no ſquare of any of the former being applied to $\dot{\rho}$, makes the breadth mediall. Let the rectangle DE be finiſhed, a then DE ſhall be $\dot{\rho}$. and b conſequently EF ($\surd$ DE) ſhall be $\dot{\rho}$, and not the ſame with any of the former. for no ſquare of the former being applyed to $\dot{\rho}$, makes the latitude BE. therefore, &c.

a ſchol. 38. 10
b 21. 10.

PROP. CXVII.



Let it be required to prove, that in ſquare figures BD, *the diameter* AC *is incommenſurable in length to the ſide* AB.

For ACq. ABq a :: 2. 1 b :: not Q. Q. c therefore AC $\overline{\sqcup}$ AB. *W.W. to be Dem.* This theoreme was of great note with the ancient philoſophers; ſo that he that underſtood it not was eſteemed by *Plato* undeſerving the name of a man, but rather to be reckoned among brutes.

a 47. 1.
b cor. 24. 8.
c 9. 10.

The End of the tenth Book.

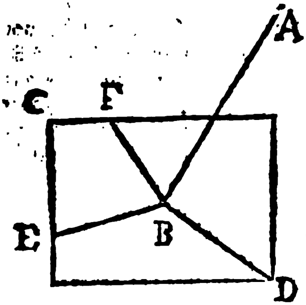
THE

THE ELEVENTH BOOK OF EUCLIDE'S ELEMENTS.

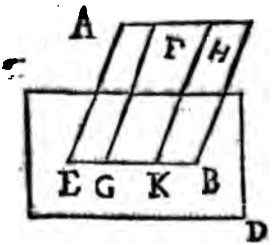
Definitions.

I. A Solid is that which hath length, breadth, and thickneſſe.

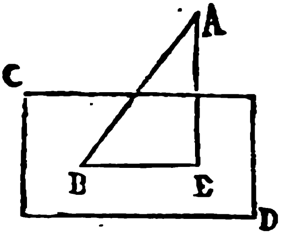
II. The terms, or extreme, of a ſolid is a Superficies.



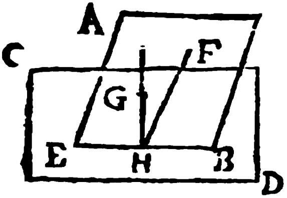
III. A right line AB is perpendicular to a Plane CD, when it makes right angles ABD, ABE, ABF with all the right lines BD, BE, BF, that touch it, and are drawn in the ſaid Plane.



IV. A Plane AB, is perpendicular to a Plane CD, when the right lines FG, HK, drawn in one Plane AB to the line of common ſection of the two Planes EB, and making right angles therewith, do alſo make right angles with the other Plane CD.



V. The tion of a ri AB to a Pla is, when a p cular AE is from A the point of that B to the pla and another drawn fro point E, which the perpendicular AE make Plane CD, to the end B of the ſaid line AB in the ſame Plane, whereby the angle is acu which is contained under the inſiſting line A the line drawn in the plane EB.



V I. The tion of a Pl to a Plane an acute F H G co under the lines F H which bein in either of the Planes AB, CD to the ſa ne of the common ſection BE, make right angl GHB, with the common ſection BE.

VII. Planes are ſaid to be inclined to othe in the ſame manner, when the ſaid angles of tion are equall one to another.

V I I I. Parallel Planes are thoſe which be longed never meet.

IX. Like ſolid figures are ſuch as are co under like planes equall in number.

X. Equall and like ſolid figures are ſuch conteined under like planes equall both in tude and magnitude.

X I. A ſolid angle is the inclination of mo two right lines which touch one another, and in the ſame ſuperficies.

Or thus;

A ſolid angle is that which is contained under more then two plane angles not being in the ſame ſuperficies, but conſiſting all at one point.

XII. A Pyramide is a ſolid figure comprehended under divers planes ſet upon one plane, (which is the baſe of the pyramide,) and gathered together to one point.

XIII. A Priſme is a ſolid figure contained under planes, whereof the two oppoſite are equall, like, and parallel; but the others are parallelograms.

XIV. A Sphere is a ſolid figure made when the diameter of a ſemicircle abiding unmoved, the ſemicircle is turned round about, till it return to the ſame place from whence it began to be moved.

Coroll.

Hence, all the rayes drawn from the center to the ſuperficies of a ſphere, are equall amongſt themſelves.

XV. The Axis of a ſphere, is that fixed right line, about which the ſemicircle is moved.

XVI. The Centre of a Sphere, is the ſame point with that of the ſemicircle.

XVII. The Diameter of a Sphere, is a right line drawn though the centre, and terminated on either ſide in the ſuperficies of the ſphere.

XVIII. A Cone is a figure made, when one ſide of a rectangled triangle (*viz.* one of thoſe that contain the right angle) remaining fixed, the triangle is turned round about till it return to the place from whence it firſt moved. And if the fixed right line be equall to the other which containeth the right angle, then the Cone is a rectangled cone; but if it be leſſe, it is an obtuſe-angled Cone; if greater, an acute-angled Cone.

XIX. The Axis of a Cone is that fix'd line about which the triangle is moved.

XX. The Baſe of a Cone is the circle which is deſcribed by the right line moved about.

XXI. A Cylinder is a figure made by the moving round of a right-angled parallelogram, one of the ſides thereof, (namely which contain the right angle) abiding fix'd, till the parallelogram be turned about to the ſame place, where it began to move.

XXII. The Axis of a Cylinder is that quieſcent right line, about which the parallelogram is turned.

XXIII. And the Baſes of a Cylinder are the circles which are deſcribed by the two oppoſite ſides in their motion.

XXIV. Like Cones and Cylinders, are they, both whoſe Axes and Diameters of their Baſes are proportionall.

XXV. A Cube is a ſolid figure contained under ſix equall ſquares.

XXVI. A Tetraedron is a ſolid figure contained under four equall and equilaterall triangles.

XXVII. An Octaedron is a ſolid figure contained under eight equall and equilaterall triangles.

XXVIII. A Dodecaedron is a ſolid figure contained under twelve equall, equilaterall and equiangular Pentagones.

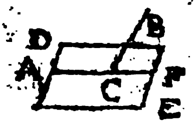
XXIX. An Icoſaedron is a ſolid figure contained under twenty equall and equilaterall triangles.

XXX. A Parallelipedon is a ſolid figure contained under ſix quadrilaterall figures, whereof thoſe which are oppoſite are parallell.

XXXI. A ſolid figure is ſaid to be inſcribed in a ſolid figure, when all the angles of the figure inſcribed are comprehended either within the angles, or in the ſides, or in the planes of the figure wherein it is inſcribed.

XXXII. Likewiſe a ſolid figure is then ſaid to be circumſcribed about a ſolid figure, when either the angles, or ſides, or planes of the circumſcribed figure touch all the angles of the figure which it contains.

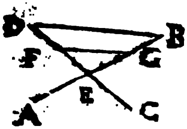
PROPOSITION I.



One part AC *of a right line cannot be in a plane ſuperficies, and another part* CB *elevated upward.*

Produce AC in the plane directly to F. If you conceive CB to be drawn ſtrait from AC, then two right lines AB, AF, have one common ſegment AC. a *Which is impoſſible.* a 10. ax. 1.

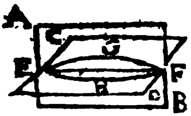
PROP. II.



If two right lines AB, CD, *cut one another, they are in the ſame plane: And every Triangle* DEB *is in one and the ſame plane.*

For imagine EFG, part of the triangle DEB, to be in one plane, and the part FDGB to be in another. then EF part of the right line ED is in a plane, and the other part elevated upwards. a *Which is Abſurd.* Therefore the triangle EDB is in one and the ſame plane; and ſo alſo are the right lines ED, EB; a wherefore the whole lines AB, DC, are in one plane. *Which was to be Demonſtrated.* a 1. 11.

PROP. III.

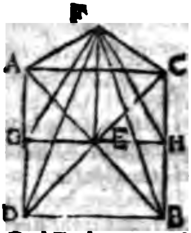


If two planes AB, CD, *cut one the other, their common ſection* EF *is a right line.*

If EF the common ſection be not a right line, a then in the plane AB draw the right line EGF, a and in the plane CD the right line EHF. therefore two right lines EGF, EHF include a ſuperficies. b *Which is Abſurd.* a 1. poſt. 1. b 14 ax. 1.

PROP.

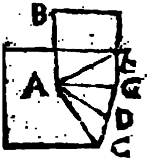
PROP. IV.



If a right line EF be at right angles erected upon two lines AB, CD, cutting one the other, at the common ſection E; it ſhall alſo be at right angles to the plane ACBD *drawn by the ſaid lines.*

Take EA, EC, EB, ED, equall one to the other, and join the right lines AC, CB, BD, AD. draw any right line GH through E, & join
a conſtr. FA, FC, FD, FB, FG, FH. Becauſe AE is *a* = EB,
b 15. 1. and DE *a* = EC, and the angle AED *b* = CEB,
c 4. 1. *c* therefore AD is = CB, *c* & likewiſe AC = DB.
d ſch. 34. 1. *d* therefore AD is parallel to CB, *d* and AC to DB.
e 29. 1. *e* wherefore the angle GAE = EBH, and the angle
f conſtr. AGE = EHB. But alſo AE *f* = EB. *g* therefore GE
= EH, *g* and AG = BH. whence by reaſon of the
g 26. 1. h 4. 1. right angles, by the hyp. and ſo equall, at E, *h* the
baſes FA, FC, FB, FD, are equall. Therefore the
triangles ADF, FBC, are equilaterall one to another.
k 8. 1. *k* and thence the angle DAF = BCF. Therefore in the triangles AGF, FBH, the ſides FG, FH
l 4. 1. *l* are equall; and ſo by conſequence the triangles
m 8. 1. FEG and FEH are mutually equilaterall. *m* there-
n 10. def. 1. fore the angles FEG, FEH are equall, and *n* ſo right
angles. In like manner, FE makes right angles with
all the lines drawn through E in the plane ADBC,
o 3. def. 11. *o* and is therefore perpendicular to the ſaid plane.

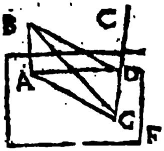
PROP. V.



If a right line AB *be erected perpendicular to three right lines* AC, AD, *touching one the other at the common ſection, thoſe three lines are in the ſame plane.*

For AC,AD, *a* are in one plane a 2. 11.
C; *a* and AD, AE, are in one plane BE. which if
u conceive to be ſeverall planes, then let their in-
ſection *b* be the right line AG; therefore becauſe b 3. 11.
by the Hypoth. is perpendicular to the right lines
E, AD. *c* and ſo to the plane FC, *d* it is alſo per- c 4. 11.
dicular to the right line AG. therefore (ſince d 3. *def.* 11.
AB is in the ſame plane with AC, AE) the
gles BAG, BAE, are right angles, and conſequent-
equall, the part & the whole. *Which is Abſurd.*

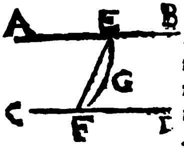
PROP. VI.



If two right lines AB, DC, *be erected perpendicular to one and the ſame plane* EF, *theſe right lines* AB, DC *are parallel one to the other.*

Draw AD, whereunto let DG
AB be perpendicular in the plane EF, and join
D, BG, AG. Being in the triangles BAD, ADG, the
gles DAB, ADG *a* are right angles, and AB *b* = a *hyp.*
G, and AD is common, *c* therefore BD is = AG. b *conſtr.*
hence in the triangles AGB, BGD, equilaterall one c 4. 1.
the other, the angle BAG is *d* = BDG: of which d 8. 1.
ing BAG is a right angle, BDG ſhall be ſo alſo.
t the angle GDC is ſuppoſed right, therefore the
ht line GD is perpendicular to the three lines
A, DB, CD. *e* which are therefore in the ſame plane e 5. 11.
herein AB is. Wherefore ſince AB and CD are in f 2. 11.
ſame plane, and the internall angles BAD,
DA, are right angles, *g* AB and CD ſhall be paral- g 28. 1.
W. W. to be Dem.

PROP. VII.

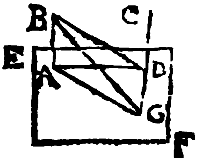


If there be two parallel right lines AB, CD, *and any points* E,F, *be taken in both of them, the line* EF *which is joined at these points, is in the ſame plane with the parallels* ABCD.

Let the plane in which AB,CD, are, be cut by another plane at the points E, F. then if EF is not in the plane ABCD, it ſhall not be the common ſection. Therefore let EGF be the common ſection; which *a* then is a right line. therefore two right lines EF,EGF, include a ſuperficies. *b Which is abſurd.*

a 3.11.
b 14.ax.1.

PROP. VIII.

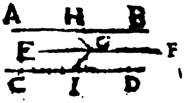


If there be two parallel right lines AB, CD, *whereof one* AB *is perpendicular to a plane* EF. *then the other* CD *ſhall be perpendicular to the ſame plane* EF.

The preparation and demonſtration of the ſixt of this Book being transferr'd hither; the angles GDA, and GDB are right angles: *a* therefore GD is perpendicular to the plane, wherein are AD,DB (*b* in which alſo AB, CD, are.) *c* therefore GD is perpendicular to CD. but the angle CDA is alſo *d* a right angle. *e* therefore CD is perpendicular to the plane EF. *W.W. to be Dem.*

a 4.11.
b 7.11.
c 3 def.11.
d 29.1.
e 4.11.

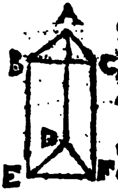
PROP. IX.



Right lines (AB, CD) which are parallel to the ſame right line EF, but not in the ſame plane with it, are alſo parallel one to the other.

In the plane of the parallels AB, EF, draw HG
perpendicular to EF; alſo in the plane of the paral-
lels EF, CD, draw IG perpendicular to EF. a there- a 4. 11.
fore EG is perpendicular to the plane wherein HG, b 8. 11.
GI are; and AH, CI are perpendicular to the ſame c 6. 11.
plane. c therefore AH and CI are parallels. *W. W. to be Dem.*

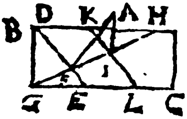
PROP. X.



If two right lines AB, AC, touching one another be parallel to two other right lines ED, DF, touching one another, and not being in the ſame plane, thoſe right lines contain equall angles, BAC, EDF.

Let AB, AC, DE, DF, be equall one
to the other, and draw AD, BC, EF, BE,
CF. Being AB, DE, a are parallels and a hyp. and conſtr.
equall, b alſo BE, AD, are parallels and b 33. 1.
equall. In like manner CF, AD, are parallels and e- c 2. ax 1. & 30. 1.
quall; c therefore alſo BE, FC, are parallels and e-
quall. d Therefore BC, EF are equall. Wherefore ſince d 33. 1.
the triangles BAC, EDF, are of equall ſides one to
the other, the angles BAC, EDF e ſhall be equall. e 8. 1.
W. W. to be Dem.

PROP. XI.



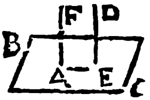
From a point given on high A to draw a right line AI perpendicular to a plane below BC.

In the plane BC draw any

a 12.1. line DE; to which from the point A *a* draw the per-
b 11. 1. pendicular AF, and *b* likewiſe FH in the plane BC
cutting the ſaid line DE at F; *a* then let fall AI per-
pendicular to FH. Which AI ſhall be perpendicular
to the plane BC.

For through I *c* let KIL be drawn parallel to DE.
c 31. 1. Becauſe DF *d* is perpendicular to AF, and FH,
d conſtr. *e* therefore DE ſhall be perpendicular to the plane
e 4. 11. IFA. and ſo alſo KL *f* is perpendicular to the ſame
f 8. 11. plane. *g* therefore the angle KIA is a right angle. but
g 3. def. 11 the angle AIF is alſo *h* a right angle. *l* therefore AI is
h conſtr. perpendicular to the plane BC. *W.W. to be Done.*
l 4. 11.

P R O P. XII.

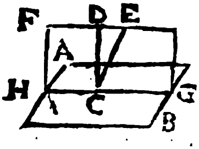


In a plane given BC, *at a point given therein* A, *to erect a perpendicular line* AF.

From ſome point without the
a 11. 11. plane, D, *a* draw DE perpendicu-
lar to the ſaid plane BC. and joining the points A,
b 31. 1. E, by a line AE, *b* draw AF parallel to DE. *c* it is ap-
c 8. 11. parent that AF is perpendicular to the plane BC.
W.W. to be Done.

This and the preceding probleme are practically performed by applying two *Squires* to the point given; as appears by 4. 11.

P R O P. XIII.

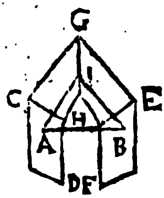


At a point given C *in a plane given* AB, *two right lines* CD, CE, *cannot be erected perpendicular on the ſame ſide.*

For both CD, and CE,
a 6. 11. *a* ſhould then be perpendicu-
lar to the plane AB, and conſequently parallels;
which is repugnant to the definition of parallel
lines.

PROP.

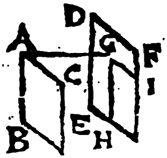
PROP. XIV.



Planes CD, FE, *to which the ſame right line* AB *is perpendicular, are parallel.*

If you deny this; then let the planes CD, FE, meet, ſo that their common ſection be the right line GH. in which take any point I, draw to it the right lines IA, IB, in the ſaid planes. where-
y in the triangle IAB, two angles IAB, IBA [a] are a hyp. and 3. def 11.
ight angles. [b] *Which is Abſurd.* b 17. 1.

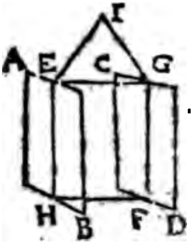
PROP. XV.



If two right lines AB, AC, *touching one the other, be parallel to two other right lines* DE, DF, *touching one the other, and not being in the ſame plane with them, the planes* BAC, EDF, *drawn by thoſe right lines are parallel one to the other.*

From A [a] draw AG perpendicular to the plane a 11. 11.
F. [b] and let GH, GI be parallel to DE, DF. [c] theſe b 31. 1.
lſo ſhall be parallel to AB, AC. Therefore ſince the c 30. 1.
ngles IGA, HGA, [d] are right angles, alſo CAG, d 3. def. 11.
AG, [e] ſhall be right angles. [f] therefore GA is per- e 29. 1.
endicular to the plane BC; but the ſame is perpen- f 4. 11.
licular to the plane EF. [h] therefore the planes BC, g conſtr.
F, are parallel. *W. W. to be Dem.* h 14. 11.

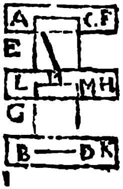
PROP. XVI.



If two parallel planes **AB**, **CD**, *be cut by ſome other plane* HEIGF, *their common ſections* EH, GF *are parallel one to the other.*

For if they be conceived to be otherwiſe; being in the ſame plane that cuts them, they will meet ſome where, if produced; ſuppoſe in I; wherefore ſince the whole lines HEI, FGI a are in the planes AD, CD, being produced, the planes alſo ſhall meet. *contrary to the Hyp.* a 1. 11.

PROP. XVII.



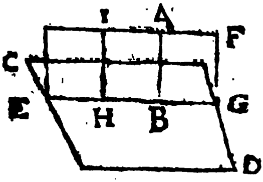
If two right lines **ALB**, **CMD**, *be cut by parallel planes* **EF**, **GH**, **IK**; *they ſhall be cut proportionally*, (AL. LB :: CM. MD.)

Let the right lines AC, BD, be drawn in the planes EF, IK; as alſo AD paſſing through the plane GH in the point N. and join NL, LM. the planes of the triangles ADC, ADB, make the ſections BD, LN, and AC, NM a parallels. Therefore AL. LB :: AN. ND b :: CM. MD. *W.W. to be Dem.*

a 16. 11.
b 2. 6.

PROP.

PROP. XVIII.

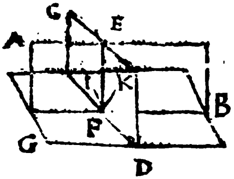


If a right line AB *be perpendicular to ſome plane* CD, *all the planes extended by that right line* AB (EF, &c.) *ſhall be perpendicular to the ſame plane* CD.

Let there be ſome plane EF drawn by AB, making the ſection EG with the plane CD; from ſome point whereof H, [a] draw HI parallel to AB in the plain EF; [b] then ſhall HI be perpendicular to the plane CD, and ſo likewiſe any other lines, that are perpendicular to EG. [c] therefore the plane EF is perpendicular to the plane CD; and by the ſame reaſon any other planes drawn by AB ſhall be perpendicular to EF. *W. W. to be Dem.*

a 31. 1.
b 8. 11.
c 4. def. 11.

PROP. XIX.

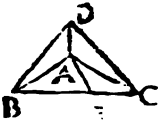


If two planes AB, CD, *cutting one the other, be perpendicular to ſome plane* GH, *their line of common ſection* EF *ſhall be perpendicular to the ſame plane* (GH.)

Becauſe the planes AB, CD, are taken perpendicular to the plane GH, it appears by 4. def. 11. that out of the point F there may be drawn in both planes AB, CD, a perpendicular to the plane GH. which ſhall be [a] but one; and therefore the common ſection of the ſaid planes. *W. W. to be Dem.*

a 13. 11.

PROP. XX.

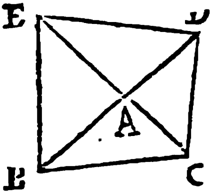


If a ſolid angle ABCD *be contained under three plane angles,* BAD, DAC, BAC, *any two of them howſoever taken are greater then the third.*

If the three angles are equall, the aſſertion is evident; if unequall, then let the greateſt be BAC; from
a 23. 1. whence *a* take away BAE = BAD, and make AD = AE; and alſo draw BEC, BD, DC.

Becauſe the ſide BA is common, and AD *b* =
b conſtr. AE; and the angle BAE *b* = BAD, *c* thence is BE
c 4. 1. = BD. but BD + DC is *d* ⊏ BC. *e* therefore DC
d 20. 1. ⊏ EC. Wherefore ſince AD *b* = AE, and the ſide
e 5. ax. 1. AC is common, and DC ⊏ EC. *f* the angle CAD
f 25. 1. ſhall be ⊏ EAC. *g* therefore the angle BAD +
g 4. ax. 1. CAD ⊏ BAC. *W. W. to be Dem.*

PROP. XXI.



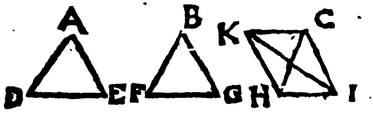
Every ſolid angle A is contained under leſſe angles then four plane right angles.

For let a plane anywiſe cutting the ſides of the ſolid angle A make a many-ſided figure BCDE, and as many triangles ABC, ACD, ADE, AEB. I denote all the angles of the polygone by X; and I term the ſumme of the angles at the baſes of the triangles Y. wherefore X
a 32. 1. and ſch. 32. 1. + 4 Right ang. *a* = Y + A. but being that,
b 20. 11. (of the angles at B) *b* the angle ABE + ABC
c 5. ax. 1. is ⊏ CBE, and the ſame is true alſo of the angles at C, at D, and at E, *c* it is manifeſt that Y is ⊏ X.

and

conſequently A ſhall be ⊐ 4 Right ang. *W.W. Dem.*

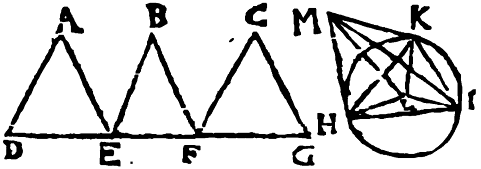
PROP. XXII.



f there be three plane angles A, B, HCI, *whereof*
howſoever taken are greater then the third, and the
t lines which contain them be equall AD, AE, FB,
then of the right lines DE, FG, HI, *coupling thoſe*
ll right lines together, it is poſsible to make a trian-

triangle may be [a] made of them, if any two be [a] 22. 1.
ter then the third': but they are ſo. For [b] make [b] 23. 1.
angle HCK = B, and CK = CH, and draw
,IK. [c] thence KH = FG. and becauſe the angle [c] 4. 1. [d] *hyp.*
I [d] ⊏ A. [e] therefore KI ⊏ DE. but KI [f] ⊐ [e] 24. 1.
+ KH (FG.) therefore DE ⊐ HI + FG. By [f] 20. 1.
like argument any two may be proved greater
the third; and conſequently [a] it is poſſible
ake a triangle of them. *W.W. to be Dem.*

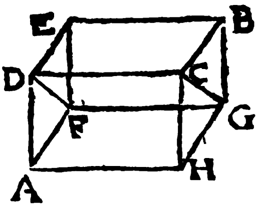
PROP. XXIII.



To make a ſolid angle MHIK *of three plane angles* A, B, C, *whereof two howſoever taken are greater then* [*] 21. 11. *the third.* * *But it is neceſſary that theſe three angles be leſſe then four right angles.*

Make AD, AE, BE, BF, CF, CG, equall one to the other; and of the ſubtended lines DE, EF, FG (that [a] 22. 11. and 23. 1. is, of the equall lines HI, IK, KH) [a] make the triangle [b] 5. 4. HKI; about which [b] deſcribe the circle LHKI. [*] See Clavius. * But becauſe AD is ⊏ HL. [c] let ADq be = HLq + [c] sch 47. 1. LMq. [d] and let LM be perpendicular to the plane of [d] 12. 11. the circle HKI. and draw HM, KM, IM. wherefore [e] 3 def 11. ſince the angle HLM [e] is a right angle, [f] 47. 1. [f] thence is [g] constr. MHq = HLq + LMq [g] = ADq. therefore MH = AD. By the ſame reaſon MK, MI, AD (that is AE, EB, &c.) are equall; therefore ſince HM = AD, and [h] constr. MI = AE, and DE [h] = HI, [k] 8. 1. [k] the angle A ſhall be = HMI, [k] as likewiſe the angle IMK = B, [k] and the angle HMK = C. wherefore a ſolid angle is made at M of the three given plane angles. *W.W. to be Done.* AD is aſſumed to be ⊏ HL. But this is maniſeſt. For if AD be = or ⊐ HL, then is the angle A [l] constr. and 8. 1. [l] = [m] or ⊏ HLI. In like manner ſhall B be equall [m] 21. 1. or ⊏ HLK, and C = or ⊏ KLI. wherefore A + B [*] 4 cor. 13. 1. + C * ſhall either equall or exceed four right angles. *contrary to the Hypoth.* therefore rather let AD be ⊏ HL. *W.W. to be Dem.*

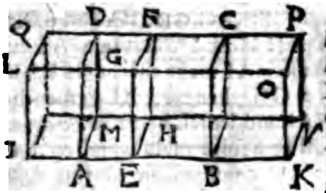
PROP. XXIV.



If a ſolid AB be contained under parallel planes, the oppoſite planes thereof (AG, DB, &c.) *are like and equall parallelograms.*

The plane AC cutting the parallel planes
AG, BB, *a* makes the a 16. 11.
ſections AH, DC, parallels. and by the ſame reaſon AD, HC are parallels. Therefore ADCH is a
pgr. By the like argument the other planes of the b 35. *def.* 1.
parallelepipedon are *b* pgrs. wherefore being AF is c 10. 11.
parallel to HG, and AD to HC, *c* the angle FAD d 34. 1.
ſhall be = CGH. therefore becauſe AF *d* = HG, e 7. 5.
and AD *d* = HC, and ſo AF. AD :: HG. HC. the g 6. 6.
triangles FAD, GHC, *g* are like and *h* equall; and h 4. 1.
conſequently the pgrs. AE, HB are like and *k* equall. k 6. *ax.* 1.
and the ſame may be ſhewn of the reſt oppoſite planes. therefore, &c.

PROP. XXV.



If a ſolid Parallelepipedon ABCD *be cut by a plane* EF *parallel to the oppoſite planes* AD, BC; *then as the baſe* AH *is to the baſe* BH, *ſo ſhall ſolid* AHD *be to ſolid* BHC.

Conceive the Parallelepipedon to be extended on either ſide. and take AI = AE, and BK = EB, and put the planes IQ, KP, parallel to the planes AD,
BC; then the pgrs. IM, AH, and *a* DL, DG, *b* and a 36. 1. and
IQ, AD, EF, &c. are *a* like and equall. *c* wherefore 1 def. 6.
the Parallelepipedon AQ is = AF; and by the ſame b 24. 11.
c 10. def. 11.

reaſon the Parallelepipedon BP = BF. therefore the ſolids IF, EP are as multiplex of the ſolids F, EC, as the baſes IH, KH, are of the baſes AH, BH. And if the baſis IH be ⊏, =, ⊐ KH, [d] likewiſe ſhall the ſolid IE be ⊏, =, ⊐ EP. [e] conſequently AH.BH :: AF.EC. *W.W. to be Dem.*

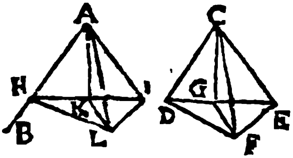
d 24.11. and 9 def. 11.
e 6. def. 5.

The ſame may be accommodated to all ſort of priſmes, whence

Coroll.

If any priſme whatſoever be cut by a plane parallel to the oppoſite planes, the ſection ſhall be a figure equall and like to the oppoſite planes.

PROP. XXVI.

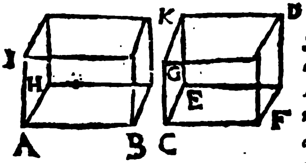


Upon a right line given AB, *and at a point given in it* A, *to make a ſolid angle* AHIL *equall to a ſolid angle given* CDEF.

From ſome point F [a] draw FG perpendicular to the plane DCE, and draw the right lines DF, FE, EG, GD, CG. Make AH = CD, and the angle HAI = DCE, and AI = CE; and in the plane HAI make the angle HAK = DCG, and AK = CG. then erect KL perpendicular to the plane HAI, and let KL be = GF. & draw AL: then AHIL ſhall be a ſolid angle equall to that given CDEF. For the conſtruction of this do's wholly reſemble the framing of that, as may eaſily appear to any that examine it.

a 11. 11.

PROP. XXVII.



Vpon a right line given AB *to deſcribe a parallelepipedon* AK, *like, and in like manner ſituate, with a ſolid parallelepipedon given* CD.

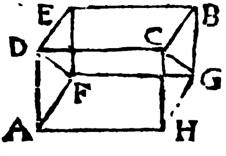
Of the plane angles, BAH, HAI, BAI, which are equall to FCE, ECG, FCG, *a* make the ſolid angle A equall to the ſolid angle C. alſo *b* make FC. CE :: BA. AH. *b* and CE. CG :: AH. AI (*c* whence of equality FC. CG :: BA. AI) and finiſh the parallelepipedon AK, which ſhall be like to that which is given.

a 26. 11.
b 12. 6.
c 22. 5.

For by the conſtruction, the Pgr. *d* BH is like to FE, and *d* HI to EG, and *d* BI to FG, & *e* ſo the oppoſites of theſe to the oppoſites of them: therefore the ſix planes of the ſolid AK are like to the ſix planes of the ſolid CD, *f* and conſequently AK, CD, are like ſolids. *W. W. to be Done.*

d 1. def. 6.
e 24. 11.
f 9. def. 11.

PROP. XXVIII.



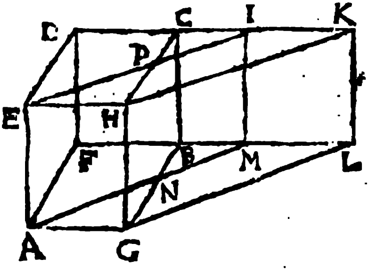
If a ſolid parallelepipedon AB *be cut by a plane* FGCD *drawn by the diagonall lines* DF, CG, *of the oppoſite planes* AE, HB, *that ſolid* AB *ſhall be equally biſected by the plane* FGCD.

For becauſe DC, FG, are *a* equall and parallels, *b* the plane FGCD is a Pgr. and being *a* the Pgrs. AE, HB, are equall and like, *b* alſo the triangles AFD, HGC, CGB, DFE are equall and like. But the Pgrs. AC, AG, are equall and like to FB and FD. therefore all the planes of the priſme FGCDAH are equall and like to all the planes of the priſme FGCDEB, and *c* conſequently this priſme is equall to that. *W. W. to be Dem.*

a 34. 11.
b 34. 1.
c 9. def. 11.

PROP.

PROP. XXIX.



Solid parallelepipedons AGHEFBCD, AGHEMLKI, *being conſtituted upon the ſame baſe* AGHB, *and* * *in the ſame height, whoſe inſiſting lines* AF, AM, *are placed in the ſame right lines* AG, FL, *are equall one to the other.*

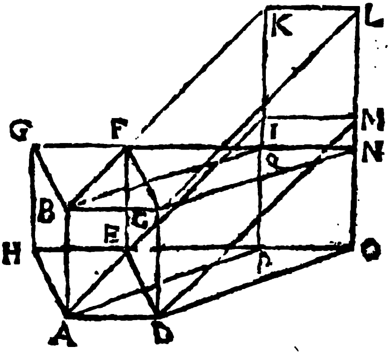
** i. e. between the parallel planes* AGHE, FLKD. *and ſo underſtand it in the follo.*

a 10. *def.* 11. *and* 35. 1.

b 3. *and* 2. *ax.* 1.

For a if from the equall priſmes AFMEDI, GBLHCK, the common priſme NBMPCI be taken away, and the ſolid AGNEHR be added, the Parallelepiped. AGHEFBCD ſhall be = AGHEMLKI. *W.W. to be Dem.*

PROP. XXX.



Solid parallelepipedons ADBCHEFG, ADCBIMLK

being

conſtituted upon the ſame baſe ADFC, *and in the height, whoſe inſiſting lines* AH, AI *are not placed ſame right lines, are equall one to the other.*

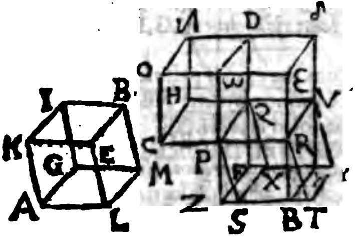
or produce the right lines HEO, GFN, and L-KIP; and draw AP, DO, BQ, CN. a then ſhall AB, HG, EF, PQ, ON be as well equall and pa-l one to the other as AD, HE, GF, BC, KL, IM, PO. *b* wherefore the parallelepipedon ADCB-NQ ſhall be equall to either parallelepipedon CBHEFG, ADCBIMLK; and *c* conſequently two are equall one to the other. *W. W. to be*

a 34. 1.

b 29. 11.

c 1. ax. 1.

P R O P. XXXI.



id parallelepipedons, ALEKGMBI, CPωOHQ- *being conſtituted upon equall baſes* ALEK, CPωO, *in the ſame height* *are equall, one to the other.*

* *by height underſtand the perpendicular drawn from the plane of the baſe to the oppoſite plane.*

ſt, let the parallelepipedons AB, CD, have the perpendicular to the baſes. and at the ſide CP produced, *a* make the Pgr. PRTS equall and o the pgr. KELA. *b* and ſo the parallelepipedon SQVYX equall and like to the parallelepipe- AB. Produce OωE, NDδ, ωPZ, DQF, ERB, TSZ, YXF; and draw Eδ, Bγ, ZF.

a 18 6.

b 27. 11. and 10 def. 11.

e planes OδN, CRVH, ZTYF. *c* are parallels o the other; *d* and the Pgrs. ALEK, CDωO, S, PRBZ are equall. Therefore ſince the paral-pedon CD.PVω :: Pgr. Cω (PRBZ) Pω :: e 25. 11.

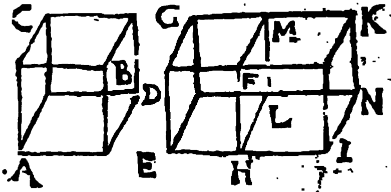
c 30. def. 11.

d hyp. and 35. 1.

parallelepipedon PRBZQVγF. PV &c; the parallele-
f 9. 5. pipedon CD f ſhall be = PRBZQVγF g = PR
g 29. 11. h conſtr. VQSTYX h = AB. *W.W. to be Dem.*

But if the parallelepipedons AB, CD, have ſides
oblique to the baſe, then on the ſame baſes and in
the ſame height place parallelepipedons whoſe ſides
k 29. 11. are perpendicular to the baſe. k They ſhall be equall
m 1. ax. 1. to one another, and thoſe that are oblique m whence
alſo the oblique parallelepipedons AB, CD, are
equall. *W.W. to be Dem.*

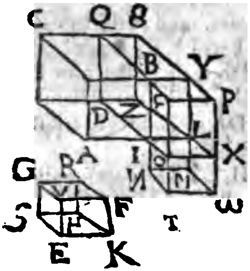
PROP. XXXII.



Solid parallelepipedons ABCD, EFGL, *of the ſame height, are one to the other, as their baſes,* AB, EF.

Produce EHI, a and make the pgr. FI = AB, and
a 45. 1. b 31. 1. b compleat the parallepp. FINM. It is clear that the
c 31. 11. parallepp. FINM. (c ABCD.) EFGL d :: FI (AB)
d 25. 11. EF. *W.W. to be Dem.*

PROP. XXXIII.



Like ſolid parallelepipedons, ABCD, EFGH, *are in tripled proportion one to the other, of that in which their homologous ſides or of like proportion* AI, EK, *are.*

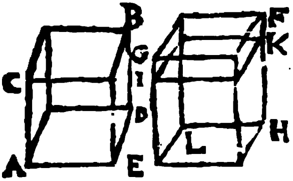
Produce the right lines AIL, DIO, BIN, and
a 3. 1. a make IL, IO, IN, equall

to EK, KH, KF, [b] and ſo the parallepp. IXMT equall b 27. 11.
nd like to the parallepp. EFGH. [c] Let the parallepps. IXPB, DLYQ be finiſhed. [d] Then ſhall be c 31. 1. d hyp.
L.IL (EK) :: DI.IO (HK) :: BI.IN.KF. [e] that is e 1. 6.
he Pgr. AD.DL :: DL.IX :: BO.IT. [f] *i.e.* the parallepp. ABCD.DLQY :: DLQY. IXBP :: IXBP.IX- f 32. 11.
T.(g EFGH.) [h] therefore the proportion of ABCD g conſtr. h 10. def. 5.
o EFGH is triple of the proportion of ABCD to k 1. 6.
LQY, [k] or of AI to EK. *W.W. to be Dem.*

Coroll.

Hence it appears that if four right lines be contiually proportionall, as the firſt is to the fourth, ſo s a parallelepipedon deſcribed on the firſt to a parallelepipedon deſcribed on the ſecond, being like nd in like manner deſcribed.

PROP. XXXIV.



In equall ſolid parallelepipedons AD-CB, EHGF, *the baſes and altitudes are reciprocall* (AD. EH :: EG. AC.) *And ſolid parallelepipedons*, AD-CB, EHGF, *whoſe baſes and altitudes are reciprocall, are equall.*

Firſt, let the ſides CB, GE be perpendicular to the baſes; then if the altitudes of the ſolids are equall, the baſes alſo ſhall be equall. and the thing is clear. But if the altitudes are unequall, from the greater EG [a] take EI = AC, and at I [b] draw the plane IK a 3. 11. b 31. 11.
parallel to the baſe EH. then c 32. 11.

1. *Hyp.* AD.EH [c] :: parallepp. ADCB.EHIK [d] :: d 17. 5.
parallepp. EHGF. EHIK [e] :: GL. IL [e] :: GE. IE. e 1. 6. f conſtr.
(*f* AC.) [g] it is plain therefore that AD.EH :: GE.AC. g 11. 5.
W.W. to be Dem. h 32. 11. k hyp.

2. *Hyp.* ADCB.EHIK [h] :: AD.EH [k] :: EG.EI [l] :: l 1. 6.
GL. IL [m] :: parallepp. EHGF. EHIK. [n] where- m 32. 11. n 9. 5.

ſince the parallelepipedons ACB = EH as before.

Moreover let the ſides be oblique to th erect right parallelepipedons upon the ſa the ſame altitude; the oblique parallelep equall to them. Wherefore ſince by the fo baſes and altitudes of theſe be reciprocall and altitudes of theſe alſo ſhall be recipr as before.

Coroll.

All theſe things are underſtood of priſ in the 29, 30, 31, 32, 33, 34. Prop. For priſms are halfes of parallelep appears *in Prop.* 28. Therefore,

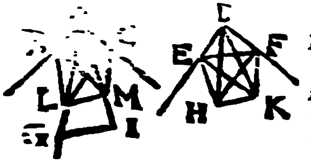
1. Triangular priſmes are of equall heigh baſes.

2. If they have the ſame or equall ba ſame altitude, they are equall.

3. If they be like, their proportion is tr of their ſides of like proportion.

4. If they be equall, their baſes and a reciprocall; and if their baſes and altitu procall, they are alſo equall.

PROP. XXXV.



If th plane an EDF, *from th thoſe ang lines* AC *levated teining equall angles with the lines firſt giv his correſpondent angle (the angle* GAB = GAC = HDF.) *and if in thoſe elevat* DH, *ſome points be taken,* G, H; *and from perpendicular lines* GI, HK, *drawn to the* EDF, *in which the angles firſt given are* *lines* AI, DK, *be drawn to the angles firſt given from the points* I, K, *which are made by the perpendiculars in the planes; thoſe right lines with the elevated lines* AG, DH *ſhall contain equall angles* GAM, HDK.

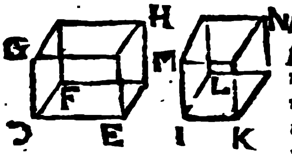
Make DH, AL, equall; and GI, LM parallels. and MC to AC. MB to AB, KF to DF, KE to DE perpendicular; and draw the right lines BC, LB, LC, and EF, HF, HE; *a* and LM is perpendicular to the plane BAC; *b* wherefore the angles LMC, LMA, LMB; and by the ſame reaſon the angles HKF, HKD, HKE are right angles. Therefore ALq *c* = LMq + AMq *c* = LMq + CMq + ACq *c* = LCq + ACq. *d* therefore the angle ACL is a right angle. Again ALq *e* = LMq + MAq *e* = LMq + BMq + BAq *e* = BLq + BAq. *d* therefore the angle ABL is alſo a right angle. By the like inference the angles DFH, DEH are right angles; *f* therefore AB = DE, *f* and BL = EH, *f* and AC = DF, and CL = FH. *g* wherefore alſo BC = EF; *g* and the angle ABC = DEF, *g* and the angle ACB = DFE. *h* whence the other right angles CBM, BCM, are equall to the other FEK, EFK. *k* therefore CM = FK, *l* and ſo alſo AM = DK. therefore if from LAq *m* = HDq be taken away AMq = DKq, *n* there remains LMq = HKq. wherefore the triangles LAM, HDK are equilaterall one to the other; *o* therefore the angle LAM = HDK. *W.W. to be Dem.*

a 8. 11.
b 3. *def.* 11.
c 47. 1.
d 48. 1.
e 47. 1.
f 26. 1.
g 4. 1.
h 1. *ax.* 1.
k 26. 1.
l 47. 1.
m *conſtr.*
n 47. 1 *and* 3 *ax.*
o 8. 1.

Coroll.

Therefore, if there be two plane angles equall, from whoſe points equall right lines be elevated on high containing equall angles with the lines firſt given, each to each; perpendiculars drawn from the extreme points of thoſe elevated lines to the planes of the angles firſt given, are equall one to the others *viz.* LM = HK.

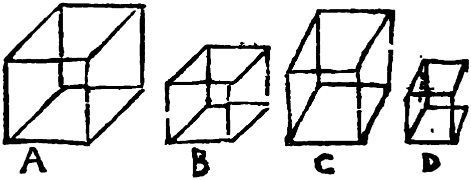
PROP. XXXVI.



If there be three right lines DE, DG, DF *proportionall, the ſolid parallelepipedon* DH *made of them, is equall to the ſolid parallelepipedon* IN *made of the middle line* DG (IL) *which is alſo equilaterall, and equiangular to the ſaid parallelepp.* DH.

a *hyp.* b 14. 6. Becauſe DE. IK [a] :: IL. DF. [b] the pgr. LK ſhall be = FE. and by reaſon of the equality of the plane angles at E and I, and of the lines GD, IM, alſo the altitudes of the parallelepipedons are equall by the prec. Coroll. [c] therefore the parallepps are equall one to the other. *W.W. to be Dem.* c 31. 11.

PROP. XXXVII.

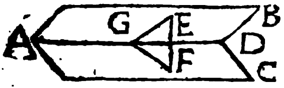


A B C D

If there be four right lines A, B. C, D, *proportionall, the ſolid parallelepipedons* A, B, C. D *being like, and in like ſort deſcribed from them, ſhall be proportionall. And if the ſolid parallelepipedons, being like and in like ſort deſcribed, be proportionall* (A. B :: C. D.) *then thoſe right lines* A, B, C, D, *ſhall be proportionall.*

a 33. 11. b *ſch.* 23. 5. For the propoſitions of the parallelepps. [a] are triple of thoſe of the lines; therefore if A. B :: C. D. [b] then ſhall the parallepp. A. parallepp. B :: parallepp. C. parallepp. D. and ſo alſo contrarily.

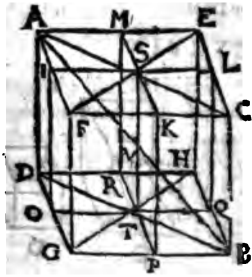
PROP. XXXVIII.



If a plane AB *be perpendicular to a plane* AC, *and a perpendicular line* EF *be drawn from a point* E *in one of the planes* (AB) *to the other plane* AC, *that perpendicular* EF *ſhall fall upon the common ſection of the planes* AD.

If it be poſſible, let F fall without the interſection AD. and in the plane AC a draw FG perpendicular a 12, 1.
to AD, and join EG. The angle FGE b is a right an- b 4. and 3. def. 11.
gle, and EFG is ſuppoſed to be ſuch alſo; therefore c 17. 1.
two right angles are in the triangle EFG. c *Which is abſurd.*

PROP. XXXIX.



If the ſides (AE, FC, AF, EC, *and* DH, GB, DG, HB) *of the oppoſite planes* AC, DB, *of a ſolid parallelepipedon* AB, *be divided into two equall parts, & planes* ILQO, PKMR, *be drawn through their ſections, the common ſection of the planes* ST, *and the diameter of the ſolid parallelepipedon* AB *ſhall divide one the other into two equall parts.*

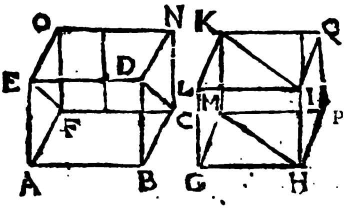
Draw the right lines SA, SC, TD, TB. Becauſe a the a 34. 1.
ſides DO, QT are equall to the ſides BQ, QT, b and b 29. 1.
the alternate angles TOD, TQB equall alſo, c the c 4. 1.
baſes DT, TB, & the angles DTO, BTQ are equall. d ſch. 15. 1.
d therefore DTB is a right line. & ſo in like manner is e 34 1.
ASC. Moreover e as well AD is parallel & equall to f 9. 11. and 1. ax.
FG e as FG to CB, & f thence AD is parallel & equal g 33. 1.
to CB; g & conſequently AC to DB. h wherefore AB h 7. 11.

and ST are in the ſame plane ABCD. There
ſince the verticall angles AVS, BVT, & the alter
angles ASV, BTV are equall; [k] and AS =
k 7 ax. 1. therefore ſhall AV be = BV, [l] and SV =
l 26. 1. *W. W. to be Dem.*

Coroll.

Hence in every parallelepipedon all the dia
ters biſect one another in one point, V.

PROP. XL.



If two priſmes ABCFED, GHMLIK, *be of*
altitude, whereof one hath its baſe ABCF *a para*
gram, and the other GHM *a triangle; and if the p*
lelogram ABCF *be double to the triangle* GHM;
priſmes ABCFED, GHML'K *are equall.*

For if the parallelepps. AN, GQ, be compl
a 31. 11. [a] they ſhall be equall, becauſe of the equality [b]
b 34. 1. and 9. ax. baſes AC, GP, & [c] of the altitudes. [d] therefore
c hyp. the priſmes, [e] the halfes thereof ſhall be equall.
d 28 11. *to be Dem.*
e 7 ax. 1.

Schol.

From the preceding demonſtrations, the dimenſ
Andr. Tacq. *triangular priſmes, and quadrangular, or parallel*
dons, is learnt; viz. *by multiplying the altitude in*
baſe.

As if the altitude be 10 foot, and the baſe
ſquare foot (the baſe may be meaſured by *Sch*
1. or by 41. 1.) then multiply 100 by 10. and

cubic foot ſhall be produced for the ſolidity of the priſme given.

For as a rectangle, ſo alſo is a right parallelepipedon produced of the altitude multiplyed into the baſe. Therefore every parallelepipedon is produced of the altitude multiplyed into the baſe, as appears by 31. of this Book.

Moreover, ſince the whole parallelepipedon is produced of the altitude drawn into the baſe, the half thereof (that is, a triangular priſme) ſhall be produced of the altitude drawn into half the baſe, namely the triangle.

An Advertiſement.

Obſ. That of thoſe letters which denote a ſolid angle, the firſt is alwayes at the point in which the angle is; but of thoſe letters which denote a pyramide, the laſt is at the ſupreme point thereof.

Ex. gr. the ſolid angle ABCD is at the point A; and the ſupreme-point of the pyramide BCDA is at the point A. and the baſe is the triangle BCD.

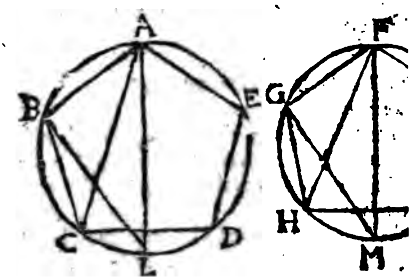
The End of the eleventh Book.

THE TWELFTH BO

OF

EUCLIDE'S ELEME

PROPOSITION I.



LIke polygonous figures ABCDE *deſcribed in circles* ABD, FG *to another, as the ſquares* d *the diameters of the circles* AI

Draw AC, BL, FH, GM

a 1 def. 6. a the angle ABC = FGH, a and AB.BC ::
b 6. 6. b therefore ſhall the angle ACB (c ALB) b
c 21. 3. (c FMG.) but the angles ABL, FGM d
d 31. 3. and ſo equall; e therefore the triangles AB
e 32. 3. are equiangular. f wherefore AB. FG ::
f cor. 4. 6. g therefore ABCDE. FGHIK :: ALq.FM
g 22. 6.

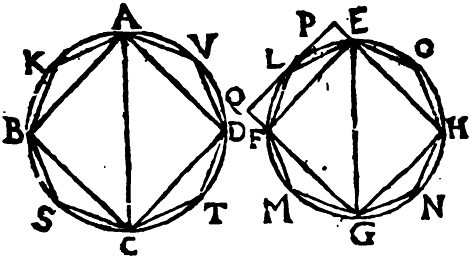
Coroll.

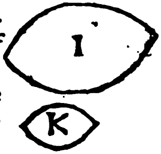
Hence (becauſe AB. FG :: AL. FM :: I &c.) the contents of like polygonous fi ſcribed in a circle are in h proportion a meters.

h 1. 12. and 22. 6.

K.

PROP. II.





Circles ABT,EFN, *are in propor-*
m one to another , as the ſquares of
eir diameters AC,EG *are.*

Suppoſe ACq. EGq :: the circle
BT. I. I ſay then I is equall to
ie circle EFN.

For firſt,if it be poſſible, let I be leſſe then the cir-
e EFN, and let K be the exceſſe or difference. In-
ribe the ſquare EFGH in the circle EFN , [a] it a ſch.7.4.
ing the half of a circumſcribed ſquare , and ſo
eater then the ſemicircle. [b] Divide equally in two b 30.3.
ie arches EF,FG,GH,HE,and at the points of the
viſions join the right lines EL, LF, &c. at L draw
ie tangent PQ ([c] which is parallel to EF) and pro- c ſch.27.3.
ice HEP, GFQ. then is the triangle ELF [d] the d 41.1.
lf of the pgr. EPQF, and ſo greater then the half
f the ſegment ELF; and in like ſort the reſt of thoſe
iangles exceed the halfs of the reſt of the ſegments.
nd if the arches EL,LF,FM, &c. be again biſected,
id the right lines joined, the triangles will likewiſe
rceed the half of the ſegments. Wherefore if the
juare EFGH be taken from the circle EFN , and
ie triangles from the other ſegments , and this be
one continually, at length [e] there will remain ſome e 1.10.
iagnitude leſſe then K. Let us have gone ſo farre,
amely to the ſegments EL,LF, FM, &c. taken to-
gether

gether leſſe then K. Therefore I ([f] the circle EFN — K) ⊐ the polyg. ELFMNHO (the circle EFN — the ſegm. EL + LF, &c.) In the circle ABT [g] conceive a like polygonon AKBSCTDV inſcribed. therefore ſince AKBSCTDV. ELFMGNHO [h] :: ACq. EGq [k] :: the circle ABT. I. and the polyg. AKBSCTDV [l] ⊐ the circle ABT. the polyg. ELFMGNHO [m] ſhall be ⊐ I. but before, I was ⊐ ELFMGNHO. which is repugnant.

f hyp. and 9. ax.
g 30. 3. and 1. poſt. 1.
h 1. 12.
k hyp.
l 9. ax. 1.
m 14. 5.

Again, if it be poſſible, let I be ⊏ the circle EPN. Therefore becauſe ACq. EGq [n] :: the circle ABT. I; and inverſely I. the circle ABT :: EGq. ACq. ſuppoſe I. the circle ABT :: the circle EFN. K. [o] therefore the circle ABT ⊏ K. [p] and EGq. ACq :: the circle EFN. K. which is ſhewn to be repugnant.

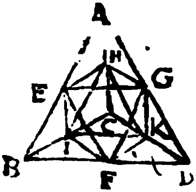
n hyp.
o 14. 5.
p 11. 5.

Therefore it muſt be concluded, that I is = to the circle EFN. *W.W. to be Dem.*

Coroll.

Hence it follows, that as a circle is to a circle, ſo is a polygonon deſcribed in one to a like polygonon deſcribed in the other.

PROP. III.

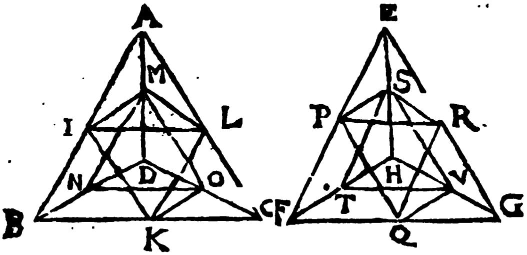


Every Pyramide ABDC *having a triangular baſe, may be divided into two pyramides* AEGH, HIKC, *equall, and like one to the other, having baſes triangular, and like to the whole* ABDC; *and into two equall priſmes,* BFGEIH, EGDIHK; *which two priſmes are greater then the half of the whole pyramide* ABDC.

Divide the ſides of the pyramide into two parts at the points E, F, G, H, I, K. and join the right lines EP, FG, GE, EI, IF, FK, KG, GH, HE. Becauſe the ſides of the pyramide are proportionally cut,

ut,[a] thence HI,AB; and GF,AB; and IF,DC; and a 2.6.
HG, DC, &c. are parallels. and conſequently HI,
G; and GH, FI are alſo parallels. therefore it is apparent that the triangles ABD, AEG, EBF, FDG,
HIK, [b] are equiangular. and that the four laſt are b 29.1.
equall: in like manner the triangles ACB, AHE, c 26.1.
EIB, HIC, FGK are equiangular; and the four laſt
are equall one to the other. Alſo the triangles
BFI, FDK, IKC, EGH; & laſtly the triangles AHG,
GDK, HKC, EFI are like and equall. Moreover the
triangles, HIK to ADB, and EGH to BDC, and
EFI to ADC, and FGK to ABC, [d] are parallel. d 15.11.
From whence it evidently follows, firſt that the pyramids AEGH, HIKC are equall, and [e] like to the e 10.def.11.
whole ABDC, and to one another. Next, that the
ſolids BFGEIH, FGDIHK are priſmes, and that of
equall height, as being placed between the parallel
planes ABD, HIK. but the baſe BFGE is [f] double f 2.ax.1.
of the baſe FDG. wherefore the ſaid priſmes are e- g 40.11.
quall; whereof the one BFGEIH is greater then the
pyramide BEFI, that is, then AEGH, the whole then
its part; & conſequently the two priſmes are greater
then the two pyramids and ſo exceed the half of the
whole pyramide ABDC. *W.W. to be Dem.*

PROP. IV.

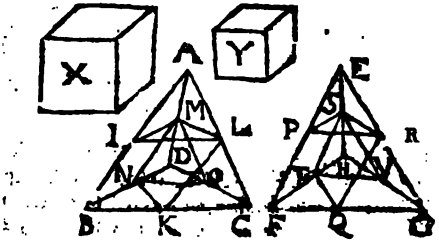


If there be two pyramids ABCD, EFGH, *of the ſame altitude, having triangular baſes* ABC, EFG, *and either of them be divided into two pyramids* (AI LM, MNOD; *and* EPRS, STVH) *equall one to th other and like to the whole, and into two equall priſme* (IBKLMN, KLCNMO; *and* PFQRST, QRGT SV;) *and if in like manner either of thoſe pyramid made by the former diviſion be divided, and this be don continually; then as the baſe of one pyramide is to the baſ of the other pyramide, ſo are all the priſmes which are i one pyramide, to all the priſmes which are in the othe pyramide, being equall in multitude.*

For (applying the conſtruction of the precedent
a 15.5. *prop.*) BC. KC *a* :: FG. QG. *b* therefore the triangle
b 22.6. c 2.6. &c. ABC is to the like triangle LKC as EFG is to *c* the
d 16.5. like RQG. therefore by permutation ABC. EFG *d* ::
e sch. 34. 11. f 7.5. LKC. RQG *e* :: the priſme KLCNMO. QRGTSV
(for theſe are of equall altitude) *f* :: IBKLMN.
g 12.5. PFQRST. *g* wherefore the triang. ABC. EFG :: the
priſm. KLCMNO + IBKLMN. the priſme QRGTSV + PFQRST. *W.W. to be Dem.*

But if the pyramids MNOD, AILM; and EPRS, STVH; be further divided in like manner the fou new priſmes made hereby ſhall be to the four produced before as the baſes MNO and AIL are to th baſes STV, and EPR; that is, as LKC to RQG, o as ABC to EFG. *b* wherefore all the priſmes of th pyramide ABCD are to all the priſmes of the pyramide EFGH as the baſe ABC is to the baſe EFG. *W.W. to be Dem.*

PROP. V.

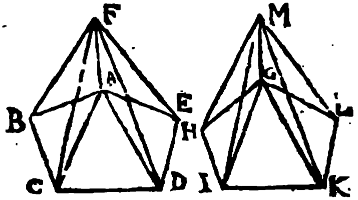


Pyramids ABCD, EFGH, *being under the ſame al-*
titude, having triangular baſes ABC, EFG, *are one to*
other as their baſes ABC, EFG; *are.*

Let the triangle ABC.EFG :: ABCD.X. I ſay X
equall to the pyramide EFGH. For if it be poſſi-
e, let X be $\sqsupset$ EFGH. and let the exceſſe be Y.
ivide the pyramide EFGH into priſmes and pyra-
ids, and the other pyramids in like manner, [a] till — a 11 10.
e pyramids left EPRS, STVH, be leſſe then the
lid Y. Therefore ſince the pyramide EFGH = X
- Y, it is manifeſt that the remaining priſmes PF-
RST, QRGTSV are greater then the ſolid X.
onceive the pyramide ABCD divided after the
me manner; [b] then will be the priſme IBKLMN + — b 4. 12.
LCNMO.PFQRST + QRGTSV :: ABC.EFG
: the pyr. ABCD.X. [d] therefore X $\sqsubset$ the priſme — c hyp.
FQRST + QRGTSV; which is contrary to that — d 14. 5.
hich was affirmed before.

Again, conceive X $\sqsubset$ the pyr. EFGH. and make
e pyr. EFGH. Y :: X. the pyr. ABCD [e] :: EFG. — e hyp. and
BC. Becauſe EFGH [f] $\sqsupset$ X, [g] thence Y $\sqsupset$ the — cor. 4 5.
r. ABCD. which is ſhewn before to be impoſſible. — f ſuppoſ.
herefore I conclude, that X is equall to the pyr. — g 14 5.
FGH. *W. W. to be Dem.*

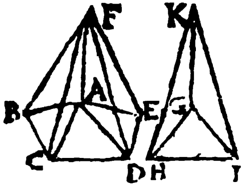
PROP. VI.



Pyramids ABCDEF, GHIKLM, *conſiſting under the ſame altitude, and having polygonous baſes* ABCDE, GHIKL, *are to one another as their baſes* ABCDE, FGHIKL *are.*

Draw the right lines AC, AD, GI, GK, then is the baſe ABC.ACD *a* :: the pyr. ABCF.ACDF. *b* therefore by compoſition, ABCD.ACD :: the pyr. ABCDF.ACDF. *a* but alſo ACD.ADE :: the pyr. ACDF. ADEF. *c* therefore of equality ABCD. ADE :: ABCDF. ADEF. and *b* thence by compoſition ABCDE.ADE :: the pyr. ABCDEF. ADEF. moreover ADE. GKL *d* :: the pyr. ADEF. GKLM; and, as before, and inverſely GKL. GHIKL :: the pyr. GKLM.GHIKLM. *c* therefore again of equality ABCDE. GHIKL :: the pyr. ABCDEF. GHIKLM. *W. W. to be Dem.*

a 5. 12.
b 18. 5.
c 22. 5.
d 5. 12.

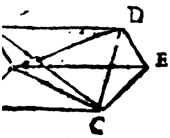


If the baſes have not ſides of equall multitude, the demonſtration will proceed thus. The baſe ABC.GHI *e* :: the pyr. ABCF. GHIK. *e* & ACD. GHI :: the pyr. ACDF. GHIK. *f* therefore the baſe ABCD. GHI :: the pyr. ABCDF. GHIK. *e* Moreover the baſe ADE.GHI :: the pyr. ADEF.GHIK. *f* therefore the baſe ABCDE. GHI :: the pyr. ABCDEF.GHIK.

e 5. 12.
24. 5.

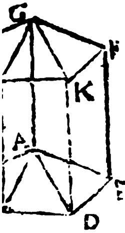
PRO

PROP. VII.



Every priſme , ABCDEF. *having a triangular baſe,may be divided into three pyramids* ACBF , ACDF , CDFE,*equall one to the other, &* *having triangular baſes.*

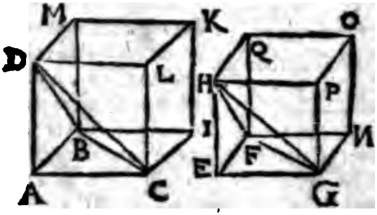
aw the diameters of the parallelograms , AC, FD. Then the triangle ACB is [a] = ACD. a 34. 1. b 5. 12.
efore the pyramids of equall height ACBF, F. are equall. In like manner the pyr. DFAC e pyr.DFEC. but ACDF and DFAC are one ſame pyramide.[c] therefore the three pyramids c 1. ax. 1.
F, ACDF, DFEC , into which the priſme is d,are equall one to the other. *W.W.to be Dem.*



Hence , every pyramide is the third part of the priſme that ha's the ſame baſe and height with it; or every priſme is treble of the pyramide that ha's the ſame baſe and height with it.

For reſolve the polygonous priſme ABCDEGHIKF into triangular priſmes ; and the pyramide ABCDEH into trian-
pyramides ; [a] then all the parts of the priſme a 7. 12.
e treble to all the parts of the pyramide,[b] con- b 1. 5.
tly the whole priſme ABCDE GHIKF is tre-
the whole pyramide ABCDEH. *W. W. to be*

PROP. VIII.



Like pyramids ABCD, EFGH, *which have triangular baſes* ABC , EFG *are in triple proportion of that in which their ſides of like proportion* AC, EG, *are.*

a 27. 11. *a* Complete the parallelepipedons ABICDMKL,
b 9 *def.* 11. EFNGHQOP, which *b* are like, and *c* ſextuple of the
c 28. 11. and 7. 12. pyramids ABCD , EFGH. *d* and therefore in the
d 15. 5. ſame proportion with them one to another, *e* that is,
e 33. 11. triple of that of the ſides of like proportion , &c.

Coroll.

Hence, alſo like polygonous pyramids have proportion tripled to that of the ſides of like proportion; as may eaſily be proved by reſolving the ſame into triangular pyramids.

PROP. IX.

See the prec. Scheme.

In equall pyramids ABCD, EFGH , *having triangular baſes* ABC , EFG , *the baſes and altitudes are reciprocall ; And pyramids having triangular baſes , whoſe altitudes and baſes are reciprocall , are equall.*

1. *Hyp.* The completed parallelepipedons ABIC-
a 28. 11. *and* 7. 12. DMKL, EFNGHQOP are *a* ſextuple of the equall
pyramids ABCD , EFGH (either of either) and ſo

equall one to the other. therefore the altitude (H.) the alt.(D) *b* :: ABIC. EFNG *c* :: ABC.EFG. *W.W. to be Dem.* b 34. 11. c 15. 5.

2. *Hyp.* The altitude (H.) the alt. (D) *d* :: ABC. EFG *e* :: ABIC. EFNG. *f* therefore the parallelepipedons ABICDMKL, EFNGHQOP are equall. *g* consequently also the pyramids ABCD, EFGH being subsextuple of the same, are equall. *W.W. to be Dem.* d *hyp.* e 15. 5. f 34. 11. g 6. ax. 1.

The ſame is applicable to polygonous pyramids; for they may alſo in like manner be reduced to triangulars.

Coroll.

Whatſoever is demonſtrated of pyramids in prop. 6, 9. *do's likewiſe agree to any ſort of priſmes; ſeeing hey are triple of the pyramids that have the ſame baſe nd altitude with them.* Therefore.

1. The proportion of priſmes of equall altitude s the ſame with that of their baſes.

2. The proportion of like priſmes is triple of hat of the ſides of like proportion.

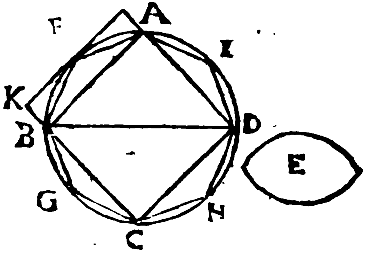
3. Equall priſmes have their baſes and altitudes eciprocall; and priſmes which are ſo reciprocall, re equall.

Schol.

From what is hitherto demonſtrated the dimenion of any priſmes and pyramids may be colected.

a The ſoliditie of a priſme is produced of the altitude multiplyed into the baſe; *b* and therefore likewiſe that of a pyramide, of the third part of the altitude multiplyed into the baſe. a 1. *cor* 12. *and ſch.* 40. 11. b 7. 12.

PROP. X.



Every Cone is the the third part of a cylinder having the ſame baſe with it ABCD, *and the altitude equall.*

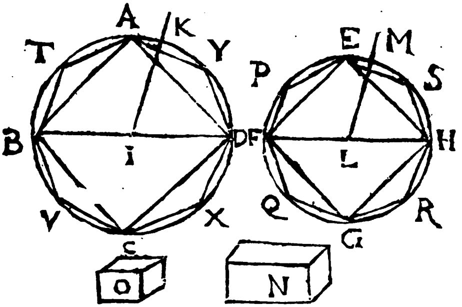
If you deny it, then firſt let ſuch cylinder be more then triple to the cone, and let the exceſſe be E. A priſme deſcribed on a ſquare in the circle ABCD *a* is ſubduple of a priſme deſcribed upon a ſquare about the circle, being equall to it and the cylinder in height. Therefore a priſme upon the ſquare ABCD exceeds the half of the cylinder. and likewiſe a priſme upon the baſe AFB, of equall height to the cylinder, *b* is greater then the half of the ſegment of the cylinder AFB. continue an equall biſection of the arches, & ſubſtract the priſmes till the remaining ſegments of the cylinder, namely at AF, FB, &c. become leſs then the ſolid E. Therefore the cyl.— ſegm. AF, FB, &c. (the priſme on the baſe AFBGCHDI) *c* is greater then the cylinder — E (*d* the triple of the cone.) therefore the pyramide, *e* a third part of the ſaid priſme (being placed on the ſame baſe, and of the ſame height) is greater then the cone of equall height on the baſe ABCD a circle, *i. e.* the part greater then the whole. *Which is Abſurd.*

See the ſecond fig. of this Book.
a *ſch 7 4. and cor 9. 12.*
b *ſch. 27. 3. and cor. 9. 12.*
c *5. ax. 1.*
d *hyp.*
e *cor. 7. 12.*

But if the cone be affirmed to be greater then the third part of the cylinder, then let the exceſſe be E. Detract the pyramids from the cone, as you did in the firſt part the priſmes from the cylinder, till ſome ſegments of the cone remain, conceive at AF, FB, BG,&c. leſſe then the ſolid E: therefore the cone — E (*f* $\frac{1}{3}$ of the cylin.) ⊐ the pyr. AFBGCHDI (the cone — ſegm. AF, FB, &c.) therefore the priſme triple to the pyramide (*viz*, of equall height, and on the ſame baſe) is greater then the cylinder on the baſe ABCD, the part then the whole. *Which is Abſ.* Wherefore it muſt be granted, that the cylinder is equall to triple of the cone. *W.W. to be Dem.*

f *hyp.*

PROP. XI.



Cylinders and Cones ABCDK, EFGHM, *being under the ſame altitude, are to one another as their baſes* ABCD, EFGH *are.*

Let the circle ABCD. the cir. EFGH :: the cone ABCDK. N. I ſay N is equall to the cone EFGHM.

For if it be poſſible, let N be ⊐ the cone EFGHM, and let the exceſſe be O. The preparation and argumentation of the prec. prop. being ſuppoſed; then ſhal O be greater then the ſegments of the cone EP, PF, FQ, &c. and ſo the ſolid N ⊐ the pyr. EPFQGRHSM. In the circle ABCD *a* make a like polygonous fig. ATBVCXDY. Becauſe the pyr. ABVYK. the pyr. EFQSM *b* :: the polyg. ATBVY. the polyg. EPFQS *c* :: the cir. ABCD. the cir. EFGH *d* :: the cone ABCDK. N. *e* thence the pyr. EPFQGRHSM *ſhall be* ⊐ N, contrary to what was affirmed before. Again conceive N ⊏ the cone EFGHM.

a 30. 3 *and* 1. *poſt.*
b 6. 12.
c *cor.* 2. 12.
d *hyp.*
e 14. 5.

& make the cone EFGHM. O :: N. the cone ABCDK *f* :: the circ. EFGH. ABCD. *g* therefore O ⊐ the cone ABCDK; *which is abſurd*, as appears by by what is ſhewn in the firſt part.

f hyp. and by inverſion.

g 14. 5.

Therefore rather admit ABCD.EFGH :: the cone ABCDK.EFGHM. *W.W. to be Dem.*

The ſame may be demonſtrated of cylinders, if cylinders and priſmes be conceived in the place of cones and pyramids. therefore, &c.

Schol.

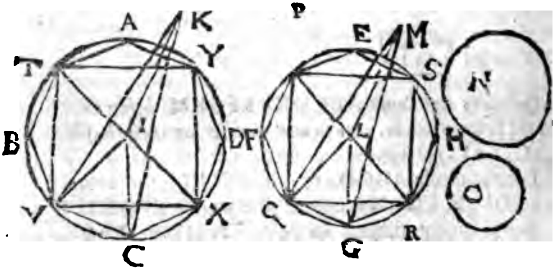
Hence, is gathered the dimenſion of all ſorts of cylinders and cones. The ſoliditie of a right cylinder is produced of the circular baſe (*a* the dimenſion whereof is to be learnt out of *Archimedes*) multiplied into the height; *b* whence in like manner that of every cylind.

a 1. Prop. de dimenſ. circ

b 11. 12.

Therefore the ſoliditie of a cone is produced of the third part of the altitude multiplyed into the baſe.

PROP. XII.



Like cones and cylinders ABCDK, EFGHM, *are in triple proportion of that of the diameters* TX, PR, *of their baſes* ABCD, EFGH.

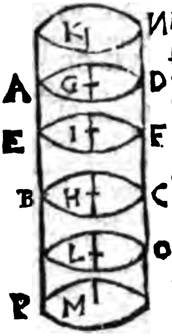
Let the cone A have to N triple proportion of TX to PR. I ſay N is = the cone EFGHM. For if it be poſſible let N be ⊐ EFGHM. and let the exceſſe be O. therefore, N ⊐ the pyr. EPFQGRHSM. Let the axes of the cones be IK, LM, and join the right lines VK, CK, VI, CI, and

QM, GM, QL, GL. Becauſe the cones are like,
[a] thence VI. IK :: QL. LM. but the angles VIK, a 14 *def.* 11.
QLM [b] are right angles. [c] therefore the triangles b 18. *def.* 11.
VIK, QLM are equiangul. [d] whence VC. VI :: QG. c 6. 6.
QL. alſo VI. VK :: QL. QM. therefore of equality d 4. 6.
VC. VK :: QG. QM. [e] moreover VK. CK :: QM. e 7. 5.
MG. therefore again of equality VC. CK :: QG. GM.
therefore the triangles VKC, QMG are like: and f 5. 6.
y the ſame reaſon the other triangles of this pyra-
nide are like to the other of that. [g] wherefore the py- g 9. *def.* 11.
amids themſelves are like. [h] But they are in triple h *cor.* 8. 12.
roportion of that of VC to QG, [k] that is, of VI to
RL, [l] or TX to PR. [m] therefore the pyr. AIBVCXD- k 4. 6.
YK. the pyr. EPFQGRHSM :: the cone ABCDK. l 15. 5.
N. [n] whence the pyr. EPFQGRHSM $\sqsupset$ N. which m *hyp.* and 11. 5.
s repugnant to what was affirmed before. n 14. 5.

Again, take N $\sqsubset$ the cone EFGHM. make the
one EFGHM. O :: N. the cone ABCDK [o] :: the
yr. EPRM. ATCK [p] :: GQ. VC thrice :: [q] PR. TX o *before.* & *inverſely.*
hrice. but O [r] is $\sqsupset$ ABCDK. which was before p *cor.* 8. 12.
hewn to be repugnant. Wherefore N = the cone q 4. 6.
EFGHM. *W. W. to be Dem.* r 14. 5.

But foraſmuch as what proportion ſoever cones
ave, alſo cylinders, being triple of them, have the
ame; therefore cylinder to cylinder ſhall have pro-
ortion triple of the diameters of the baſes.

PROP. XIII.

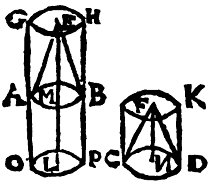


If a cylinder ABCD *be divided by a plane* EF *parallel to the oppoſite planes* BC, AD, *then as one cylinder* AEFD *is to the other cylinder* EBCF, *ſo is the axis* GI *to the axis* IH.

The axis being produced, *a* take GK = GI, and HL = IH = LM. and conceive planes drawn at the points K, L, M, parallel to the circles AD, BC, *b* therefore the cylinder FD = the cyl. AN. and the cyl. EC *b* = BO *b* = OP. therefore the cylinder EN is as multiplex of the cylinder ED as the axis IK is of the axis IG. and in like manner the cylinder FP is as multiplex of the cylinder BF, as the axis IM is of the axis IH. but as IK is $=, \sqsubset, \sqsupset$ IM, *c* ſo is the cylinder EN $=, \sqsubset, \sqsupset$ FP. *d* therefore the cylinder AEFD. the cyl. EBCF :: GI. IH. *W. W. to be Dem.*

a 3. 1.
b 11. 12.
c 11, 12.
d 6. *def.* 5.

PROP. XIV.



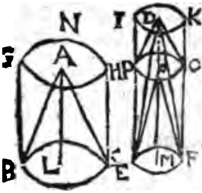
Cones AEB, CFD, *and cylinders* AH, CK, *conſiſting upon equall baſes* AB, CD, *are to one another as their altitudes* ME, NF *are.*

The cylinder HA, and the axis EM being produced, take ML = FN; and at the point L draw a plane parallel to the baſe AB. *a* then ſhall the cyl. AP be = CK. *b* but the cyl. AH. AP (CK) :: ME. ML (NF.) *W. W. to be Dem.*

The ſame may be affirmed of cones ſubtriple of cylinders; * as alſo of priſmes and pyramids.

a 11. 12.
b 13, 12.
* *apply* 9, *and* 7. 12.

PROP. XV.



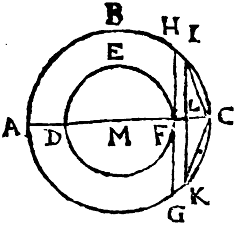
In equall cones BAC, EDF, *and cylinders* BH, EK, *the baſes and altitudes are reciprocall* (BC. EF :: MD. LA:) *And cones and cylinders, whoſe baſes and altitudes are reciprocall, are equall one to the other.*

If the altitudes be equall then the baſes are equall too, nd the thing is evident. If unequall, then take away O = LA.

1. *Hyp.* Then is MD. MO ([a] LA) [b] :: the cyl. a 14. 12.
K. ([c] BH) EQ [d] :: the cir. BC. EF. *Which was* b *conſtr.* c *hyp.*
be Dem. d 11. 12.
2. *Hyp.* BC. EF [e] :: DM. OM (LA) [f] :: the cyl. EK. e *hyp.* f 11. 12.
Q [g] :: BC. EF [h] :: BH. EQ. [k] Therefore the cylind. g 11. 5.
K = BH. *Which was to be Dem.* h 11. 12. k 9. 5.

The ſame argument may be uſed for cones.

PROP. XVI.



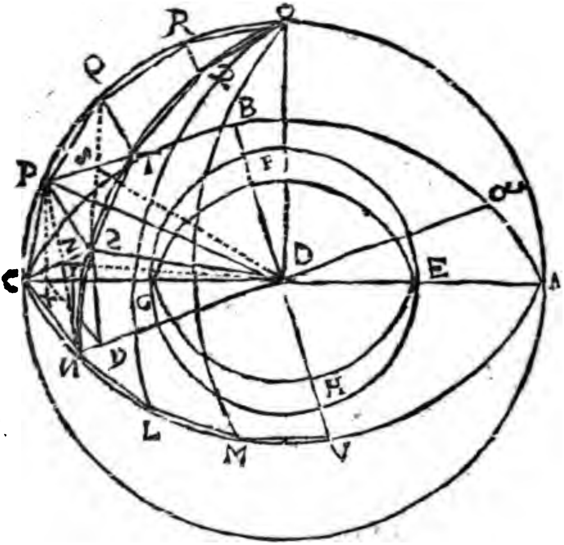
Two unequall circles ABCG, DEF, *having the ſame centre* M, *to inſcribe in the greater circle* ABCG *a polygonous figure of equall and even ſides, which ſhall not touch the leſſer circle* DEF.

Through the center M draw the line AC cutting he circle DEF in F, from whence raiſe a perpendicular FH. [a] divide the ſemicircle ABC into two e- a 30. 3.
uall parts; and the half thereof BC alſo; and ſo do
ontinually, [b] till the arch IC become leſſe then the b 1. 10.
rch HC. from I let fall the perpendicular IL. It

is manifeſt that the arch IC meaſures the whole circle, and that the number of arches is even, and ſo that the ſubtended line IC is the ſide[e] of the poly-
c ſch. 16.4. gonon that may be inſcribed without touching the
d cor 16.3. leſſer circle DEF. For HG[d] touches the circle DEF,
e 28.1. *e* to which IK is parallel, and placed outwardly;
f 34.def.1. *f* whereſore IK does not touch the circle DEF; much leſſe do CI, CK, and the other ſides of the polygonon more remote from the center. *W. W. to be Done.*

Coroll. Obſerve that IK touches not the circle DEF.

PROP. XVII.



Two ſpheres ABCV, EFGH, *conſiſting about the ſame center* D, *being given, to inſcribe a ſolide of many ſides (or Polyedron) in the greater ſphere* ABCV, *which ſhall*

ſhall not touch the ſuperficies of the leſſer ſphere
EFGH.

Let both the ſpheres be cut by a plane paſſing by
the center making the circles EFGH, ABCV; and
the diameters AC, BV drawn, cutting perpendicu-
larly. In the circle ABCV, [a] inſcribe the equilaterall a 16. 12.
polygone VMLNC, &c. not touching the circle
EFGH: then draw the diameter Nα, and erect DO
perpendicular to the plane ABC. by DO, and by
the diameters AC, Nα, conceive planes DOC,
DON erected, which ſhall be [b] perpendicular to the b 18. 11.
circle ABCV, and ſo in the ſuperficies of the ſphere
make [c] the quadrants DOC, DON. In which let c cor. 33. 6.
the right lines CP, PQ, QR, RO, NS, ST, Tγ, γO
[d] be fitted, equall, and of equall multitude with CN, d 4. 1.
NL, &c. make the ſame conſtruction in the other
quadrants OL, OM, &c. and in the whole ſphere.
Then I ſay the thing required is done.

From the points P, S, to the plane ABCV draw
the perpendiculars PX, SY, [e] which ſhall fall on the e 38. 11.
ſections AC Nα. Therefore becauſe both [f] the right f 12. ax.
angles PXC, SYN, [g] and PCX, SNY inſiſting on g 27. 3.
[h] equall circumferences, [f] are equall, the triangles h 32. 1.
alſo PCX, SNY [h] are equiangular. Wherefore being
PC [k] = SN, [l] alſo is PX = SY, [l] and XC = YN; k conſtr.
[m] whence DX = DY. [n] and therefore DX. XC :: l 26. 1.
DY. YN. [o] therefore YX, NC are parallels. but be- m 3. ax. 1.
cauſe PX, SY are equall, and ſince being perpendicu- n 7. 5.
lar to the ſame plane ABCV, they are alſo [p] paral- o 2. 6.
lels, [q] therefore YX, SP ſhall be equall and parallels. p 6. 11.
[r] whence SP, NC, are parallel one to the other; and q 33. 1.
ſo the [ſ] quadrilaterall NCPS, and by the ſame rea- r 9. 12.
ſon SPQT, TQRG, as alſo the [t] triangle γRO are ſ 7. 11.
ſo many planes. In like manner the whole ſphere t 2. 11.
may be ſhewn full of ſuch quadrilateralls and trian-
gles. wherefore the figure inſcribed is a polyedron.

From the center D [u] draw DZ perpendicular to
the plane NCPS; and join ZN, ZC, ZS, ZP. Be- u 11. 11.
cauſe DN. NC [x] :: DY. YX. thence NC is [y] $\sqsubset$ YX x 4. 6.
y 14. 5.

(SP,) and likewiſe SP ⊏ TQ, and TQ ⊏ γI
And becauſe the angles DZC, DZN, DZS, DZ
z 3 def. 11. z are right, and the ſides DC, DN, DS, DP, a equa
a 15 def. 11. b 47. 1. and DZ common, b thence ZC, ZN, ZS, ZP are
quall one to the other; and conſequently about t
c 15. def. 4. 1 quadrilarerall NCPS c a circle may be deſcribed,
d conſtr. which (becauſe NS, NC, CP, are d equall, and N
e 28. 3. ⊏ SP) NC e ſubtends more then the quadran
f 31. 9. f therefore the ang. NZC at the center is obtu
g 12. 2. g therefore NCq ⊏ 2 ZCq (ZCq + ZNq.) I
NI be drawn perpendicular to AC. therefore ſir
h 32. 1. the angle ADN (h DNC + DCN) k is obtuſe, t
k 9. ax. 1. half of it DCN ſhall be greater then the half o
l 5. 1. right angle; and ſo that which remains of the rig
n 19. 1. ang. CNI ſhall be leſſe then it. n whence IN ⊏ I
o 47. 1 therefore NCq (NIq + ICq) o ⊐ 2 INq. the
p 47. 1. q cor. 19. 12. fore IN ⊏ ZC. and conſequently DZ p ⊏ D
but the point I is q without the ſphere EFGH. &
much more the point Z. wherefore the plane N
r 47. 1. PS, (whoſe r next point to the center is Z) do
not touch the ſphere EFGH. And if a perpendic
lar Dδ be drawn to the plane SPQT, the point δ,
ſo alſo the plane SPQT is yet further remov
from the center. which is alſo true of the otl
planes of the polyedron. Therefore the polyedr
ORQPCN, &c. inſcribed in the greater ſphere, do
not touch the leſſer. *W. W. to be Done.*

Coroll.

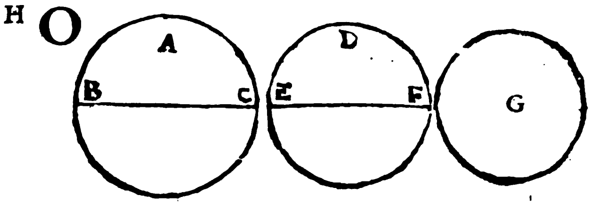
Hence it follows, *that if in any other ſphere a ſo
polyedron, like to the abovesaid ſolid polyedron, be
ſcribed, the proportion of the polyedron in one ſpher
the polyedron in the other is triple of that of the dia
ters of the ſpheres.*

For if right lines be drawn from the center
the ſpheres to all the angles of the baſes of the ſ
polyedrons, then the polyedrons will be divided
to pyramids equall in number and like; whoſe

n

mologous ſides are ſemidiameters of the ſpheres; as appears, if the leſſer of theſe ſpheres be conceived deſcribed within the greater about the ſame center, For the right lines drawn from the center of the ſphere to the angles of the baſes will agree one to the other by reaſon of the likeneſſe of the baſes; and ſo will like pyramids be made. Wherefore ſince every pyramide in one ſphere to every pyramide like it in the other ſphere [a] ha's proportion triple to that[a] *cor.* 8. 12. of the homologous ſides, that is, of the ſemidiameters of the ſpheres; and [b] as one pyramide is to one [b] 12. 5. pyramide, ſo all the pyramids, that is, the ſolid polyedron compoſed of theſe, are to all the pyramids, that is, the ſolid polyedron compoſed of the others; therefore the polyedron of one ſphere ſhall have to the polyedron of the other ſphere, proportion triple of that of the ſemidiameters, [c] and ſo of the diameters of the ſpheres. [c] 15. 5.

PROP. XVIII.



Spheres BAC, EDF, *are in triple proportion one to the other of that in which their diameters* BC, EF, *are.*

Let the ſphere BAC be to the ſphere G in triple proportion of that of the diameter BC to the diameter EF. I ſay G = EDF. For if it be poſſible, let G be $\sqsupset$ EDF. and conceive the ſphere G concentricall with EDF. In the ſphere EDF [a] inſcribe a [a] 17. 12. polyedron not touching the ſphere G, and a like po-

polyedron in the ſphere BAC. Theſe polyedrons [b] are in triple proportion of the diameters BC, EF, [c] that is, of the ſphere BAC to G. [d] Conſequently the ſphere G is greater then the polyedron inſcribed in the ſphere EDF, the part then the whole.

Again, if it be poſſible, let the ſphere G be $\sqsubset$ EDF. and as the ſphere EDF is to another ſphere H, ſo let G be to BAC, [e] that is, in triple proportion of the diameter EF to BC. therefore ſince BAC [f] $\sqsubset$ H, we ſhall incurre the abſurdity of the firſt part. wherefore rather the ſphere G $=$ EDF. *W.W. to be Demonſtrated.*

[b] *cor.* 17.12.
[c] *hyp.*
[d] 14.5.
[e] *hyp. inverſ.*
[f] 14.5.

Coroll.

Hence, As one ſphere is to another ſphere, ſo is a polyedron deſcribed in that to a like polyedron deſcribed in this.

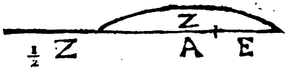
The End of the twelfth Book.

THE

THE THIRTEENTH BOOK OF EUCLIDE'S ELEMENTS.

PROP. I.

If a right line z *be divided according to extream and mean proportion* (z.a :: a.e.) *the ſquare of the half of the whole line* z, *and of the greater ſegment* a, *as one line, is quintuple to that which is deſcribed of half of that whole line* z.



I ſay Q. $a + \frac{1}{2} z = 5$ Q: $\frac{1}{2}$ z. [a] that is, $aa + \frac{1}{4} zz + za = zz + \frac{1}{4} zz$. [b] or $aa + za = zz$. For $ze + za$ [c] $= zz$. and ze [d] $=$ aa. [e] therefore $aa + za = zz$. *W.W. to be Dem.*

a 4. 2.
b 3. ax. 1.
c 2. 2.
d hyp. and 16. 6.
e 2. ax. and 1. ax.

PROP. II.

See the 1. *Scheme.*

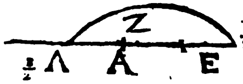
If a right line $\frac{1}{2}$ z + a *be in power quintuple to a ſegment of it ſelf* $\frac{1}{2}$ z. *the line double of the ſaid ſegment* (z) *being divided according to extreme and mean proportion, the greaer ſegment is* (a) *the other part of the right line at firſt given* $\frac{1}{2}$ z + a.

I ſay z.a :: a.e. For becauſe by the hyp. [*] $aa + \frac{1}{4} zz + za = zz + \frac{1}{4} zz$; or $aa + za = zz$ [a] $= ze + za$, [b] thence ſhall aa be $=$ ze. [c] wherefore z. a :: a. e. *W.W. to be Dem.*

* 4. 2.
a 2. 2.
b 3. ax. 1.
c 17. 6.

PROP. III.

If a right line z *be divided according to e*
mean proportion (z. a :: a.e.) *the line made*
ſegment e *and half of the greater ſegment* a,
quintuple to the ſquare, which is deſcribed
line of the greater ſegment a.

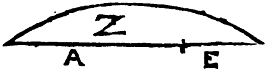


a 4. 2.
b 3. ax.
c 3. 2.
d hyp. and 17. 6.

I ſay Q:e + $\frac{1}{2}$ a. a that is ee
ea = aa + $\frac{1}{4}$ aa
ea = aa. For ee + ea c = ze d = aa. I
to be Dem.

PROP. IV.

If a right line z *be cut according to extrem*
proportion (z.a :: a.e) *the ſquare made o*
line z, & *that made of the leſſer ſegment* e, *b*
are triple of the ſquare made of the greater ſ



a 4. 1.
b 3. 2.
c 17. 6.
d 2. ax.

I ſay zz
aa. a or aa
ae + ee =
ae + ee b =
aa. d therefore aa + 2 ae + 2 ee = 3 aa
be Dem.

PROP. V.

If a right
cut according
and mean prop
and a line AD, *equall to the greater ſegment*
to it, the whole right line DB *is divided acc*
treme and mean proportion; and the great
the right line AB *given at the beginning.*

a hyp.

For becauſe AB.AD a :: AC. CB. an
ſion AD.AB :: CB. AC. therefore by c
DB.AB :: AB.AC (AD.) *W.W. to be De*

Schol.

But if **BD**. B**A** :: BA. **AD**. then ſhall be **BA**. **AD** :: **AD**. BA − AD. For by diviſion is BD − **BA** (AD.)BA :: BA − AD.AD.therefore inverſely **BA**. **AD** :: AD.**BA** − **AD.**

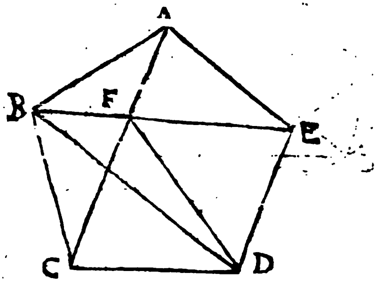
PROP. VI.

If a rationall right line AB *be cut according to extreme and mean proportion in* C, *either of the ſegments* (AC, CB) *is an irrationall line of that kind which is called apotome or reſiduall.*

To the greater ſegment AC [a] adde AD $= \frac{1}{2}$ AB. [b] therefore DCq = 5 DAq. [c] therefore DCq $\sqcap\!\sqcup$ DAq. conſequently [d] ſince AB, [e] and ſo the half thereof DA are ρ, likewiſe DC is ρ. But becauſe 5. 1 :: not Q.Q. [f] thence is DC $\sqcap\!\sqcup$ DA. [g] therefore DC − AD, that is, AC, is a reſiduall line. Further, becauſe ACq [h] = AB x BC, and AB is ρ, [k] likewiſe BC is a reſiduall line. *W.W.to be Dem.*

a 3. 1.
b 1. 13.
c 6. 10.
d *hyp.*
e *ſch.* 12. 10.
f 9. 10.
g 74. 10.
h 17. 6.
k 98. 10.

PROP. VII.



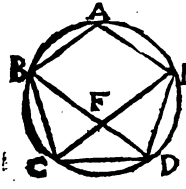
If three angles of an equilaterall Pentagone ABCDE, *whether they follow in order*, (EAB, ABC, BCD,) *or not*, (EAB, BCD, CDE) *be equall, the pentagone* ABCDE *ſhall be equiangular.*

Let the right lines BE, AC, BD, be
the equall angles in order.

Being the ſides EA, AB, BC, CD,
a hyp. b 4. 1. cluded angles [a] are equall, [b] therefore ſh
c 4 and 5. 1. BE, AC, BD, [c] & the angles AEB, ABE,
d 6. 1. e 3. ax. 1 be equall. [d] wherefore BF = FA, [e] and
ly FC = FE; therefore the triangles
f 8. 1. are equilaterall one to the other: [f] whe
g 2. ax. 1. FCD = FED. [g] conſequently the a
BCD. In like manner the ang. CDE is
reſt; wherefore the pentagone is equian
be Dem.

But if the angles EAB, BCD, CDE, v
h 4. 1. in order, be ſuppoſed equall, [h] then ſh
k 5. 1. AEB be = BDC, and BE = BD. [k] a
l 2. ax. ang. BED = BDE. [l] conſequently th
AED = CDE. therefore becauſe th
D, in order, are equall, as before, the pe
be equiangular. *W. W. to be Dem.*

PROP. VIII.

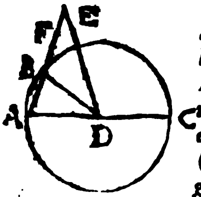


If in an equilat
angular Pentagone
right lines BD, C
angles BCD, CD
order, thoſe lines do
according to extre
proportion; and the
ments BF *or* EF *a*
ſide of the Pentagone BC.

[a] Deſcribe about the pentagon
a 14. 4. b 28. 3. ABD. [b] The arch ED is = BC, [c] there
c 27. 3. FCD = FDC. [d] therefore the ang
d 32. 1. e 33. 6. FCD (FCD + FDC.) But the arch
ED, and conſequently the angle BCF
f 6. 1. = BFC. [f] wherefore BF = BC. *Which*
Moreover becauſe the triangles BCD,
g 27. 3. h 4. 6. [g] equiangular, [h] therefore BD. DC. (

(BF.) FD. and likewiſe EC. EF :: EF. FC. *W.W. to be Dem.*

PROP. IX.



If the ſide of an Hexagone BE, *and the ſide of a Decagone* AB, *both deſcribed in the ſame circle* ABC, *be added together the whole right line* AE *is cut according to extreme and mean proportion* (AE. BE :: BE. AB,) *and the greater ſegment thereof is the ſide of the Hexagone* BE.

Draw the diameter ADC, and join the right lines DB, DE. Becauſe the ang. BDC a = 4 BDA, and the ang. BDC b = 2 DBA (DAB + DBA) thence ſhall DBA (b BDE + BED) c be = 2 BDA d = 2 BDE. whence the ang. DBA or DEB e = ADE. Therefore the triangles ADE, ADB, are equiangular: f wherefore AE. AD (g BE) :: AD. (BE.) AB. *W.W. to be Dem.*

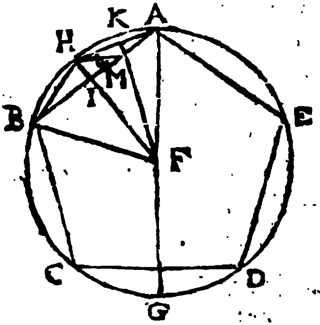
a hyp. and 27. 3.
b 32. 1.
c 7 ax. 1.
d 5. 1.
e 1 ax. 1.
f 4. 6.
g cor. 15. 4.

Coroll.

Hence, If the ſide of a hexagone in a circle be cut according to extreme and mean proportion; the greater ſegment thereof ſhall be the ſide of the Decagone in the ſame circle.

PROP. X.

If an equilaterall Pentagone ABCDE *be deſcribed in a circle* ABCE, *the ſide of the pentagone* AB *containeth in power both the ſide of a hexagone* FB, *and the ſide of a decagone* AH *deſcribed in the ſame circle.*



Draw the diameter AG. and biſect equally the arch AH in K. and draw FK, FH, FB, BH, HM.

a 28.3. and 3. ax. The ſemicircle AG — the arch AC [a] $= AG - AD$. that is, the arch CG [b] $= GD = AH = HB$. b hyp. and 7. ax. therefore the arch $BCG = 2\,BHK$; [c] and ſo the c 33.6. ang. $BFG = 2\,BFK$. [d] but the ang. $BFG = 2\,BAG$. d 20.3. [e] therefore the ang. $BFK = BAG$. Wherefore the e 1. ax. 1. f 32.1. triangles BFM, FAB, [f] are equiangular. [g] whence g 4.6. h 17.6. AB. BF :: BF. BM. [h] therefore $AB \times BM = BFq$. k 27.3. Moreover, the ang. AFK [k] = HFK, and FA = FH. m 4.1. [m] wherefore $AL = LH$, [m] and the angles FLA, FLH are equall, and ſo right angles. therefore the ang. n 27.3. LHM [m] = LAM [n] = HBA. therefore the trian- o 32.1. p 4.6. gles AHB, AMH, [o] are equiangular. wherefore AB. q 17.6. AH :: AH. AM. [q] therefore $AB \times AM = AHq$. So r 2.2. ſ 2. ax. that ſeeing ABq [r] $= AB \times BM + AB \times AM$, [ſ] thence $ABq = BFq + AHq$. *W. W. to be Dem.*

Coroll.

1. Hence, a right line (FK) which being drawn from the center (F) divides an arch (HA) into two equall ſegments, do's alſo divide the right line (HA) ſub-

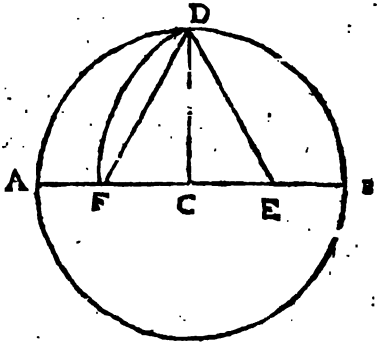
btending that arch, perpendicularly into two uall ſegments.

2. The diameter of a circle (A G) drawn from any gle (A) of a pentagone, do's divide equally in o both the arch (CD) which the ſide of the pengone oppoſite to that angle ſubtends, and al- the oppoſite ſide it ſelf (CD) and that perpendilarly.

Schol.

Here, according to our promiſe, we ſhall lay down a dy praxis of the 11. prop. *of the* 4. *Book.*

Probl.



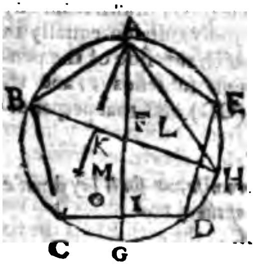
o find out the ſide of a pentagone to be inſcribed in a le AD 3.

Draw the diameter AB, to which erect a perpenılar CD at the center C, divide CB equally in ınd make EF = ED. then DF ſhall be the ſide he pentagone.

'or BF x FC + ECq [a] = EFq [b] = EDq [c] = q + ECq. [d] therefore BF x FC = DCq or BCq. ıerefore BF.BC :: BC. FC. therefore ſince BC ıe ſide of a hexagone, [f] FC ſhall be the ſide of a ıgone. Conſequently DF [h] = √ DCq + FCq :he ſide of a pentagone. *W.W. to be Done.*

a 6. 2.
b conſtr.
c 47. 1.
d 3. ax.
e 17. 6.
f 9. 13.
g 10. 13.
h 47. 1.

PROP. XI.



If in a circle ABCD, *whoſe diameter is rationall* AG, *an equilaterall pentagone be inſcribed* ABCDE; *the ſide of the pentagone* AB *is an irrationall line, of that kind which is called a minor line.*

Draw the diameter BFH, and the right lines AC, AH; and * make FL $= \frac{1}{4}$ of the ray FH; and CM $= \frac{1}{4}$ CA.

* 10. 6.

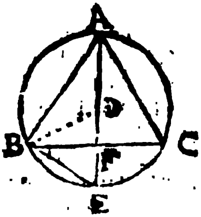
a cor. 10. 13.
b 32. 1.
c 4. 6.
d 16. 5.
e 18. 5.
f 22. 6.
g 1. 13.

Becauſe the angles AKF, AIC, are [a] right angles, & CAI commune, the triangles AKF, AIC, are [b] equiangular: [c] therefore CI. FK [c] :: CA. FA (FB) [d] :: CM. FL. therefore by permutation FK. FL :: CI. CM [d] :: CD. CK (2 CM) and ſo by [e] compoſition CD + CK. CK :: KL. FL. [f] conſequently Q: CD + CK ([g] 5 CKq.) CKq :: KLq. FLq. therefore KLq = 5 FLq. wherefore if BH (ρ́) be taken 8, FH ſhall be 4, FL 1, and FLq 1, BL 5, and BLq 25, KLq 5. by which it appears that BL and KL are ρ́ [h] ⊥, [k] and ſo BK is a reſiduall, and KL its congruent or adjoining line. but being BLq − KLq = 20, [l] thence BL ⊐⊏ √BLq − KLq. [m] whence BK ſhall be a fourth reſiduall line. Therefore becauſe ABq [m] is = HB x BK, [n] ſhall AB be a minor line. *Which was to be Dem.*

h 9. 10.
k 74. 10.
l 9. 10.
m cor 8. 6. and 17. 6.
n 95. 10.

Prop.

PROP. XII.



If in a circle ABEC *an equilaterall triangle* ABC *be inſcribed, the ſide of that triangle* AB *is in power triple to the line* AD *drawn from* D *the centre of the circle to the circumference.*

The diameter being extended to E, draw BE. Becauſe the arch BE a = EC, the arch BE is the ſixt part of the circumference. b therefore BE = DE. Hence AEq c = 4 DEq (4 BEq) d = ABq + BEq (+ ADq.) e conſequently ABq = 3 ADq. *W.W. to be Dem.*

a cor. 10, 13.
b cor. 15. 4.
c 4. 2.
d 47. 1.
e 3. ax. 1.

Coroll.

1. AEq. ABq :: 4. 3.

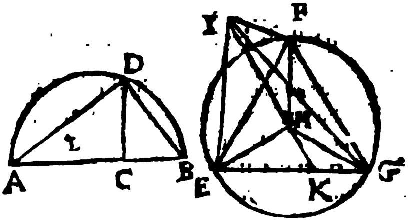
2. ABq. AFq :: 4. 3. f For ABq. AFq :: AEq. ABq.

3. DF = FE. For the triangle EBD g is equilaterall, h and BF perpendicular to ED. therefore EF = FD.

4. Hence, AF = DE + DF = 3 DF.

f cor. 8. 6. and 22. 6.
g cor. 15. 4.
h cor. 3. 3.

PROP. XIII.



To deſcribe a pyramide EGFI, *and comprehend it in a ſphere given: and to demonſtrate that the diameter of the ſphere* AB *is in power ſeſquialtera of the ſide* EF *of the pyramide* EGFI.

a 10. 6. About AB deſcribe the ſemicircle ADB; [a] and
let AC be = 2 CB. from the point C erect the perpendicular line CD; and join AD, DB. then at the
intervall of the ray HE = CD deſcribe the circle
b cor. 15. 4. HEFG. [b] wherein inſcribe the equilaterall triangle
c 12. 11. EFG. from H [c] erect IH = CA perpendicular to
d 3. 1. the plane EFG. produce IH to K, [d] ſo that IK =
AB; and join the right lines IE, IF, IG. Then EFGI
ſhall be the pyramide required.

For becauſe the angles ACD, IHE, IHF, IHG,
e conſtr. [e] are right angles; & CD, HE, HF, HG [e] equall, [e] &
f 47. 1. IH = AC; [f] therefore AD, IE, IF, IG ſhall be equall
g 20. 6. among themſelves. But being AC (2 CB.) CB [g] ::
ACq. CDq. thence ſhall ACq be = 2 CDq. there-
h 2. ax. fore ADq [f] = ACq + CDq [h] = 3 CDq = 3 HEq
k 12. 13. [k] = EFq. [l] therefore AD, EF, IE, IF, IG are equall,
l 1. ax. 1. and ſo the pyramide EFGI is equilateral. But if the
point C be placed upon H, and AC upon HI; the
m 8. ax. right lines AB, IK, [m] ſhall agree, as being equall.
Wherefore the ſemicircle ADB being drawn about
n 15. def. 1. the axis AB or IK [n] ſhall paſſe by the points E, F,
* 31. def. 11. G, [*] and ſo the pyramide EFGI ſhall be inſcribed
in a ſphere. *W. W. to be Done.*

o cor. 8. 6. Alſo it is manifeſt that BAq. ADq [o] :: BA. AC [p] ::
p conſtr. 3. 2. *W. W. to be Dem.*

Coroll.

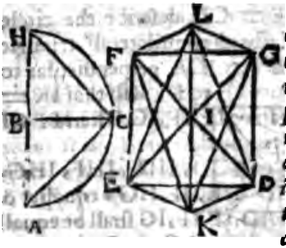
1. ABq. HEq :: 9. 2. For if ABq be put 9, then
q 12. 13. ACq (EFq) ſhall be 9. [q] conſequently HEq ſhall
be 2.

2. If L be the center, then ſhall AB. LC :: 6. 1.
r conſtr. For if AB be put 6, then AL ſhall be 3. [r] and thence
AC 4. wherefore LC ſhall be 1. Hence

3. AB. HI :: 6. 4 :: 3. 2. whence

4. ABq. HIq :: 9. 4.

PROP. XIV.



To deſcribe an Octaedron KEFGDL, *and comprehend it in the given ſphere, wherein a pyramide is: and to demonſtrate that* AH *the diameter of the ſphere is in power double of* AC *the ſide of that Octaedron.*

About AH deſcribe the ſemicircle ACH. and from the center B erect the perpendicular BC. draw AC, HC. then upon ED = AC a make the ſquare a 46. 1.
EFGD, whoſe diameters DF, EG, cut in the center I. from I draw IL = AB b perpendicular to the b 12. 11.
plane EFGD. produce IL, c till IK = IL. and join c 3. 1.
KE, KF, KG, KD, LE, LF, LG, LD; then ſhall KEFGDL be the Octaedron required.

For AB, BH, FI, IE, &c. being ſemidiameters of equall ſquares are equall one to the other. d whence d 4. 1.
the baſes LF, LE, FE, &c. of the rightangled triangles LIE LIF, FIE, &c. are equall, and conſequently the eight triangles LFE, LFG, LGD, LDE, KEF, KFG, KGD, KDE, are equilaterall, e and make an e 27. def. 11.
Octaedron, which may be inſcribed in a ſphere, whoſe center is I, and IL or AB the radius. (becauſe AB, IL, IF, IK, &c. f are equall.) *W. W. to be* f conſtr.
Done. Moreover, it is evident that AHq (LKq) g = g 47. 1.
2 ACq (2 LDq.) *W. W. to be Dem.*

Coroll.

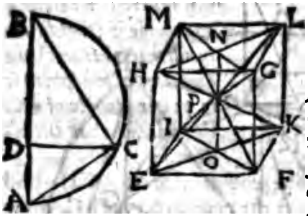
1. Hence it is manifeſt, that in the octaedron the three diameters EG, FD, LK doe cut one the other perpendicularly in the center of the ſphere.

2. Alſo that the three planes EFGD, LEKG, LFKD are ſquares, cutting one another perpendicularly.

3. The Octaedron is divided into two like and equall pyramids EFGDL, and EFGDK, whoſe common baſe is the ſquare EFGD.

15. 11. 4. Laſtly, it follows that the oppoſite baſes of the octaedron are parallel one to the other.

PROP. XV.



To deſcribe a cube EFGHIKLM, *and comprehend it in the ſame ſphere wherein the former figures were; and to demonſtrate that* AB *the diameter of the ſphere is in power triple to* EF *the ſide of that cube.*

a 10. 6. Upon AB deſcribe a ſemicircle ACB; a and make
b 46. 11. $AB = 3\,DA$ from D raiſe the perpendicular DC, & join BC and AC. Then upon $EF = AC$ b make the ſquare EFGH, upon whoſe plane let the right lines EI, FK HM, GL, ſtand perpendicular, being equall to EF, & connect them with the right lines IK, KL, LM, IM. The ſolid EFGHIKLM is a cube, as is ſufficiently apparent from the conſtruction.

In the oppoſite ſquares EFKI, HGLM, draw the diameters EK, FI, HL, MG, by which let the planes EKLH, FIMG be drawn, cutting one another in the
c cor. 39 11. line NO. which c ſhall divide equally in two parts
d 15 def. 11. and 14 def. 11. the diameters of the cube EL, FM, GI, HK, in P the center of the cube. d therefore P ſhall be the center of a ſphere paſſing by the angular points of the cube.
e 47. 1 Moreover, ELq $e = EKq + KLq$ $e = 3\,KLq$, f for
f conſtr
g cor. 8. 6. 3 ACq. but ABq. ACq g :: BA. DA f :: 3. 1. h there-
h 14 5. fore $AB = EL$. wherefore we have made a cube, &c. *W.W. to be Done.*

Coroll.

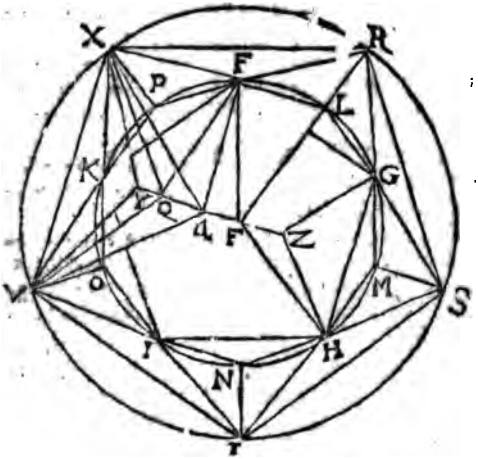
1. Hence it is manifeſt, that all the diameters of the cube are equall one to another, and do equally biſect one another in the center of the ſphere. And by the ſame means the right lines which conjoin the

centers of the oppoſite ſquares are equally biſected in the ſame center.

2. The diameter of a ſphere containeth in power the ſide of a tetraedron and of a cube, *viz.* ABq [k] = [l] BCq + [m] ACq.

k 47. 1.
l 13, 15.
m 15. 13.

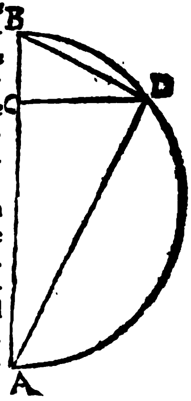
PROP. XVI.



To deſcribe an Icoſaedron ZGHIKFYVXRST, *and encompaſſe it in the ſphere, wherein were contained the foreſaid ſolids; and to demonſtrate that* FG *the ſide of the Iſocaedron is that irrationall line, which is called a minor line.*

Upon AB the diameter of a ſphere deſcribe the ſemicircle ADB; & [a] make AB = 5 BC. then from C erect CD perpendicular, and draw AD and BD. At the diſtance EF = BD deſcribe the circle EFKNG;

a 10. 6.



b 11.4 NG; b wherein inſcribe the equilaterall pentagone
FKIHG. Divide equally in two parts the arches FG,
GH, &c. and join the right lines FL, LG, &c. being
the ſides of a decagone. Then c erect EQ, LR, MS,
c 12.11. NT, OV, PX equall to EF, and perpendicular to the
plane FKNG; and connect RS, ST, TV, VX, XR; as
alſo FX, FR, GR, GS, HS, ST, HT, IT, IV, KV, KX.
Laſtly, produce EQ, and take $QY = FL$, and $EZ = FL$. and conceive the right lines ZG, ZH, ZI,
ZK, ZF to be drawn; as alſo YV, YX, YR, YS,
YT. Then I ſay the Icoſaedron required is
made.

d conſtr. For becauſe EQ, LR, MS, NT, OV, PX, are d e-
e 6.11. quall and e parallel, alſo thoſe lines that join them
f 33.1. EL, QR, EM, QS, EN, QT, EO, QV, EP, QX, f are
equall & parallel. And thence likewiſe LM (or FG)
g 15.11. RS, MN, ST, &c. are equall one to the other. g there-
h 1.def.3. fore the plane drawn by EL, EM, &c. is diſtant e-
qually from the plane paſſing by QR, QS, &c. h and
the circle QXRSTV drawn from the center Q is e-
quall to the circle EPLMNO; and RSTVX is an
equilaterall pentagone. But EF, EG, EH, &c. and
QX, QR, QS &c. being conceived to be drawn; then
k 47.1. becauſe $FRq \overset{k}{=} FLq + LRq$, l or $EFq \overset{m}{=} FGq$,
l conſtr.
m 10.13. n therefore FR, FG, and ſo all RS, FG, FR, RG, GS,
n ſch 48.1. GH, &c. ſhall be equall one to the other. and conſe-
and 1 ax. quently the ten triangles RFX, RFG, RGS, &c. are
equilaterall and equall. Moreover, becauſe XQY is a
o cor.14.11. o right angle, therefore $XYq \overset{p}{=} QXq + QYq \overset{q}{=}$
p 47.1.
q 10.13. VXq or FGq. wherefore XY; VX, and likewiſe YV,
YT, YS, YR, ZG, ZH, &c. are equall. Therefore o-
ther ten triangles are made, equilaterall and equall
both to one another, and to the ten former; and ſo
an Icoſaedron is made.

Moreover, divide equally EQ in α, draw the right
r 15.def.1. lines αF, αX, αV; and becauſe $QX \overset{r}{=} QV$, and αQ
the common ſide, and FQX, EQV are right angles,
ſ 4.1. ſ therefore ſhall αX be = αV; and by the ſame rea-
ſon all the lines αX, αR, αS, αT, αV, αF, αG, αH, αI, αK
are

are equall. But becauſe $ZQ.QE$[t] $:: QE.ZE$. therefore $Z\alpha q$[u] $= 5\,E\alpha q$[x] $= EQq\,(EEq) + E\alpha q$[y] $= \alpha Fq$. therefore $Z\alpha = \alpha F$.[z] in like manner $\alpha F = Y\alpha$. therefore the ſphere, whoſe center is α, and αF the ray, ſhall paſſe by the 12 angular points of the Icoſaedron.

t 9. 13.
u 3. 13.
x 4. 2.
y 47. 1.
z 15. 5.

Laſtly, becauſe $Z\alpha.\alpha E :: ZY.QE$;[a] and ſo $Z\alpha q.\alpha Eq :: ZYq.QEq$.[b] therefore $ZYq = 5\,QEq$, or $5\,BDq =$: but $ABq.BDq$[c] $:: AB.BC :: 5.1$.[d] therefore $ZY = AB$. *W.W. to be Done.*

Therefore if AB be put ῥ,[e] then $EF = \sqrt{ABq}$ ſhall be alſo ῥ. and conſequently FG the ſide of the pentagone, and likewiſe of the Icoſaedron,[f] is a minor line. *Which was to be Demonſtrated.*

a 22. 6.
b 14. 5.
c cor. 8. 6.
d 1. cor. 1.
e ſch. 12. 10.
f 11. 13.

Coroll.

1. From hence is inferred, that the diameter of the ſphere is in power quintuple of the ſemidiameter of the circle encompaſſing the five ſides of the Icoſaedron.

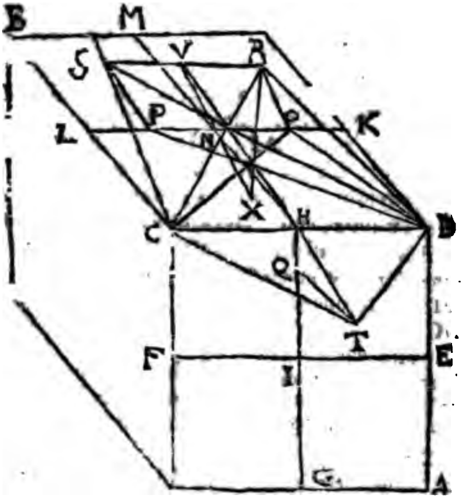
2. Alſo it is manifeſt that the diameter of the ſphere is compoſed of the ſide of a hexagone, that is, of the ſemidiameter, and two ſides of the decagone of a circle encompaſſing the five ſides of the Icoſaedron.

3. It appears likewiſe that the oppoſite ſides of an Icoſaedron, ſuch as RX, HI, are parallels. For RX[a] is parall. to LP,[b] parall. to HI.

a 33. 1.
b ſch. 16. 3.

PROP.

PROP. XVII.



To deſcribe a Dodecaedron, and comprehend it in the ſphere wherein the former figures were comprehended: and to demonſtrate that the ſide RS *of the Dodecaedron is an irrationall line of that ſort which is called an apotome or reſiduall line.*

Let AB be a cube inſcribed in the given ſphere,
and let all the ſides thereof be divided equally in
the points E, H, F, G, K, L, &c. and join the right
a 30. 6. lines KL, MH, HG, EF. a make HI.IQ :: IQ.QH;
and take NO, NP, = IQ. then erect OR, PS, perpendicular to the plane DB, and QT to the plane
AC; and let OR, PS, QT, be equall to IQ, NO,
NP. whence DR, RS, SC, CT, DT, being connected,
DRSCT ſhall be a pentagone of the dodecaedron
required. For draw NV parallel to OR, and having
drawn NV out as farre as the center of the cube X,

join the right lines DS, DO, DP, DR, CP, HV, HI, RX. Becauſe DOq [a] $= LKq$ ([b] KNq) $+ KOq$ [c] $= 3\,ONq$ ($3\,ORq$) [d] thence $DRq = 4\,ORq$ [e] $= OPq$, or RSq. therefore $DR = RS$. By the ſame reaſon DR, RS, SC, CT, TP are equall. But becauſe OR [f] is = and [g] parallel to PS, therefore RS, OP, and [h] conſequently RS, DC ſhall be alſo parallels. [h] therefore theſe with them that conjoin them DK, CS, VH, are in one and the ſame plane. Moreover, becauſe HI. IQ [k] :: IQ. (TQ) QH [k] :: HN. NV, and both TQ, HN, and QH, NV [k] are perpendicular to the ſame plane, [l] and ſo likewiſe parallels, [m] THV ſhall be a right line. [n] therefore the Trapezium DRSC, and the triangle DTS are in one plane extended by the right lines DC, TV. [o] therefore DCTSR is a pentagone, and that alſo equilaterall, by what is ſhewn already. Furthermore, becauſe PK. KN :: KN. NP; and DSq [p] $= DPq + PSq$ (PNq) [p] $= DKq + PKq + NPq$, [q] thence $DSq = DKq + 3\,KNq = 4\,DKq$ ($4\,DHq$) [r] $= DCq$. therefore $DS = DC$: whence the triangles DRS, DCT, are equilaterall one to another. [s] therefore the angle $DRS = DTC$, and likewiſe the ang. $CSR = DCT$. therefore the pentagone DTCSR is alſo equiangular. Moreover, becauſe AX, DX, CX, &c. are ſemidiameters of the cube, [t] thence is $XN = IH$ or KN, [u] and ſo $XV = KP$; wherefore becauſe RVX, is a [x] right angle, [z] thence $RXq = XVq + RVq$ (NPq) $= KPq + NPq$ [a] $= 3\,KNq$ [b] $= AXq$ or DXq, &c. therefore RX, AX, DX, and by the ſame reaſon XS, XT, AX, are equall one to another. And if by the ſame method, whereby the pentagone DTCSR was made, twelve like pentagons, touching the twelve ſides of the cube, be made, they ſhall compoſe a Dodecaedron, and a ſphere ~~paſſing by their angular points~~, whoſe ray is AX or RX, ſhall comprehend that Dodecaedron. *W. W. to be Done.*

Laſtly, becauſe KN. NO [c] :: NO. OK, [d] thence

KL.

[a] 47. 1.
[b] 7. ax. 1.
[c] 4. 13.
[d] 47. 1.
[e] 4. 2.
[f] *conſtr.* 96. 11.
[g] 33. 1.
[h] 9. 1.
[k] 7. 11.
[l] 6. 11.
[m] 32. 6.
[n] 1. & 2. 11.
[o] 5. 13.
[p] 47. 1.
[q] 1. ax. 2. & 4. 13.
[r] 4. 2.
[s] 8. 1.
[t] 15. 13.
[u] 1. ax. 1.
[x] 29. 1.
[z] 47. 1.
[a] 4. 13.
[b] 15. 13.
[c] *conſtr.*
[d] 15. 5.

KL.OP :: OP.OK + PL. Therefore if AB the diameter of the ſphere be ſuppoſed ρ, then ſhall KL [e] $= \sqrt{\frac{AB}{3}}$ [f] be alſo ρ. [g] whence OP or RS the ſide of the dodecaedron ſhall be a reſiduall line. *W.W. to be Dem.*

e 15. 13.
f ſch. 12. 10
g 6. 13.

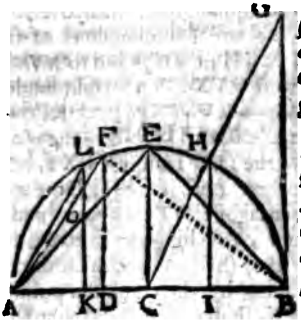
Coroll.

From this demonſtration it follows, 1. that if the ſide of a cube be cut in extreme and mean proportion, the greater ſegment ſhall be the ſide of the dodecaedron deſcribed in the ſame ſphere.

2. If the leſſer ſegment of a right line, cut in extreme and mean proportion, be the ſide of the dodecaedron, the greater ſegment ſhall be the ſide of the cube inſcribed in the ſame ſphere.

3. It is manifeſt alſo, that the ſide of the cube is equall to the right line which ſubtendeth the angle of a pentagone of the dodecaedron inſcribed in the ſame ſphere.

PROP. XVIII.



To find out the ſides of the precedent five figures, and compare them together.

Let AB be the diameter of the ſphere given, and AEB the ſemicircle, and let AC be [a] $= \frac{1}{2}$ AB, and AD [b] $= \frac{1}{3}$ AB. then erect the perpendiculars CE, DF, and BG = AB. join AF, AE, BE, BF, CG; and let fall the perpendicular HI from H; and

a 10. 1.
b 10. 6.

and CK being taken equall to CI, from K erect the perpendicular KL, and join AL. Laſtly, c make AF. AO :: AO.OF. c 30. 6.

Therefore 3.2 d :: AB.BD e :: ABq. BFq the ſide of a Tetraedron. and 2. 1 :: a AB. AC :: ABq. BBq f the ſide of an Octaedron. d *conſtr.* e *cor.* 8. 6. f 14. 13.

Alſo 3. 1 d :: AB. AD e :: ABq. AFq, g the ſide of an Hexaedron. g 15. 13. h *conſtr.* k *cor.* 17. 13.

Moreover, becauſe AF. AO h :: AO. OF. k thence ſhall AO be the ſide of a Dodecaedron. Laſtly, BG, (2 BC.) BC l :: HI. IC. m therefore HI = 2 CI n = KI. therefore HIq o = 4 CIq. conſequently CHq p = 5 CIq. q therefore ABq = 5 KIq. r therefore KI or HI is a ray of a circle encloſing the pentagone of an Icoſaedron; & AK or IB r is the ſide of a decagone inſcribed in the ſame circle. s whence AL ſhall be the ſide of a pentagone; t and alſo the ſide of an Icoſaedron. Whereby it appears that BF, BE, AE are ρ ⊐. and AL, AO ρ ⊐, and BF ⊏ BE, and BE ⊏ AF, and AF ⊏ AO. And becauſe 3 AFq = ABq s = 5 KLq, and AF x AO ⊏ AF x OF, u and ſo AF x AO + AF x OF ⊏ 2 AF x OF, y that is, AFq ⊏ z 2 AOq. a thence ſhall 3 AFq (5 KLq) be ⊏ 6 AOq. conſequently KL ⊏ AO; and much rather AL ⊏ AO. l 4 6 m 14. 5. n *conſtr.* o 4. 2. p 47. 1. q 15. 5. r *cor.* 16. 13. s 10. 13. t 16. 13. u 1. 6. x 4 *ax.* 1. y 1. 2. z 17. 6. a 47. 1.

That we may expreſſe theſe ſides in numbers; If AB be ſuppoſed √ 60, then, reducing what is already ſhewn to ſupputation, BF = √ 40, & BE = √ 30, & AF = √ 20. Alſo AL = √: 30 − √ 180 (for AK = √ 15 − √ 3 and KL (HI) = √ 12:) Laſtly AO = √: 30 − √ 500 (√ 25 − √ 5.)

Schol.

It is very apparent that beſides the five aforeſaid figures, there cannot be deſcribed any other regular ſolid figure (viz ſuch as may be contained under ordinate and equall plane figures.)

For three plane angles at leaſt are required to the conſtituting of a ſolid angle; [a] all which muſt be leſſe then four right angles. [b] But 6 angles of an equilaterall triangle, 4 of a ſquare, and ſix of a hexagon, do ſeverally equall 4 right angles; & 4 of a pentagon, 3 of a heptagon, 3 of an octagone, &c. do exceed 4 right angles: Therefore only of 3, 4, or 5 equilaterall triangles, of 3 ſquares, or 3 pentagones, it is poſſible to make a ſolid angle. Wherefore beſides the five above mentioned, there cannot be any other regular bodies.

a 21. 11.
b See ſchol. 32. 1.

Out of P. Herigon.

The Proportions of the ſphere and the five regular figures inſcribed in the ſame.

Let the diameter of the ſphere be 2. then ſhall

The Peripherie or circumference of the greater circle, be $6\,\underline{|28318}$.

The ſuperficies of the greater circle, $3\,\underline{|14159}$.

The ſuperficies of the ſphere, $12\,\underline{|56637}$.

The ſolidity of the ſphere, $4\,\underline{|1879}$.

The ſide of the tetraedron, $4\,\underline{|62299}$.

The ſuperficies of the tetraedron, 4|6188.

The ſolidity of the tetraedron, 0|15132.

The ſide of the Hexaedron, 1|1547.

The ſuperficies of the hexaedron, 8.

The ſolidity of the hexaedron, 1|5398.

The ſide of the Octaedron, 1|41421.

The ſuperficies of the octaedron, 6|9282.

The ſolidity of the octaedron, 1|33333.

The ſide of the Dodecaedron, 0|71364.

The ſuperficies of the dodecaedron, 10|51462.

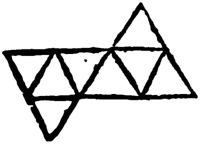
The ſolidity of the dodecaedron, 2|78516.

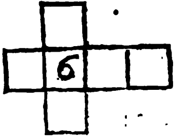
The ſide of the Icoſaedron, 1|05146.

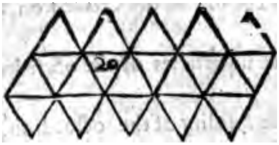
The ſuperficies of the Icoſaedron, 9|57454.

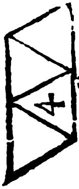
The ſolidity of the Icoſaedron, 2|53615.

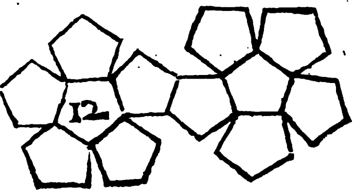
If five equilaterall and equiangular figures, like theſe in the ſchemes beneath, be made of paper, and rightly folded, they will repreſent the five regular bodies.







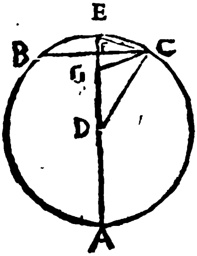




The End of the thirteenth Book.

THE FOURTEENTH BOOK OF EUCLIDE'S ELEMENTS.

PROP. I.

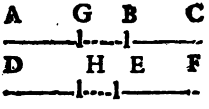


A Perpendicular line DF *drawn from* D *the center of a circle* ABC *to* BC *the ſide of a pentagone inſcribed in the ſaid circle, is the half of theſe two lines taken together,* viz. *of the ſide of the hexagone* DE, *and the ſide of the decagone* EC *inſcribed in the ſame circle* ABC.

Take FG = FE, and draw CG: [a] Then CE is = CG. therefore the ang. CGE [b] = CEG [b] = ECD. therefore the ang. ECG [c] = EDC [d] = $\frac{1}{4}$ ADC [e] = $\frac{1}{2}$ CED ($\frac{1}{2}$ ECD.) [f] conſequently the angle GCD = ECG = EDC. [g] wherefore DG = GC (CE.) therefore DF = CE (DG) + EF = $\frac{DE + CE}{2}$. *W.W. to be Dem.*

a 4. 1.
b 5. 1.
c 32. 1.
d *hyp.* and 33. 6.
e 20. 3.
f 7. *ax.*
g 6. 1.

PROP. II.



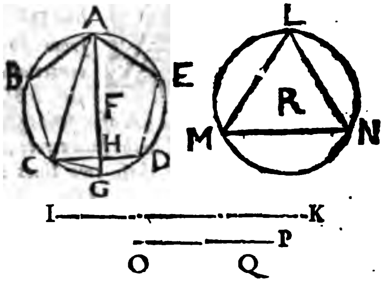
If two right lines AB, DE, *be cut according to extreme and mean proportion* (AB. AG :: AG. GB. *and* DE. DH :: DH. HE.) *they ſhall be cut after the ſame manner,* viz. *into the ſame proportions* (AG. GB :: DH. HE.)

Take

a 17.6.
b 8. 2.
c 2. ax. 1.
d 22. 5. and 22. 6.
e 22. 5.
f 17. 5.

Take BC = BG; and EF = EH. Then AB x BG is *a* = 5 AGq. wherefore ACq *b* = 4 ABG + AGq *c* = 5 AGq. In like manner ſhall DFq be = 5 DHq. *d* therefore AC. AG :: DF. DH. whence by addition AC + AG. AG :: DF + DH. DH. that is, 2 AB. AG :: 2 DE. DH. *e* conſequently AB. AG :: DE. DH; *f* whence by diviſion AG. GB :: DH. HE. *W. W. to be Dem.*

PROP. III.



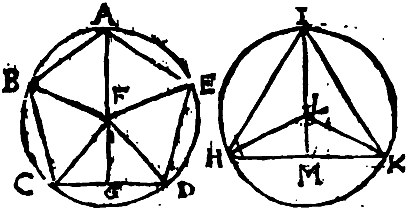
The ſame circle ABD *comprehends both* ABCDE *the pentagone of a Dodecaedron, and* LMN *the triangle of an Icoſaedron inſcribed in the ſame ſphere.*

a ſch. 47. 1.
b 30. 6.
c 47. 1.
d 4. 2.
e 10. 13.
f 1, & 3. ax.
g 8. 13.
h 2 13. & 16. 5.
k 22. 6. and 4 5.
l 15 13.
m conſtr.
n cor. 16. 13.
o 12. 13.
p 10. 13.
q 15. 5.
* before.
r 1 ax 1. and ſch. 48. 1.
ſ 1. def. 3.

Draw the diameter AG, and the right lines AC, CG. and let IK be the diameter of the ſphere, *a* and IKq = 5 OPq. *b* & make OP. OQ :: OQ. QP. Becauſe ACq + CGq *c* = AGq *d* = 4 FGq; & ABq *e* = FGq = CGq. *f* thence ACq + ABq = 5 FGq. moreover, becauſe CA. AB *g* :: AB. CA — AB; and OP. OQ :: OQ. QP. *h* and ſo CA. OP :: AB. OQ. *k* therefore 3 ACq. (*l* IKq.) 5 OPq (*m* IKq) :: 3 ABq. 5 OQq. therefore 3 ABq = 5 OQq. But becauſe ML *n* is the ſide of a pentagon inſcribed in the circle, whoſe ray is OP, thence 15 RMq *o* = 5 MLq *p* = 5 OPq + 5 OQq = * 3 ACq + 3 ABq *q* = 15 FGq. *r* therefore RM = FG. *ſ* and conſequently the circle ABD is = to the circle LMN. *W. W. to be Dem.*

PROP.

PROP. IV.



If from F the center of a circle encompaſsing the pentagone of a dodecaedron ABCDE, *a perpendicular line* FG *be drawn to one ſide of the pentagone* CD; *the rectangle contained under the ſaid ſide* CD *and the perpendicular* FG, *being thirty times taken, is equall to the ſuperficies of the Dodecaedron. Alſo,*

If from the center L *of a circle incloſing the triangle of an Icoſaedron* HIK, *a perpendicular line* LM *be drawn to one ſide of the triangle* HK, *the rectangle contained under the ſaid ſide* HK, *& the perpendicular* LM, *being thirty times taken, ſhall be equall to the ſuperficies of the Icoſaedron.*

Draw FA, FB, FC, FD, FE. [a] then ſhall the triangles CFD, DFE, EFA, AFB, BFC be equall. but CD x FG [b] = 2 triangles CFD. therefore 30 CD x GF [c] = 60 CFD [d] = 12 pentagones ABCDE [e] = to the ſuperficies of the dodecaedron. *W.W. to be Dem.*

Draw LI, LH, LK; then HK x LM [f] is = 2 triang. LHK. therefore 30 HK x LM [g] = 60 HLK = 20 HIK [h] = to the ſuperficies of the Icoſaedron. *W.W. to be Dem.*

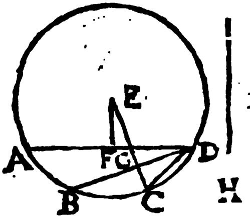
[a] 8. 1.
[b] 41. 1.
[c] 15. 5.
[d] 6 ax.
[e] 17. 3.
[f] 41. 1.
[g] 15. 5.
[h] 16. 13.

Coroll.

CD x FG. HK x LM [k] :: the ſuperficies of the dodecaedron to the ſuperf. of the Icoſaedron.

[k] 15. 5.

PROP. V.

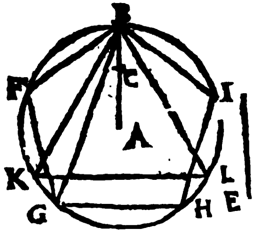


The ſuperficies of a Dodecaedron hath to the ſuperficies of an Icoſaedron inſcribed in the ſame ſphere, the ſame proportion that H *the ſide of a cube hath to* AD *the ſide of an Icoſaedron.*

Let the circle ABCD a 3. 14. [a] encloſe both the pentagone of a dodecaedron, and the triangle of an Icoſaedron; whoſe ſides are BD, AD. upon which from the center E let fall the perpendiculars EF, EGC; and draw CD.

b 9. 13. Becauſe EC + CD. EC [b] :: EC. CD. thence EG.
c 1. 14. d cor. 12. 13. ([c] $\frac{1}{2}$ EC + CD.) EF ([d] $\frac{1}{2}$ EC) [e] :: EF. EG − EF
e 15. 5. f cor. 17. 13. ($\frac{1}{2}$ CD.) but H. BD [f] :: BD. H − BD. [g] therefore
g 2. 14 h 1. 6. H. BD :: EG. EF. conſequently H x EF = BD x
k 7. 5. EG. wherefore ſince H. AD [g] :: H x EF. AD x EF.
l cor 4. 14. [k] thence ſhall be H. AD [h] :: BD x EG. AD x EF [l] ::
the ſuperficies of a dodecaedron to the ſuperficies of an Icoſaedron. *W.W. to be Dem.*

PROP. VI.



If a right line AB *be cut in extreme and mean proportion, then as the right line* BF, *containing in power that which is made of the whole line* AB, *and that which is made of the greater ſegment* AC, *is to the right line* E *containing in power that which is made of the whole line* AB, *and that which is made of the leſſer ſegment* BC; *ſo is the ſide of the cube* BG *to the ſide of an Icoſaedron* BK *inſcribed in the ſame ſphere with the cube.*

In the circle, whoſe ſemidiameter is AB, inſcribe BFGHI the pentagone of a dodecaedron, and BKL the triangle of an Icoſaedron. *a* wherefore BG ſhall be the ſide of a cube inſcribed in the ſame ſphere. therefore BKq *b* = 3 ABq; and Eq *c* = 3 ACq. therefore BKq. Eq *d* :: ABq. ACq *e* :: BGq. BFq. wherefore by inverſion BGq. I Kq :: BFq. Eq. *f* whence BG.BK :: BF. E. *W.W. to be Dem.*

a cor. 17. 13.
b 12. 13.
c 4. 13.
d 15. 6.
e 2. 14.
f 22. 6.

PROP. VII.

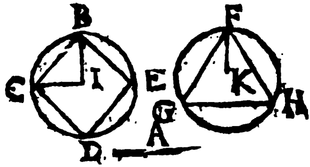
A Dodecaedron is to an Icoſaedron, as the ſide of a Cube is to the ſide of an Icoſaedron, inſcribed in one and the ſame ſphere.

Becauſe *a* the ſame circle comprehends both the pentagone of a dodecaedron, and the triangle of an Icoſaedron, *b* the perpendiculars drawn from the center of the ſphere to the planes of the pentagone and triangle, ſhall be equall one to another. Therefore if the dodecaedron and Iſocaedron be conceived divided into pyramids, right lines being drawn from the center of the ſphere to all the angles, the altitudes of all the pyramids ſhall be equall one to

a 3. 14.
b 47. 1.

c 5, and 6. 12. to the other. Wherefore ſince the pyramids c of equall height are one to another as their baſes, & the ſuperficies of the dodecaedron is equall to twelve pentagones, and the ſuperficies of the Icocaedron to twenty triangles, the dodecaedron ſhall be to the Icoſaedron, as the ſuperficies of the dodecaedron is to the ſuperficies of the Icoſaedron, d that is, as d 5. 14. the ſide of the cube is to the ſide of the Icoſaedron.

PROP. VIII.



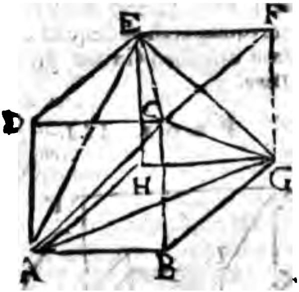
The ſame circle BCDE *comprehends both the ſquare of the cube* BCDE; *and the triangle of the octaedron* FGH *inſcribed in one and the ſame ſphere.*

a 15. 13. b 47. 1. c 14. 13. d 12. 13. e 2. def. 3. Let A be the diameter of the ſphere. Becauſe Aq a = 3 BCq b = 6 BIq; and alſo Aq c = 2 GFq d = 6 KFq; thence ſhall BI be = KF. e therefore the circle CBED = GFH. *W.W. to be Demonſtrated.*

The End of the fourteenth Book.

THE FIFTEENTH BOOK OF EUCLIDE'S ELEMENTS.

PROPOSITION I.



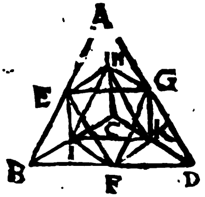
IN *a cube given* ABGHDCFE *to deſcribe a pyramide* AGEC.

From the angle C draw the diameters CA, CG, CE; and connect them with the diameters AG, GE, EA. All which are [a] equall among themſelves, as being the diameters of equall ſquares: therefore the triangles CAG, CGE, CEA, EAG are equilaterall and equall; and conſequently AGEC is a pyramide, which inſiſts upon the angles of the cube, and therefore [b] is inſcribed in the ſame. *W.W. to be Done.*

[a] 47. 1.

[b] 31. def. 11.

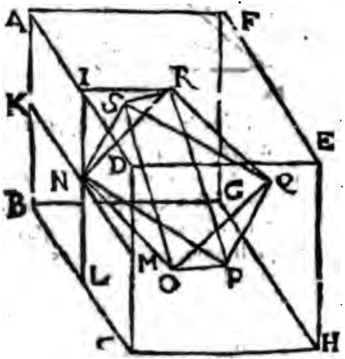
PROP. II.



In a pyramide given ABDC *to deſcribe an octaedron* EGKIFH.

a 10. 1. a Biſect the ſides of the pyramide in the points, E, I, F, K, G, H, which join with the right lines EF, FG, GE,
b 4. 11 &c. All theſe are b equall one to the other; conſequently the 8 triangles EHI, IHK, &c. are equilaterall and equall, and ſo make
c 27. def. 11. c an octaedron deſcribed d in the given pyramide.
d 31. def. 11. *W.W. to be Done.*

PROP. III.

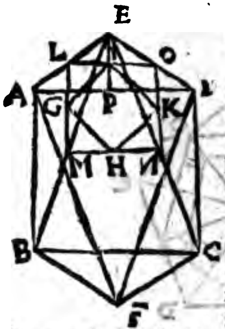


In a cube given CHGBDEFA *to deſcribe an octaedron* NPQSOR.

* 8. 4. Connect * the centers of the ſquares N, P, Q, S, O, R with the twelve right lines NP, PQ, QS, &c.
a 4. 1. which are a equall among themſelves; and ſo make
b 31. and 27. def. 11. 8 equilaterall and equall triangles: wherefore b the Octaedron NPQSOR b is inſcribed in the cube. *W.W. to be Done.*

PROP.

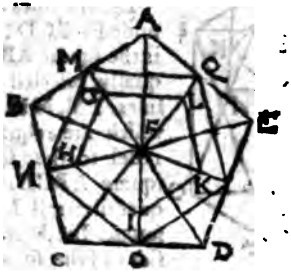
PROP. IV.



In an Octaedron given ABCDEF, *to inſcribe a cube.*

Let the ſides of the pyramide EABCD, whoſe baſe
is the ſquare ABCD, be equally
biſected by the right
lines, LM, MN, NO, OL,
which are *a* equall and *b* parallel a 4.1.
to the ſides of the b 2.6.
ſquare ABCD. *c* then the c 29.def.1.
quadrilaterall LMNO is a
ſquare. In like manner, if the
ſides of the ſquare LMNO
be equally biſected in the points G, H, K, I, & GH,
HK, KI, IG connected, GHKI ſhall be a ſquare.
And if in the other 5 pyramides of the octaedron,
the centers of the triangles be in the ſame ſort conjoined
with right lines, then other ſquares will be
deſcribed like and equall to the ſquare GHKI.
wherefore ſix ſuch ſquares ſhall make a cube, which
ſhall be deſcribed within an octaedron, *d* being its d 31.def.1
eight angles touch the eight baſes of the octaedron
in their centers. *W.W. to be Done.*

PROP. V.



In an Icoſaedron given to inſcribe a Dodecaedron.

Let ABCDEF be a pyramide of the Icoſaedron, whoſe baſe is the pentagone ABCDE; and the centers of the triangles G, H, I, K, L; which connect with the right lines GH, HI, IK, KL, LG. Then GHIKL ſhall be a pentagone of the dodecaedron to be inſcribed. * 5.4.

For the right lines FM, FN, FO, FP, FQ, paſſing by the centers of the triangles , *a* do equally divide their baſes into two parts. *b* therefore the right lines MN, NO, OP, PQ, QM *c* are equall one to the other; *d* whence alſo the angles MFN, NFO, OFP, PFQ, QFM are equall. therefore the pentagone GHIKL is equiangular. *e* and conſequently equilaterall, being FG, FH, FI, FK, FL *f* are equall. And if in the other eleven pyramids of the Icoſaedron, the centers of the triangles be in like ſort conjoined with right lines , then will pentagones equall and like to the pentagone GHIKL be deſcribed. Wherefore 12 of ſuch pentagones ſhall conſtitute a dodecaedron; which alſo ſhall be deſcribed

a cor.3.3.
b 4.1.
c 4.1.
d 8.1.
e 4.1.
f 12.13.

in the Icoſaedron, ſeeing the twenty angles of the dodecaedron conſiſt upon the centers of the twenty baſes of the Icoſaedron. Whereby it appears that we have deſcribed a dodecaedron in an Iſocaedron given. *Which was to be Done.*

FINIS.

[illegible]